VESF
Virgo EGO Scientific Forum

BH/NS-NS mergers

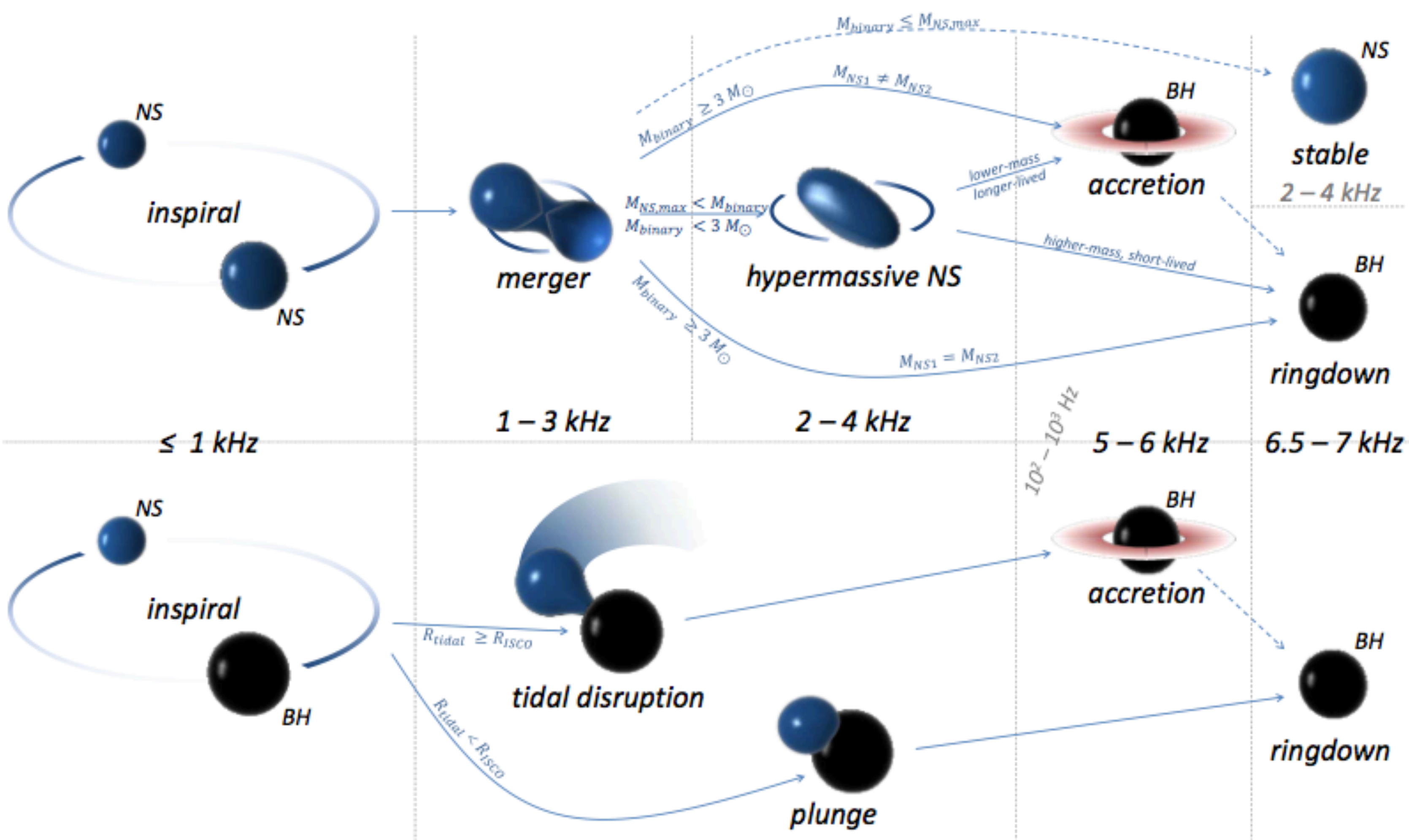
Andrea Maselli

Einstein Telescope Meeting 2013

Hannover, Max Planck Institute

22 October, 2013

Coalescing binaries



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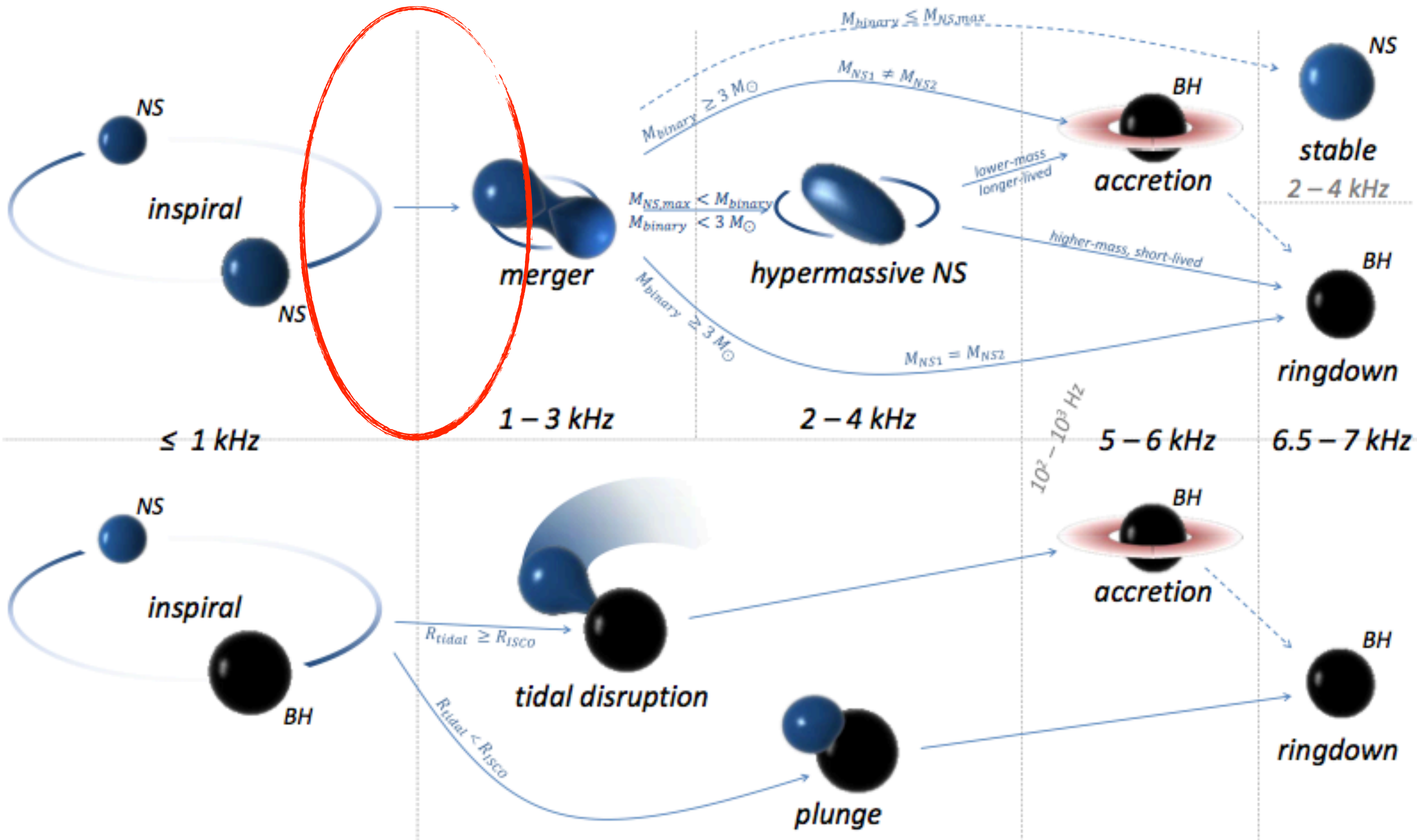

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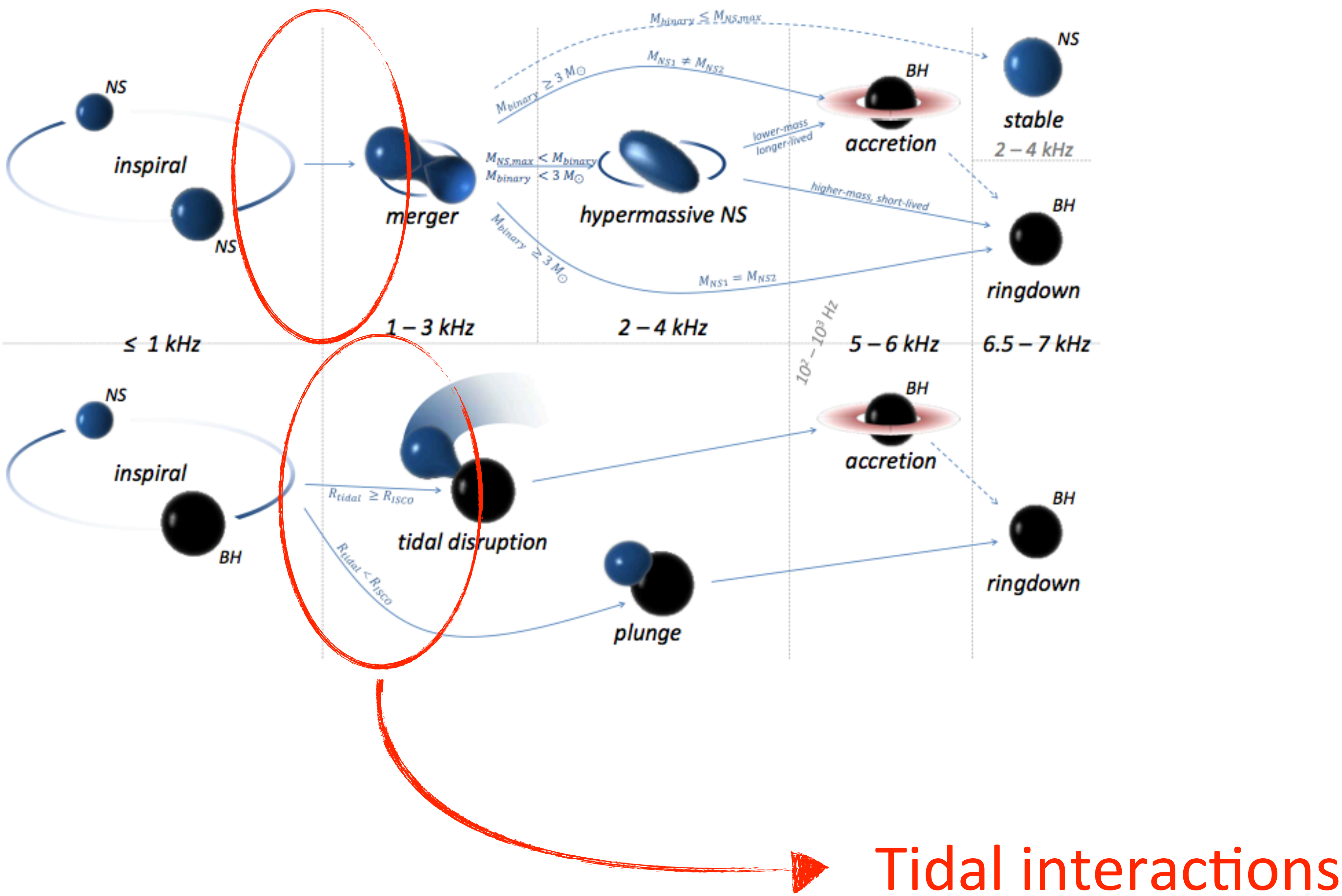

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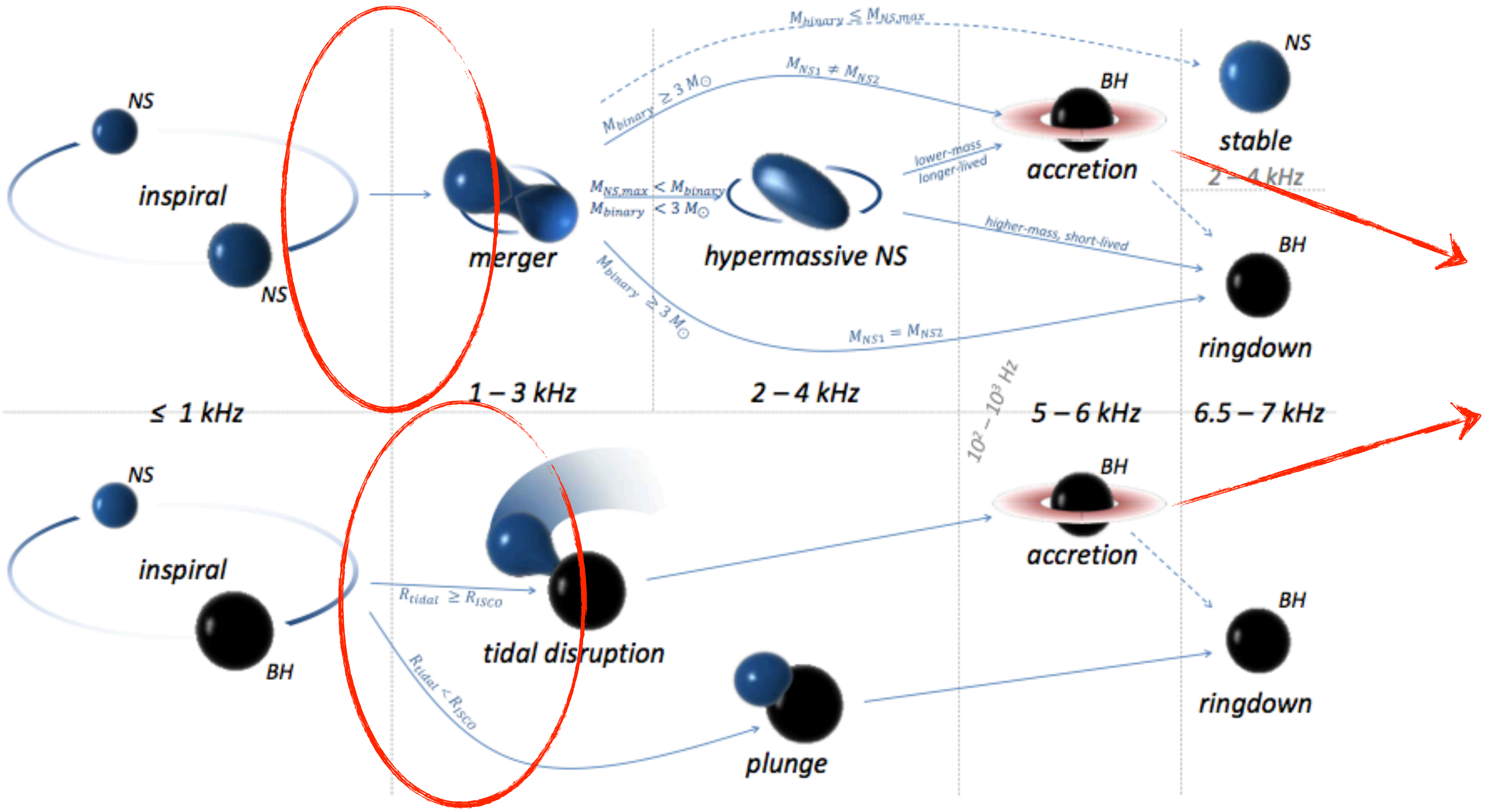
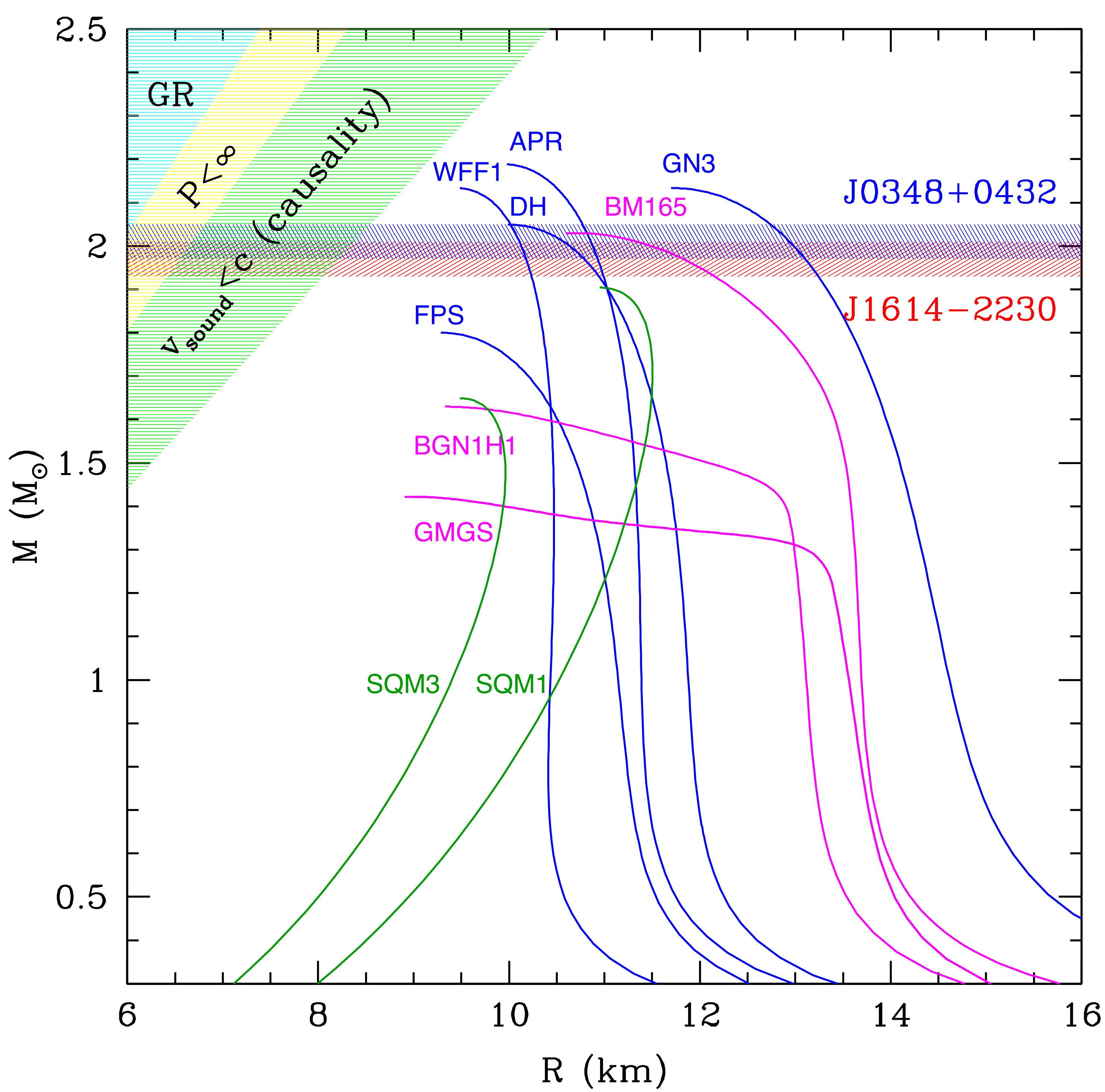


Engine of SGRBs

Tidal interactions

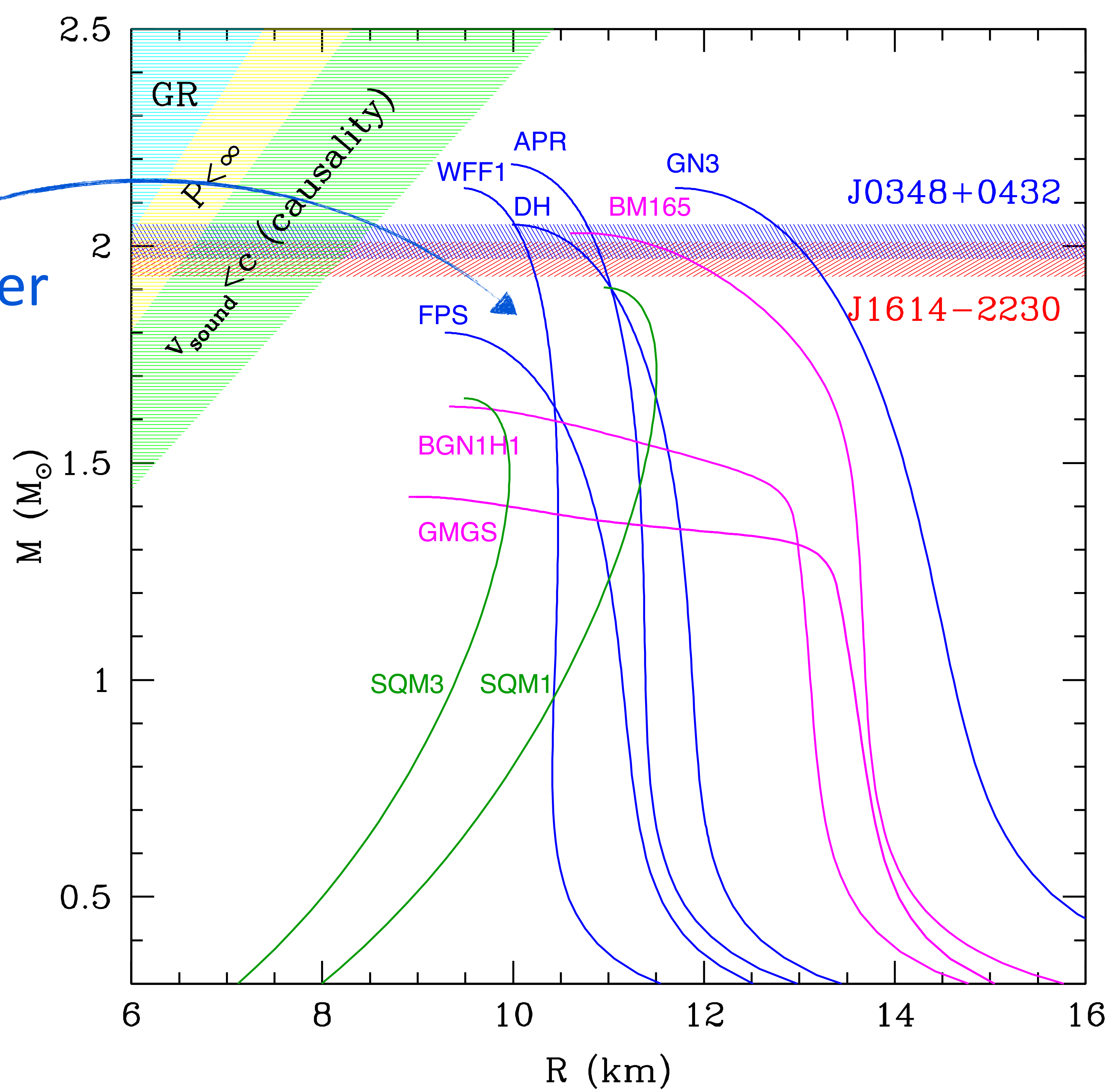
The NS structure



(credits M. Fortin)

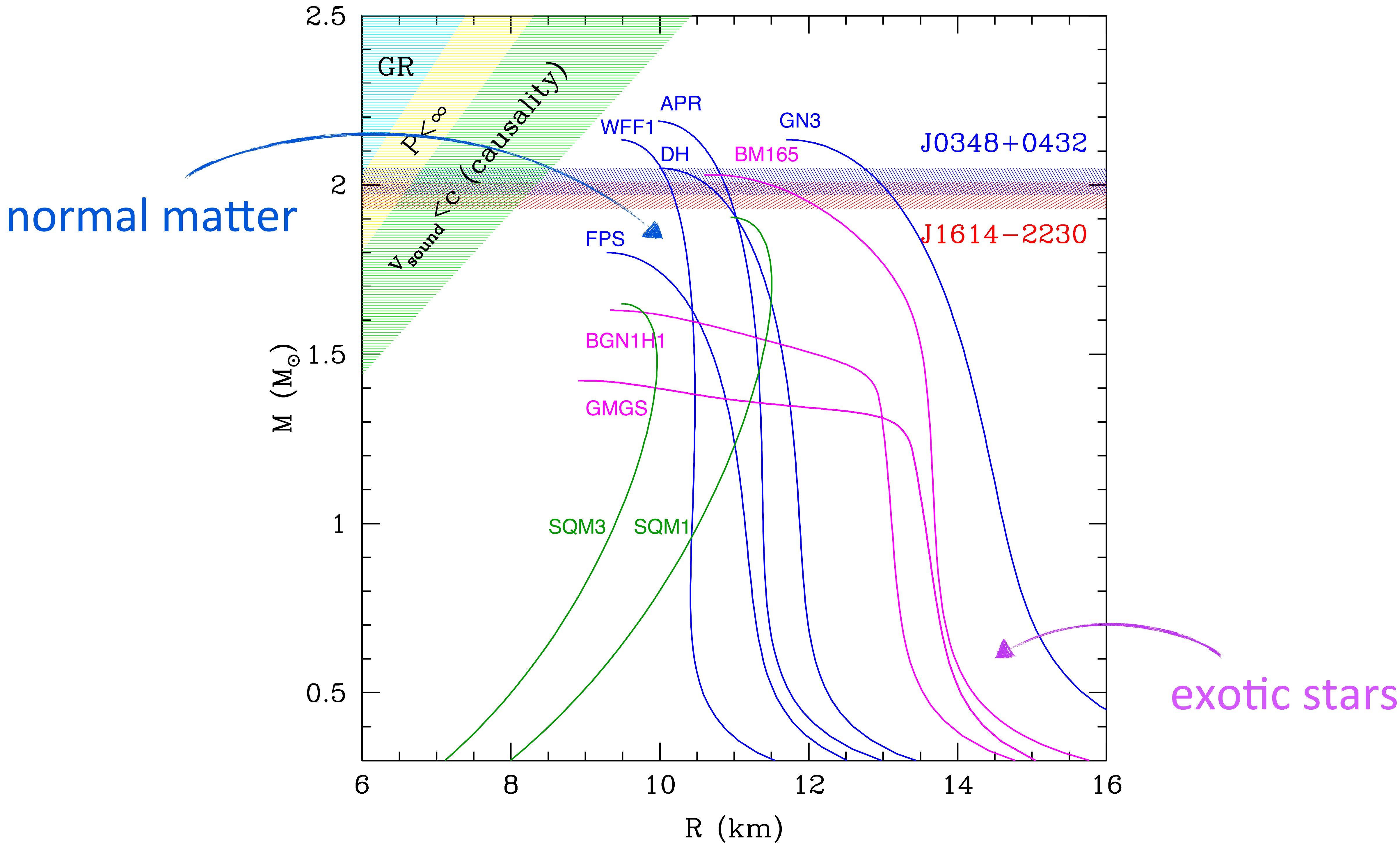
The NS structure

normal matter



(credits M. Fortin)

The NS structure



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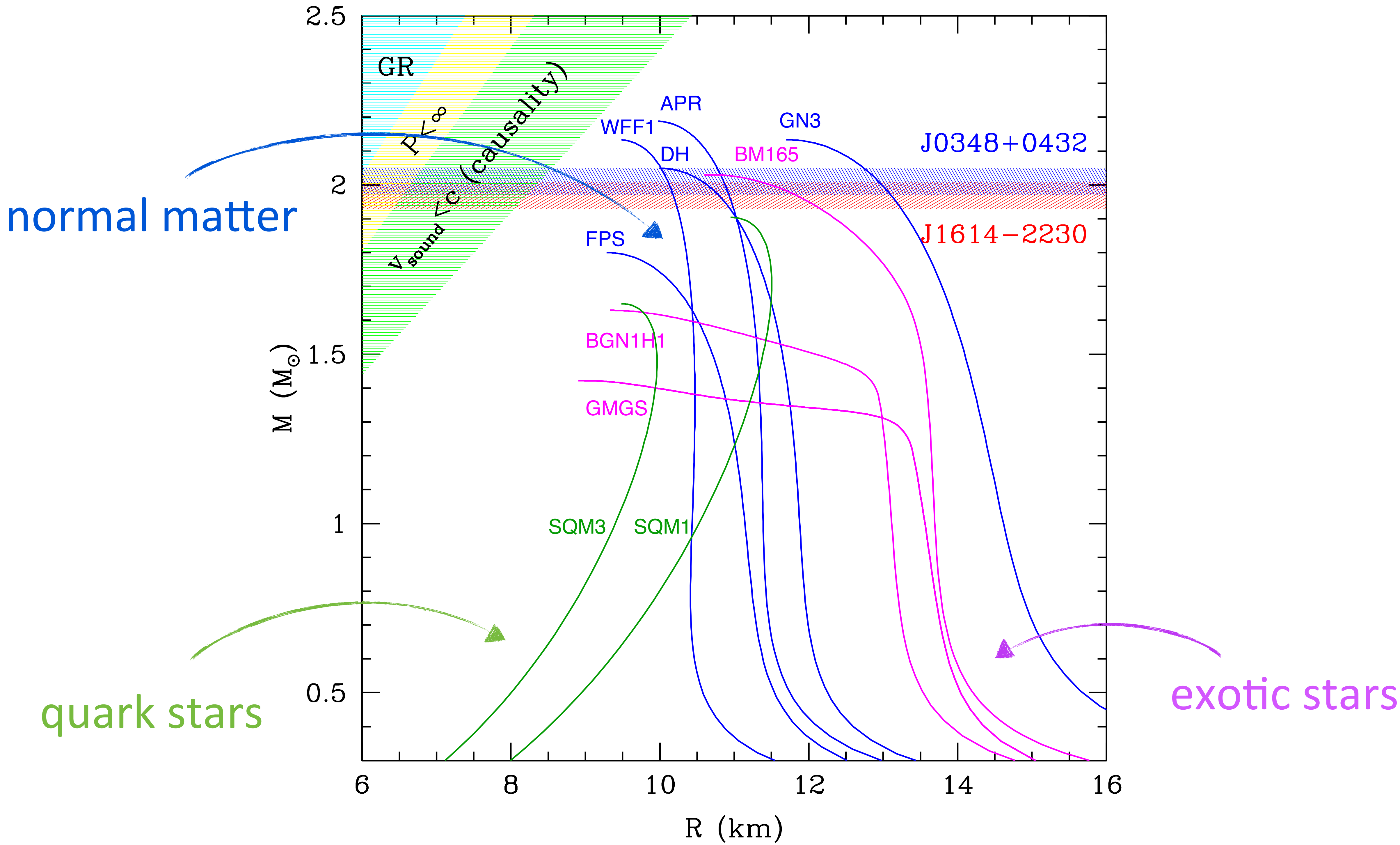

(credits M. Fortin)

Ingredient 1: Love numbers

Tidal deformations are included in the PN framework through the Love numbers theory, based on adiabatic approximation

T.Damour, A.Nagar, PRD 81, 084016 (2010)
J.Vines, E.Flanagan, T.Hinderer, PRD 83, 084051 (2011)

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$$Q_{ij} = \lambda_2 C_{ij}$$

λ_2 is the tidal deformability, related to the dimensionless Love number k_2

$$\lambda_2 = \frac{2}{3} k_2 R_{\rm NS}^5$$

- ❑ Computed by GR perturbation theory
- ❑ Depends only on the NS compactness $\mathcal{C} = M_{\rm NS}/R_{\rm NS}$

EoS

The gravitational signal

Tidal effects add linearly into the GW phase $\quad h^{lm} \simeq A^{lm} e^{i\psi^{lm}(f)}$

$$\psi = \psi_{\rm PP} + \psi_{\rm T}$$

- The point-particle term depends on masses and spins $\quad x = \left(\frac{Gm\omega}{c^3}\right)^{2/3}$

$$\psi_{\rm PP} \propto x^{-5/2} \left\{ 1 - \left(\frac{743}{336} + \frac{11}{4}\nu \right) x - \left[16\pi - \frac{113}{3}\xi + \frac{38}{3}\nu(\xi_1 + \xi_2) \right] x^{3/2} + \ldots + \mathcal{O}(x^{7/2}) \right.$$

Newt 1 PN 1.5 PN

- The *tidal* phase includes quadrupolar deformation through the Love number

$$\psi_{\rm T} \propto x^{-5/2} \bar{\lambda}_2 \left[\left(24 \frac{m}{m_1} + 264 \frac{m_2}{m_1} \right) x^5 + \mathcal{O}(x^6) + 1 \leftrightarrow 2 \right.$$

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$\mathcal{O}(10^5)$ 5 PN $= 1/c^{10}$

Ingredient 2: $I \ \lambda\text{ove} \ Q$ (forever)

The *exterior* NS properties depend on the *interior* structure, i.e. on the equation of state

- ☐ How fast the star can spin ⟶ I
- ☐ How much the star can be deformed ⟶ $Q \ , \ \lambda$

E-M and GW observations are not enough accurate to select between different EOS

- ☐ Degeneracies in the extraction of NS parameters

  - ☑ Spins and quadrupole moment in the GW phase

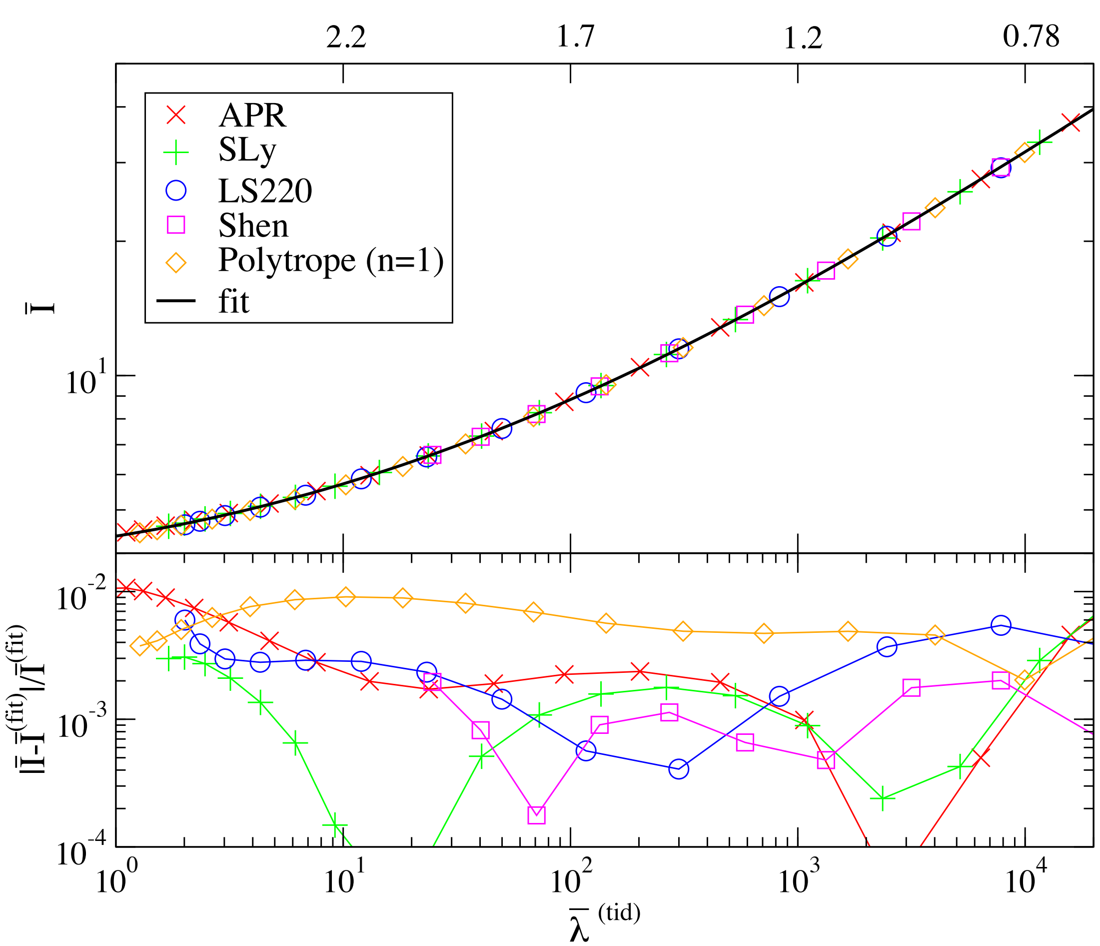
  - ☑ Correlations between EOS and modified gravity corrections, prevent robust GR test which are *internal* structure independent

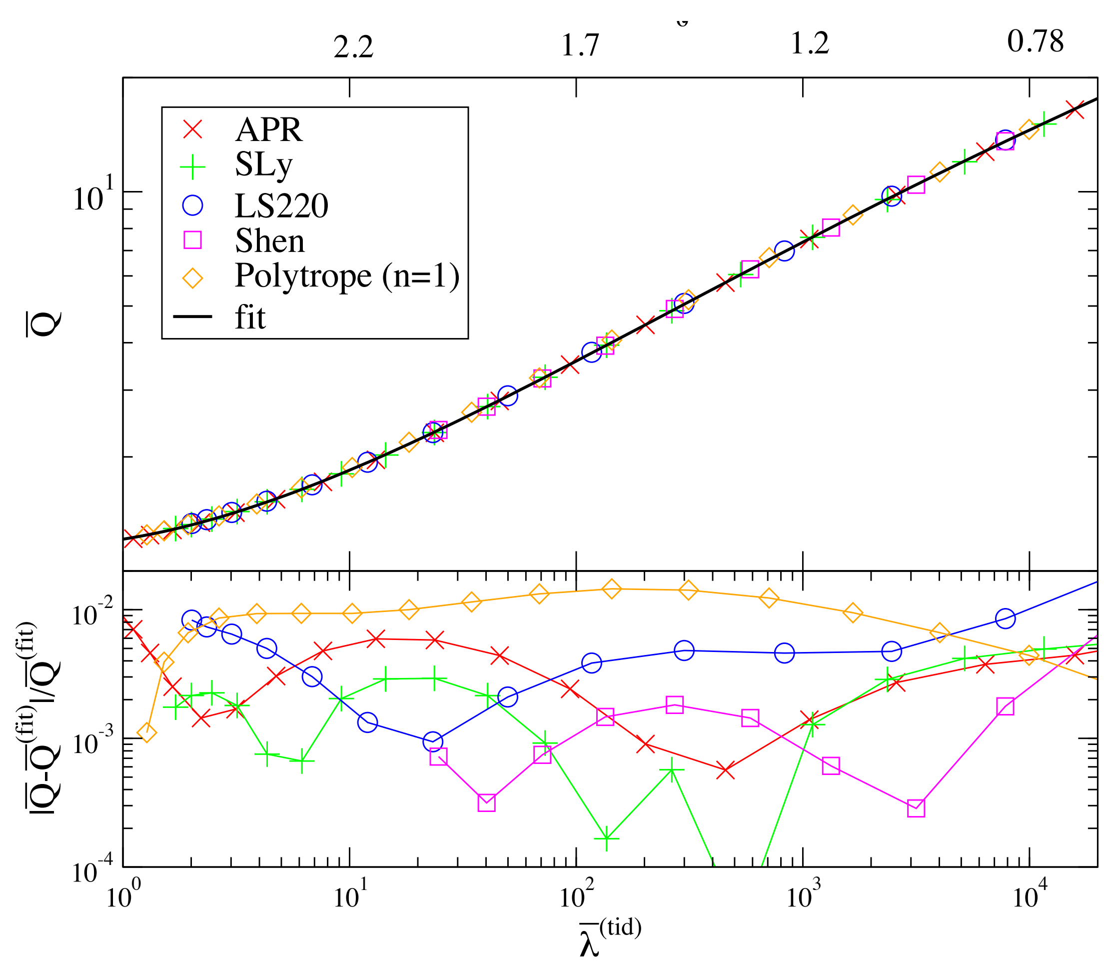
I λove Q (forever)

Degeneracies can be broken by the I-Love-Q *universal* relations

K.Yagi, N.Yunes, Science 341, 6144 (2013)

Analytic relations between I, Q and λ, insensitive to the star EOS

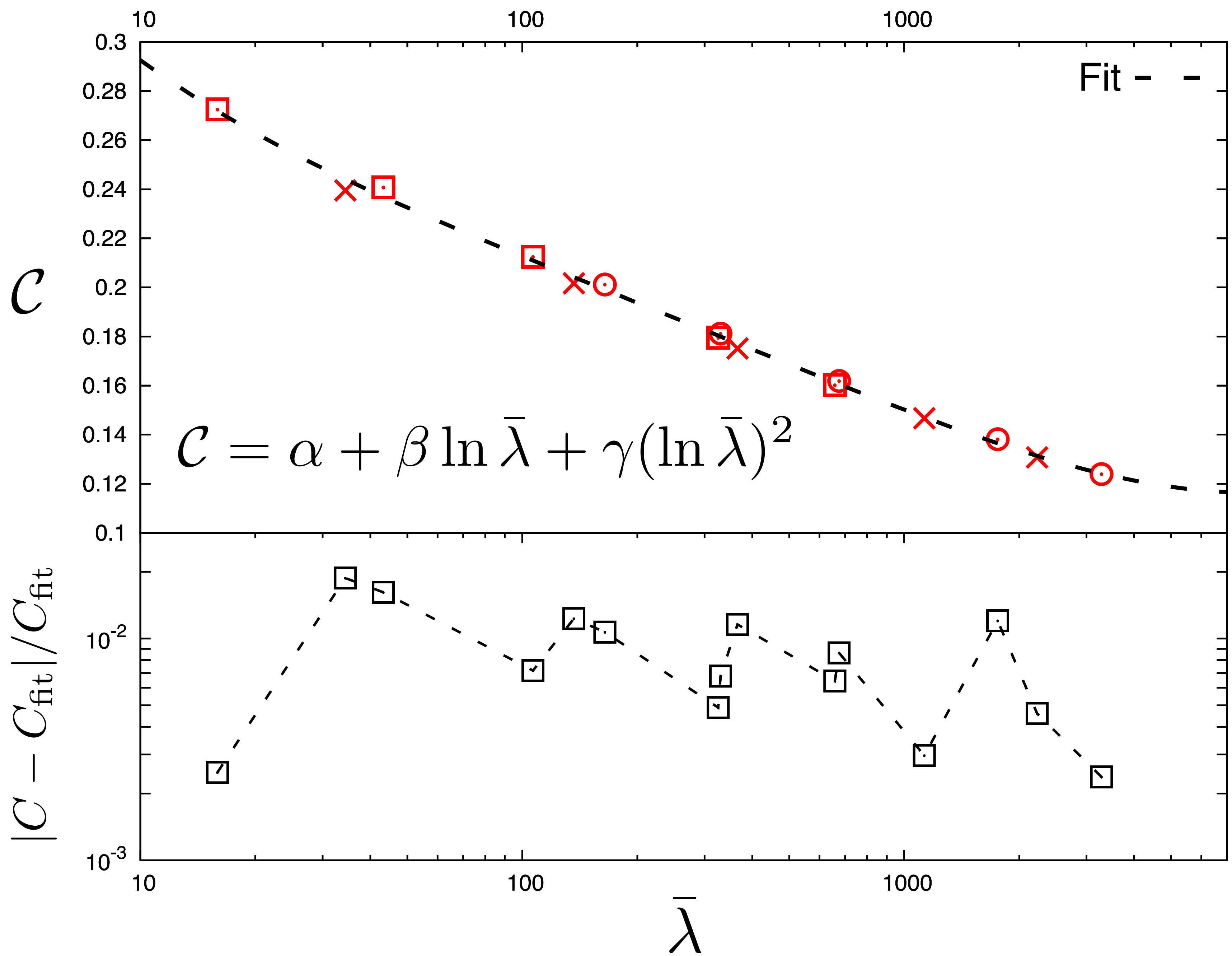




$\mathcal{C} - \lambda ove$

We find a new relation between $\bar{\lambda}$ and the NS compactness

***A.M.** et al, PRD 88, 023007 (2013)*

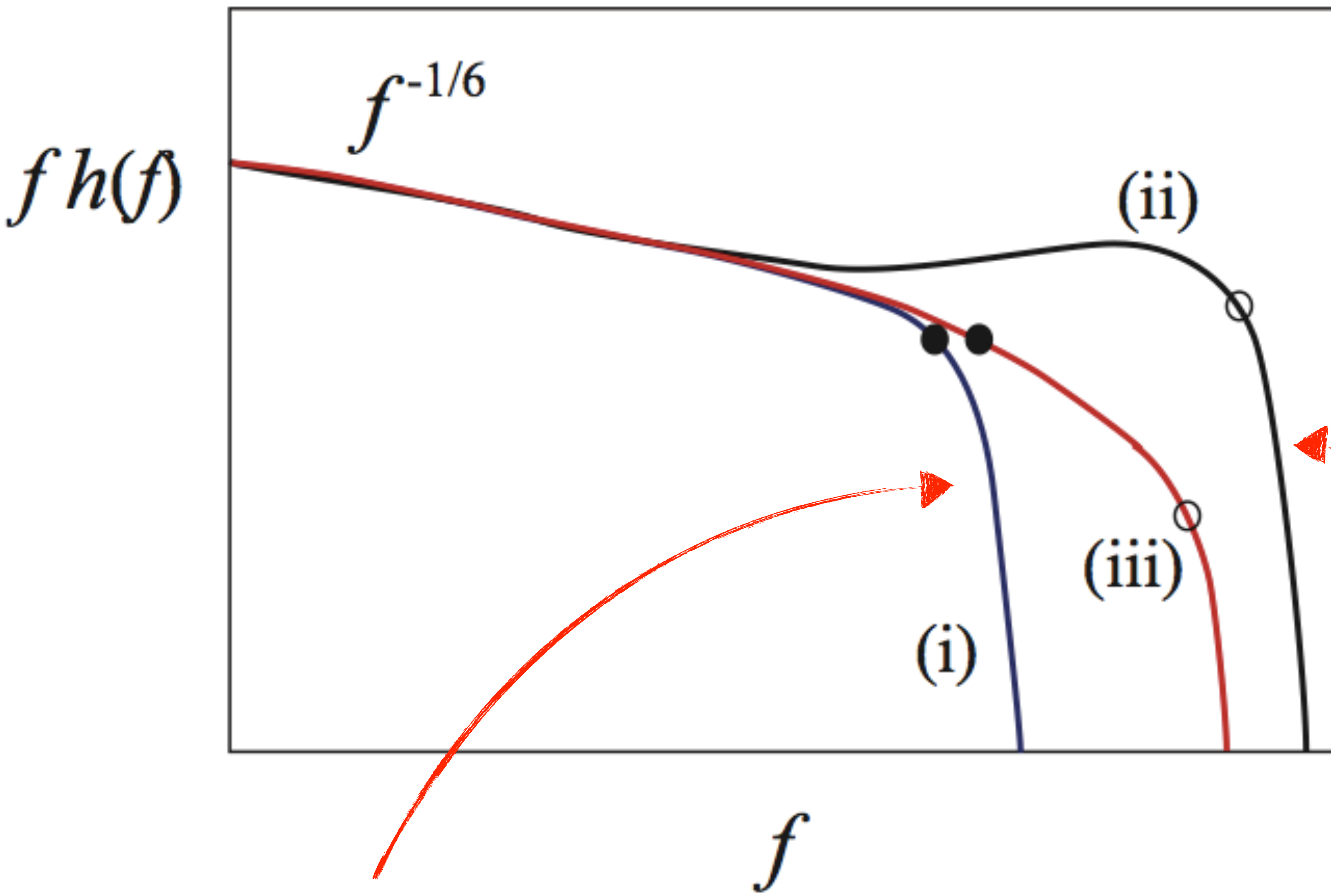


accurate more than 2%

Ingredient 3: tidal cut-off

Tidal disruption imprints on gravitational wave signals

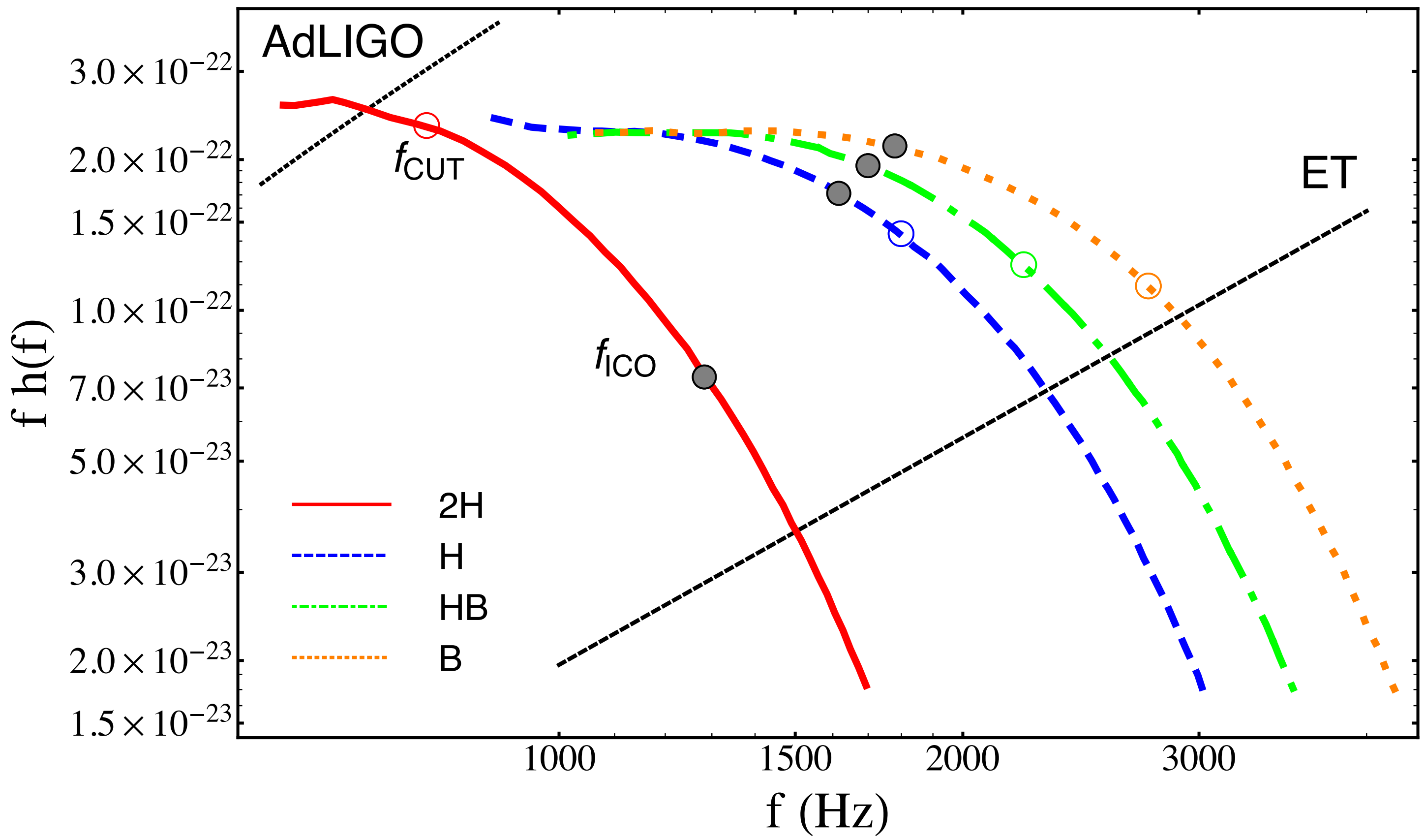
Kyutoku et al., PRD 84, 064018 (2011)



Tidal disruption at $f_{\rm cut}$
+ disk mass

NS is swallowed
by the BH

Ingredient 3: tidal cut-off



Kyutoku et al., PRD 82, 044049 (2010)

$$\ln(m f_{\rm cut}) = (3.87 \pm 0.12) \ln \mathcal{C} + (4.03 \pm 0.22)$$

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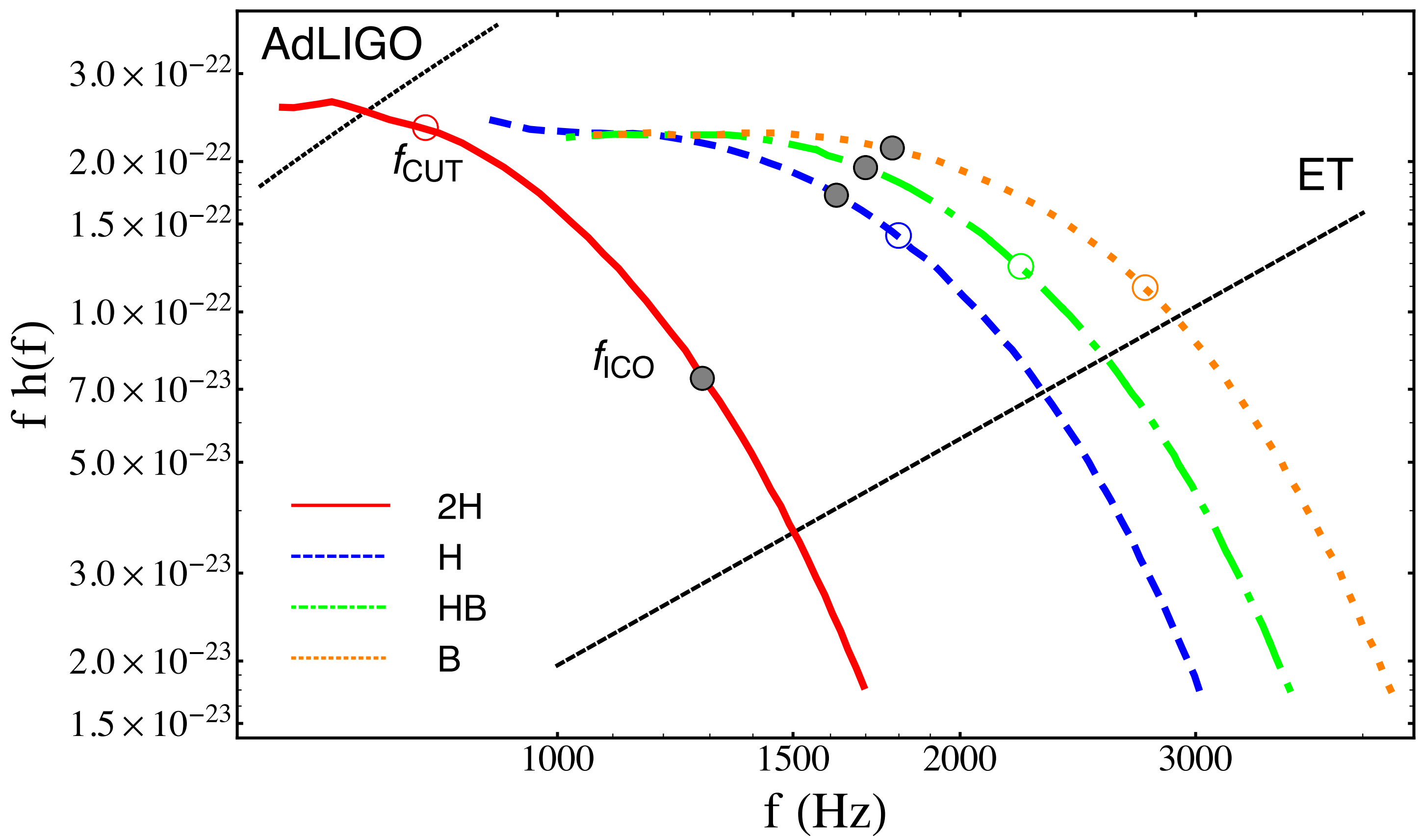

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ET

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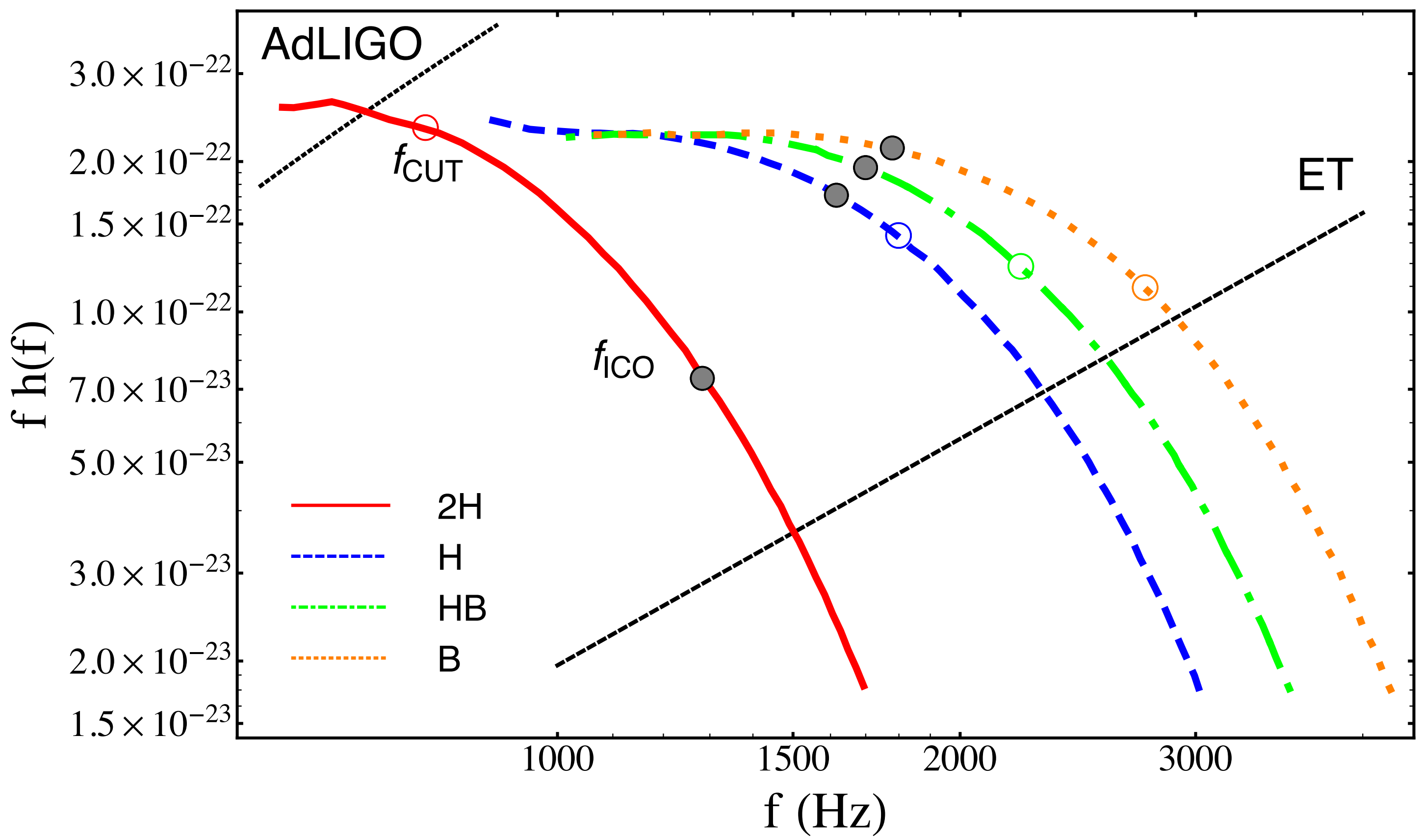

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ET

- We modify the PN template to reproduce the tidal disruption

$$\bar{h}_{\rm PN}(f) = \begin{cases} h_{\rm 3PN} & f < f_{\rm cut} \\ h_{\rm 3PN} \times \Theta(f, f_{\rm cut}) & f_{\rm cut} \le f \le f_{\rm cut} + \delta f \\ 0 & f > f_{\rm cut} + \delta f \end{cases} \qquad \Theta(f, f_{\rm cut}) = e^{-\alpha(f/f_{\rm cut} - 1)}$$

The strategies

- We assume the set of unknown parameters $\theta = (t_c, \phi_c, \ln \mathcal{M}, \ln \nu, \lambda, f_{\rm cut})$
  - We use the fit $f_{\rm cut}(\mathcal{C})$ to compute the error $\sigma_{\mathcal{C}_{\rm cut}}$
  - We employ the universal relation $\mathcal{C}(\lambda)$ and estimate $\sigma_{\mathcal{C}_\lambda}$
  - We combine the two information to get a weighted mean of $\mathcal{C}$ and $\sigma_{\mathcal{C}}$
- We express all the information on the EOS in one parameter, λ
  - We use the two semi-analytical fits to compute a 5 x 5 Fisher matrix for the parameters $\theta = (t_c, \phi_c, \ln \mathcal{M}, \ln \nu, \lambda)$

__A.Maselli.__ L.Gualtieri and V.Ferrari, ArXiv: 1310.5381 (2013)

Results: strategy I

model	$\sigma_{\ln \tilde{\lambda}}$ (%)	$\sigma_{\ln f_{\rm cut}}$ (%)
2H_100_120	1.3	3.6
2H_500_120	6.0	14
2H_1000_120	10	22
2H_2000_120	15	27
2H_100_135	1.5	6.6
2H_500_135	6.7	26
2H_1000_135	11	40
2H_2000_135	17	49

model	$\sigma_{\ln \mathcal{C}_{\rm cut}}$ (%)	$\sigma_{\ln \mathcal{C}_\lambda}$ (%)	$\sigma_{\ln \mathcal{C}}$ (%)
2H_100_120	8.8	3.0	2.9
2H_500_120	9.5	3.2	3.0
2H_1000_120	10	3.5	3.3
2H_2000_120	11	4.1	3.9
2H_100_135	8.7	3.0	2.8
2H_500_120	11	3.2	3.1
2H_1000_120	13	3.6	3.5
2H_2000_120	15	4.1	4.0

- Relative error on the NS compactness of the order of 3-4%
- $\sigma_{\mathcal{C}_\lambda}$ has a mild dependence on the luminosity distance
- The reduction on $\sigma_{\mathcal{C}}$ due to the inclusion of the cut-off frequency is marginal

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The use of $f_{\rm cut}$ does not significantly improve the estimate of the stellar compactness

Results: strategy II

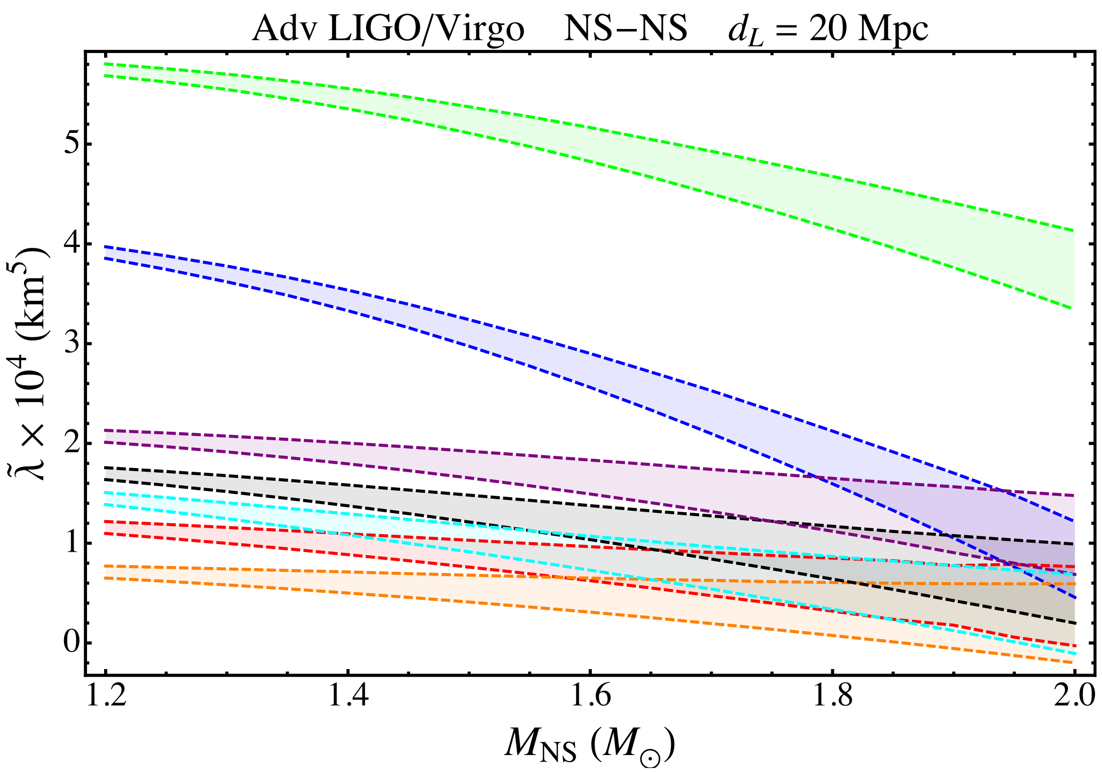
model	$\sigma_{\ln\tilde{\lambda}}$ (%)	$\sigma_{\ln\mathcal{C}}$ (%)
2H_100_120	1.3	3.0
2H_500_120	5.3	3.2
2H_1000_120	8.2	3.4
2H_2000_120	10	3.6
2H_100_135	1.5	3.0
2H_500_135	6.1	3.2
2H_1000_135	9.5	3.4
2H_2000_135	12	3.7

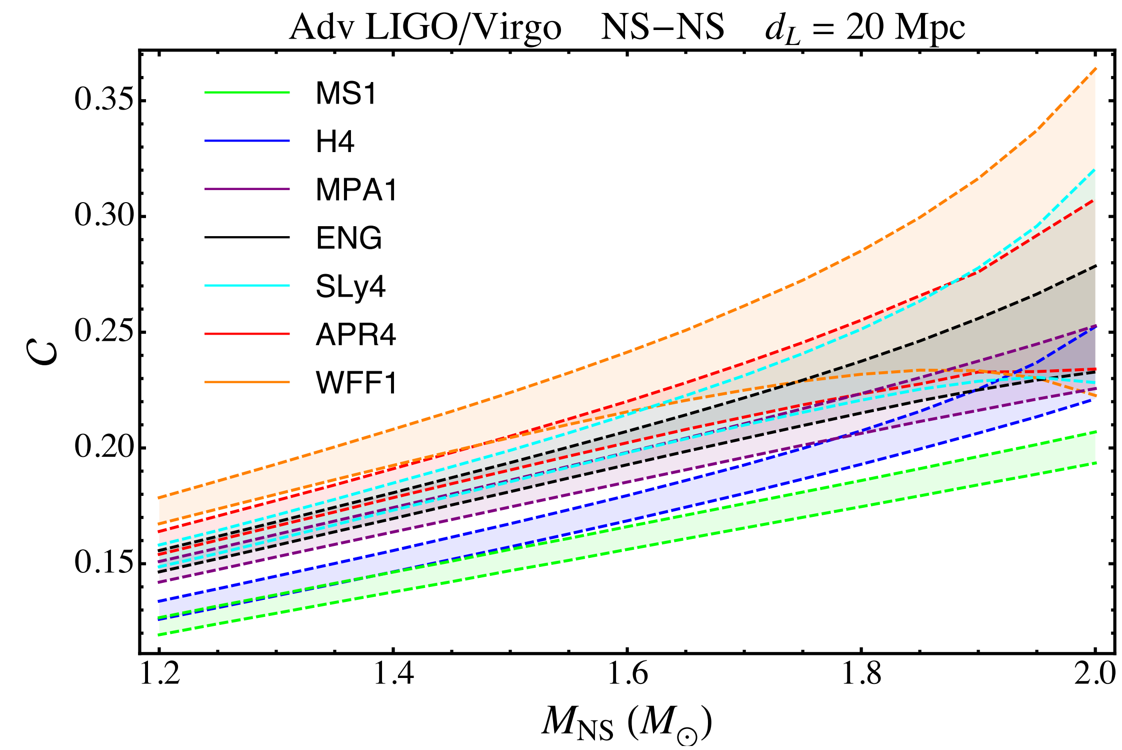
$\sigma_{\ln\tilde{\lambda}}$ (%)
1.3
6.0
10
15
1.5
6.7
11
17

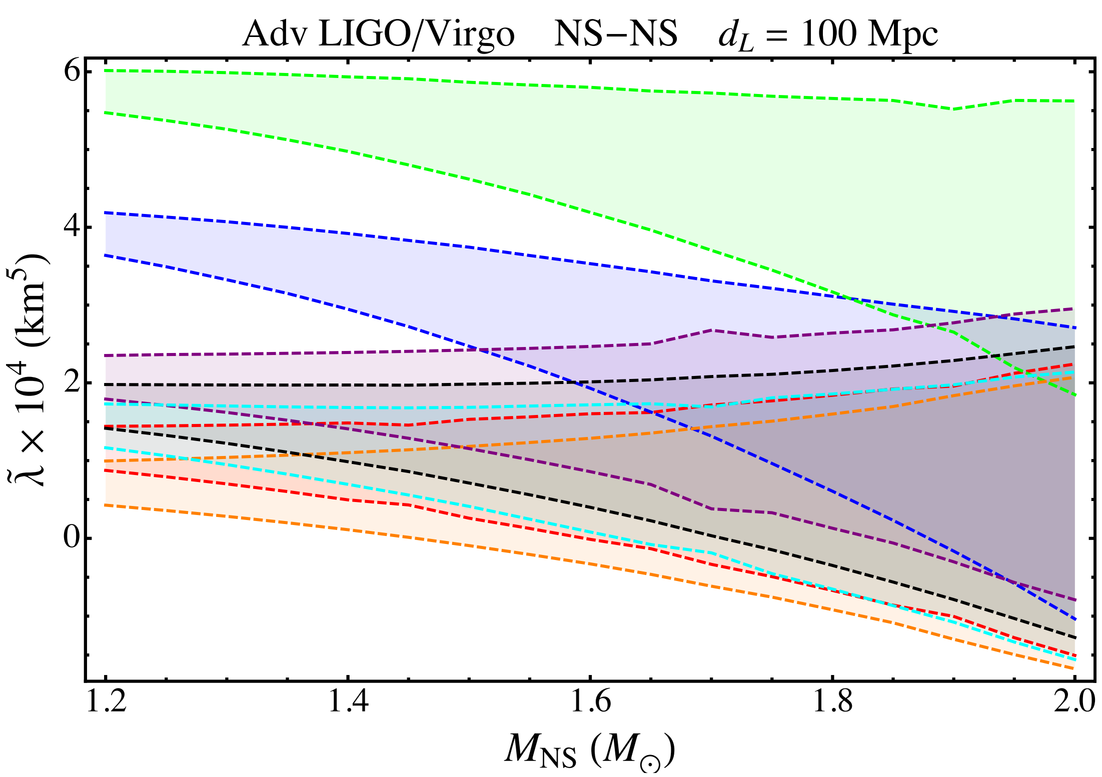
Only λ

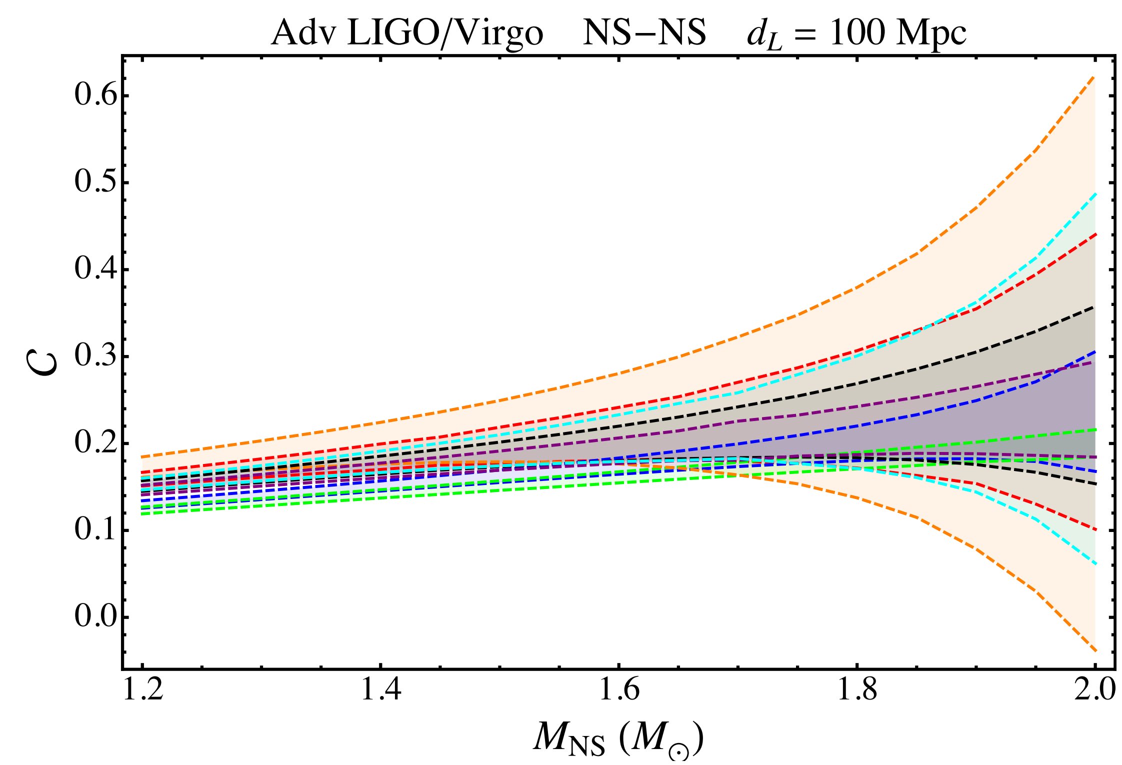
- [ ] The error on the NS compactness does not vary remarkably
- [ ] Unlike $\mathcal{C}$ the error on λ has a large spread
- [x] The accuracy on the tidal deformability improves, with a reduction on $\sigma_{\tilde{\lambda}}$ of the order of 30% for more distant sources

Adv Virgo/LIGO: NS-NS λ or $\mathcal{C}$?

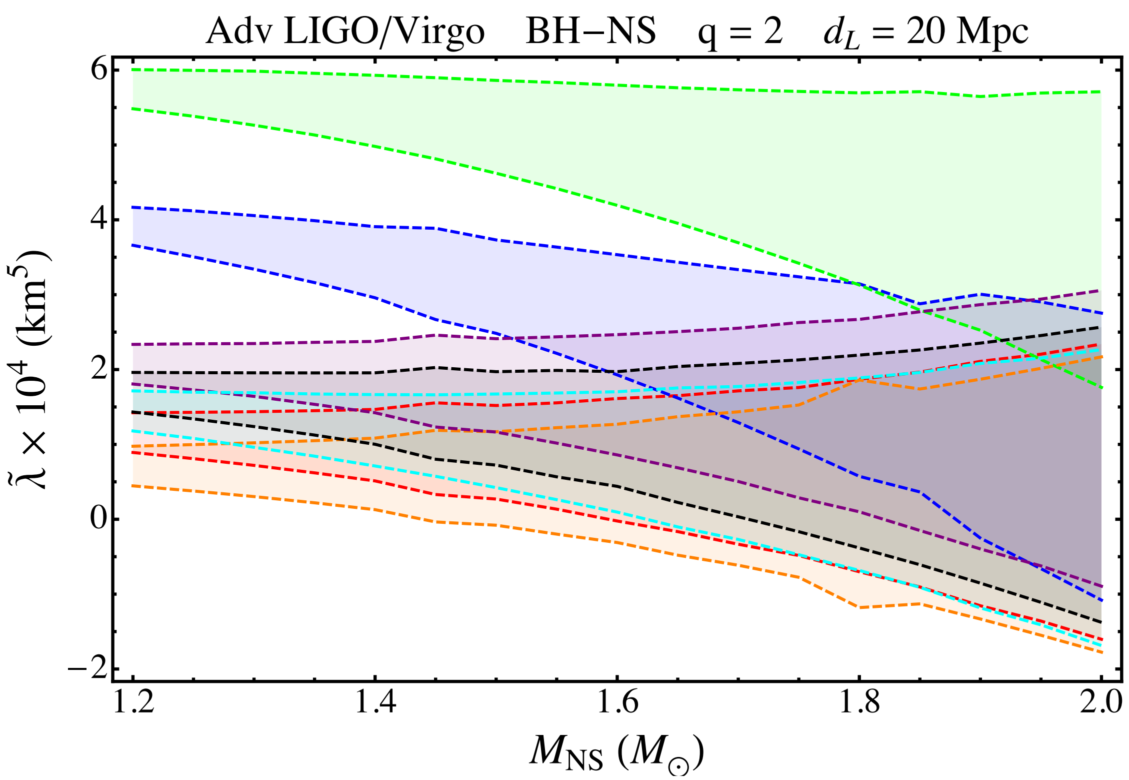


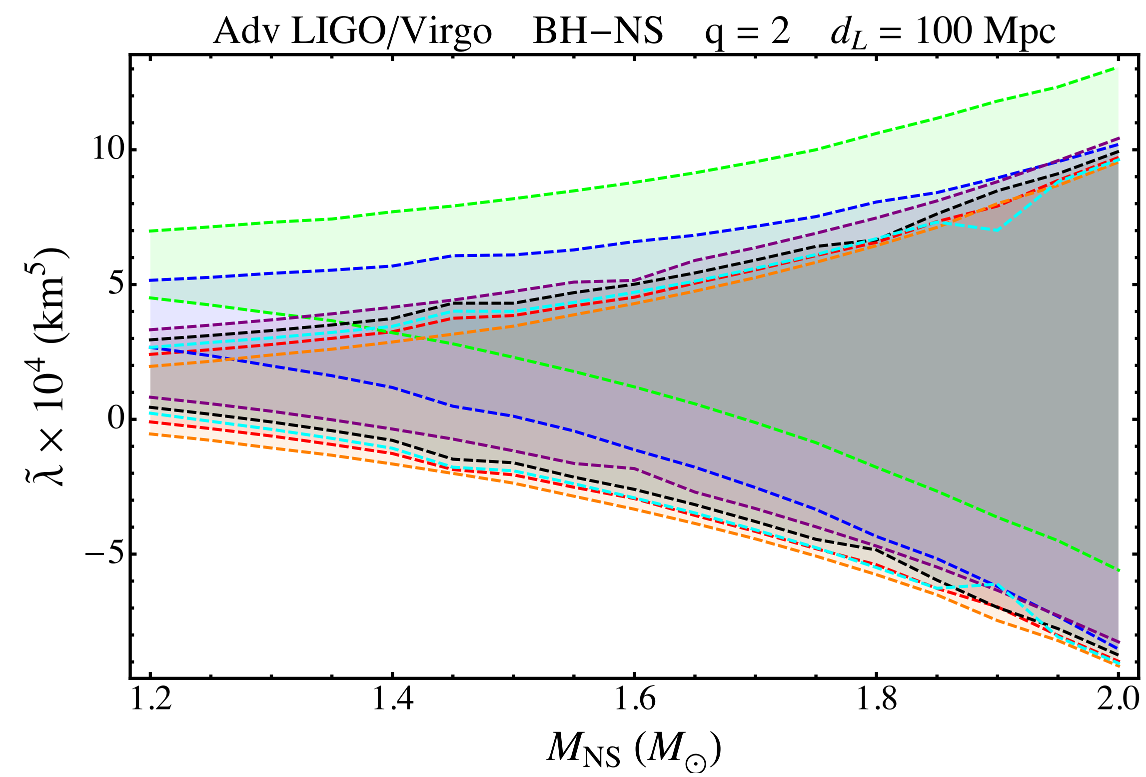






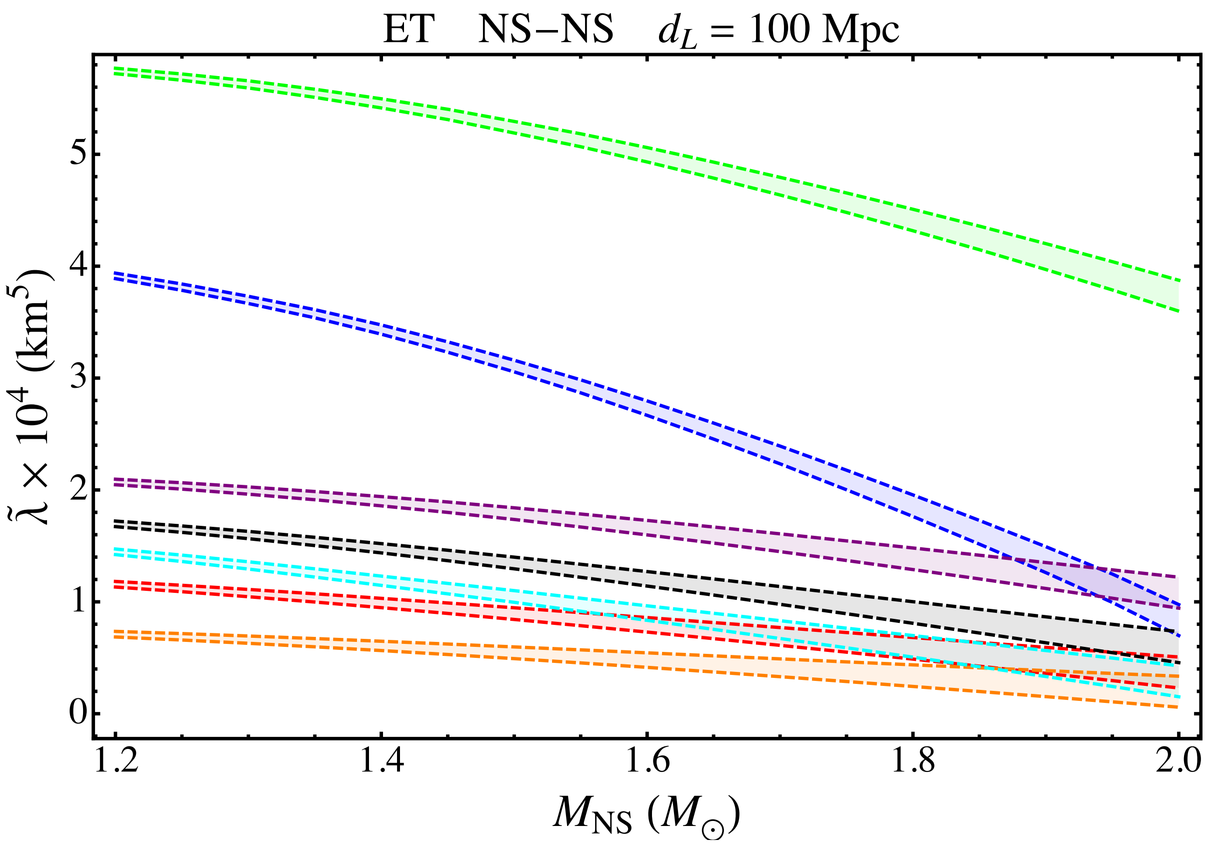
Adv Virgo/LIGO: BH-NS

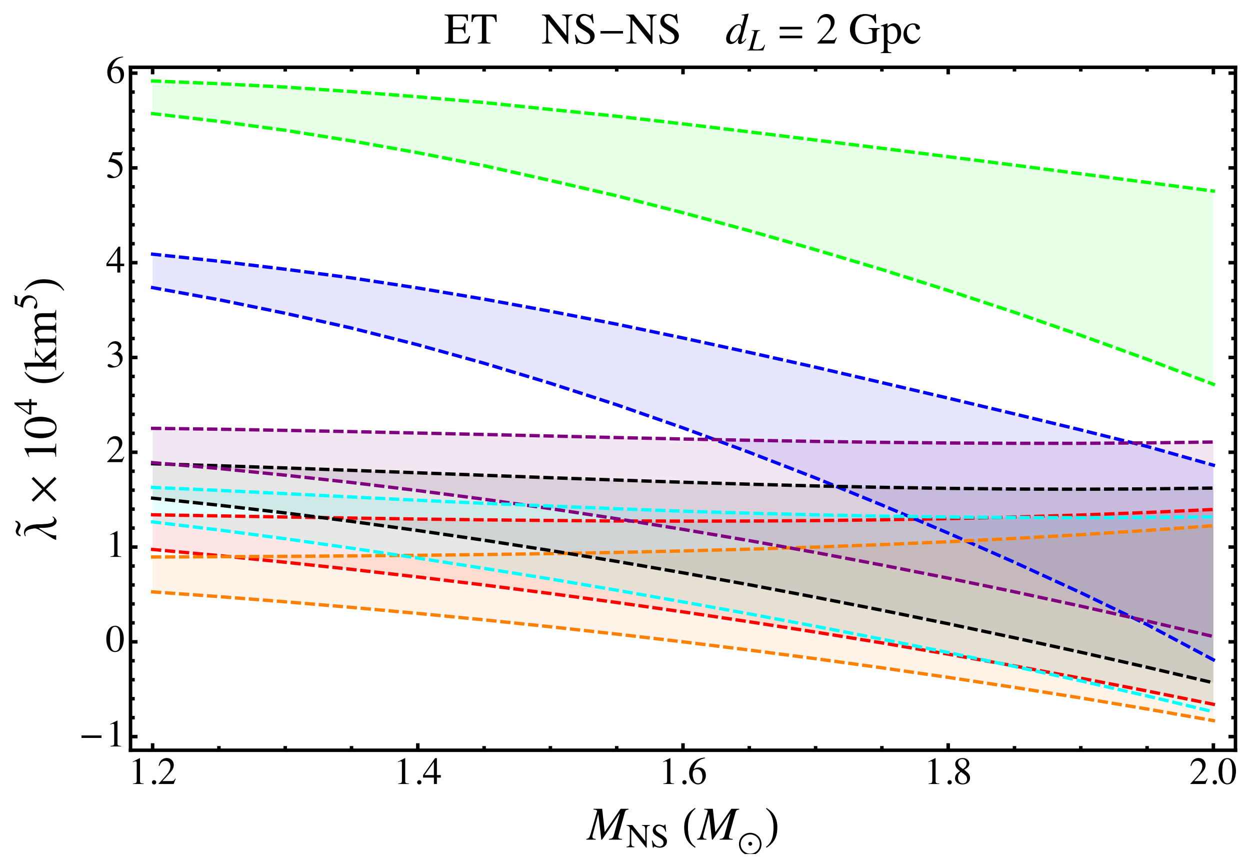




- λ is a better indicator than $\mathcal{C}$
- For BNS systems Adv detectors may constraint the EOS for low NS masses $\lesssim 1.5 M_\odot$ and stiff equation of state
- For BH-NS binaries Adv detectors may gain information on the NS composition only for close sources

The Einstein Telescope: NS-NS





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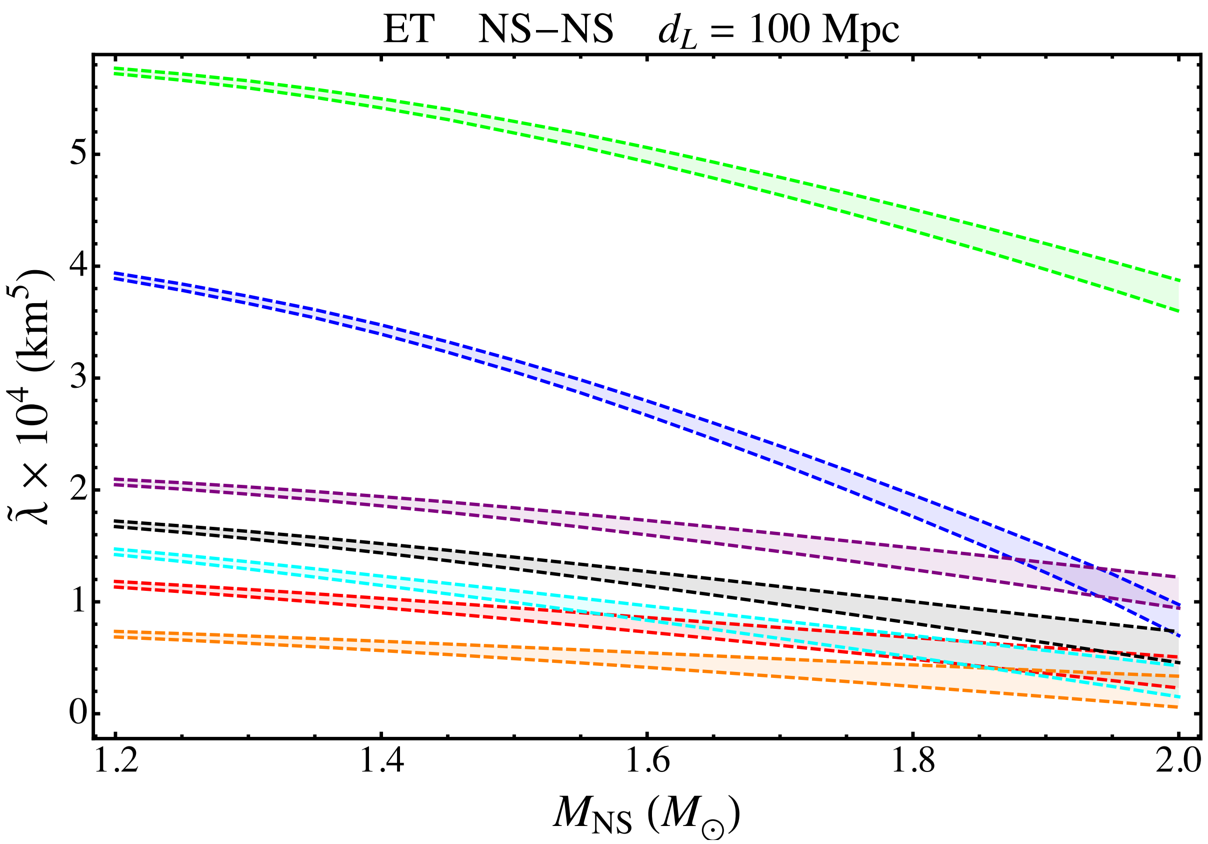
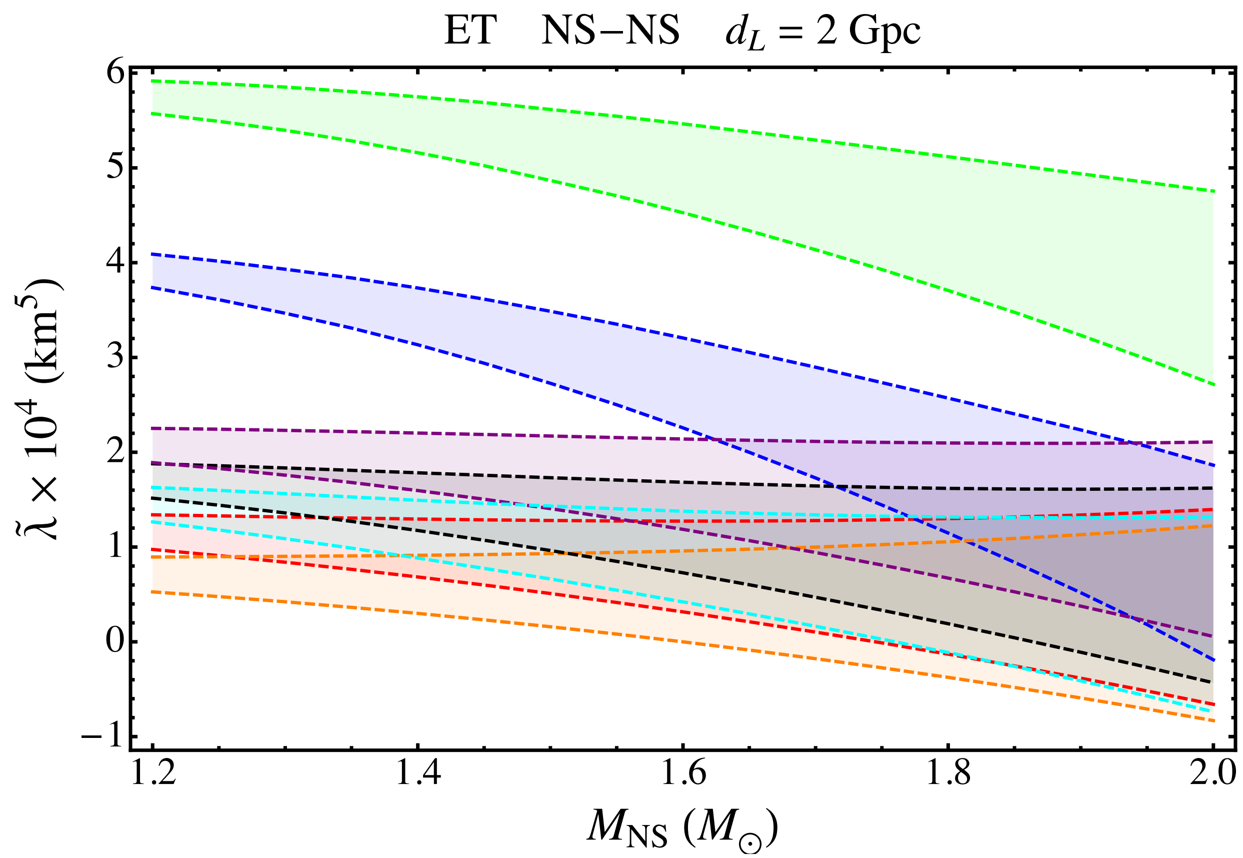
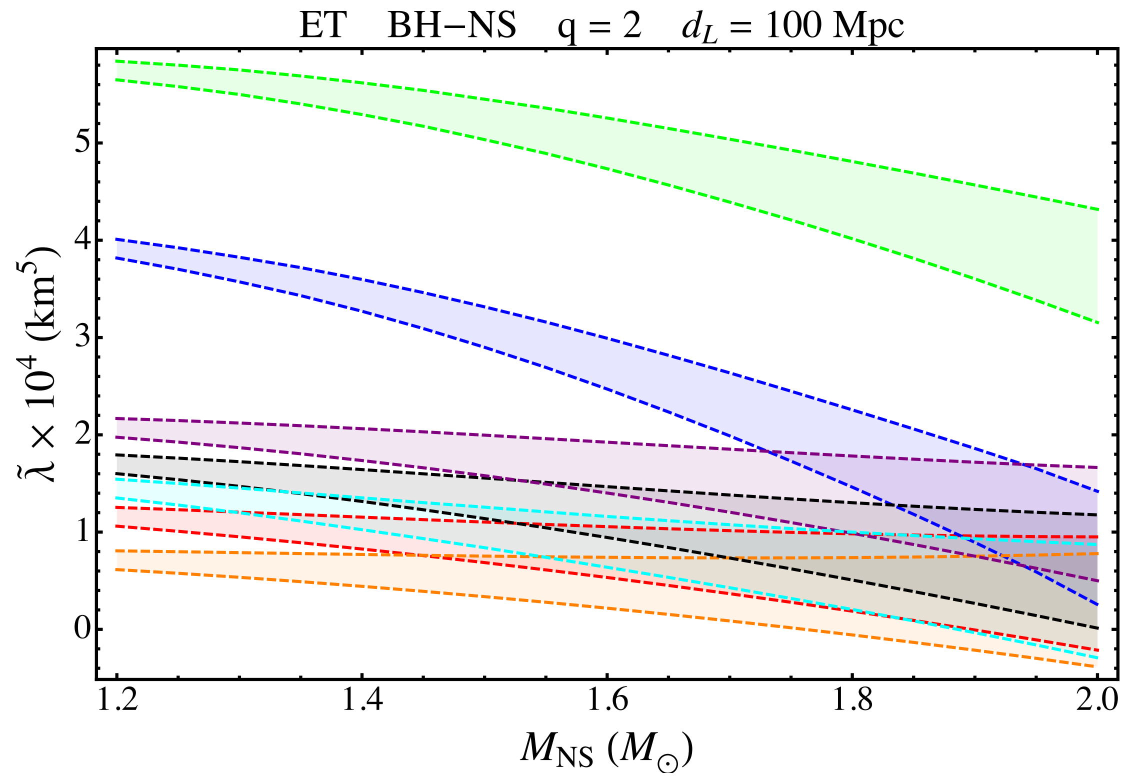
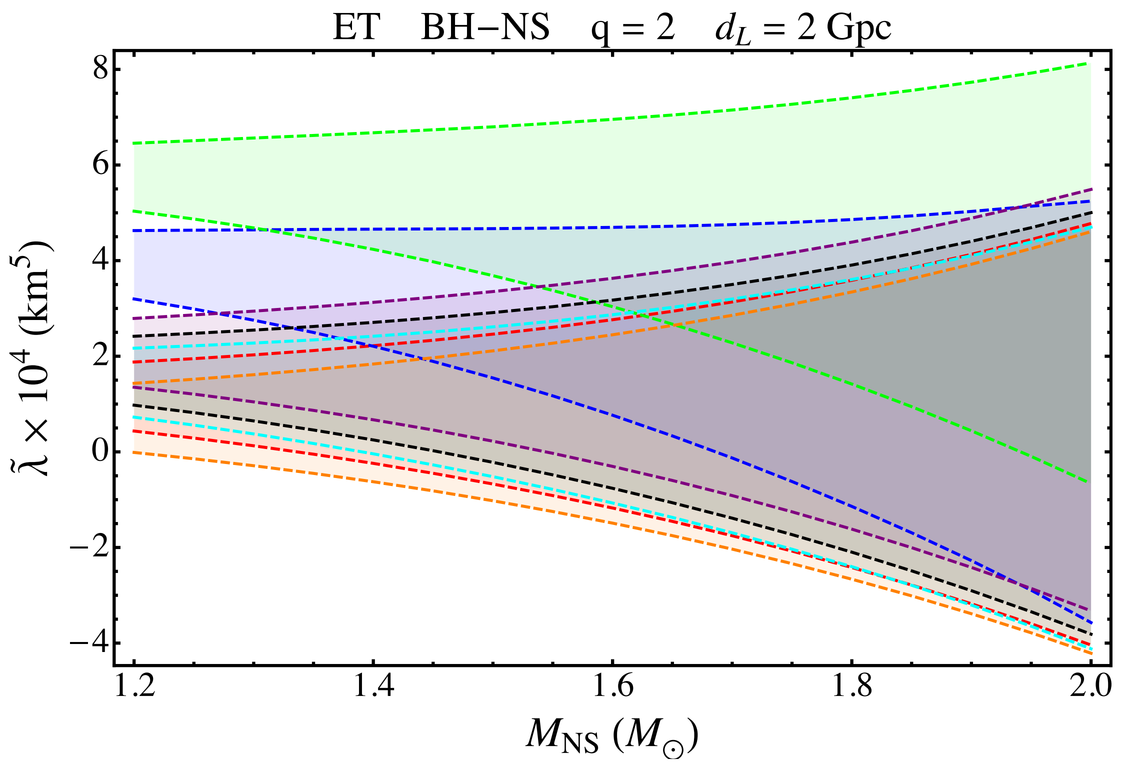


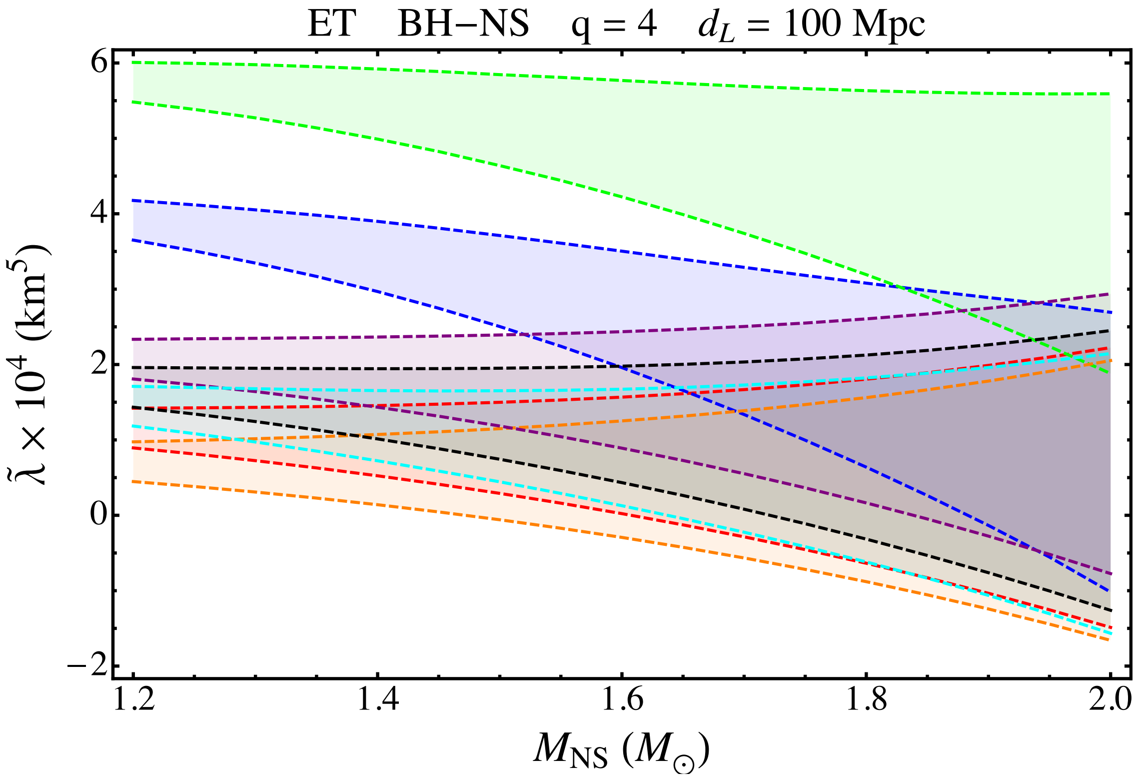


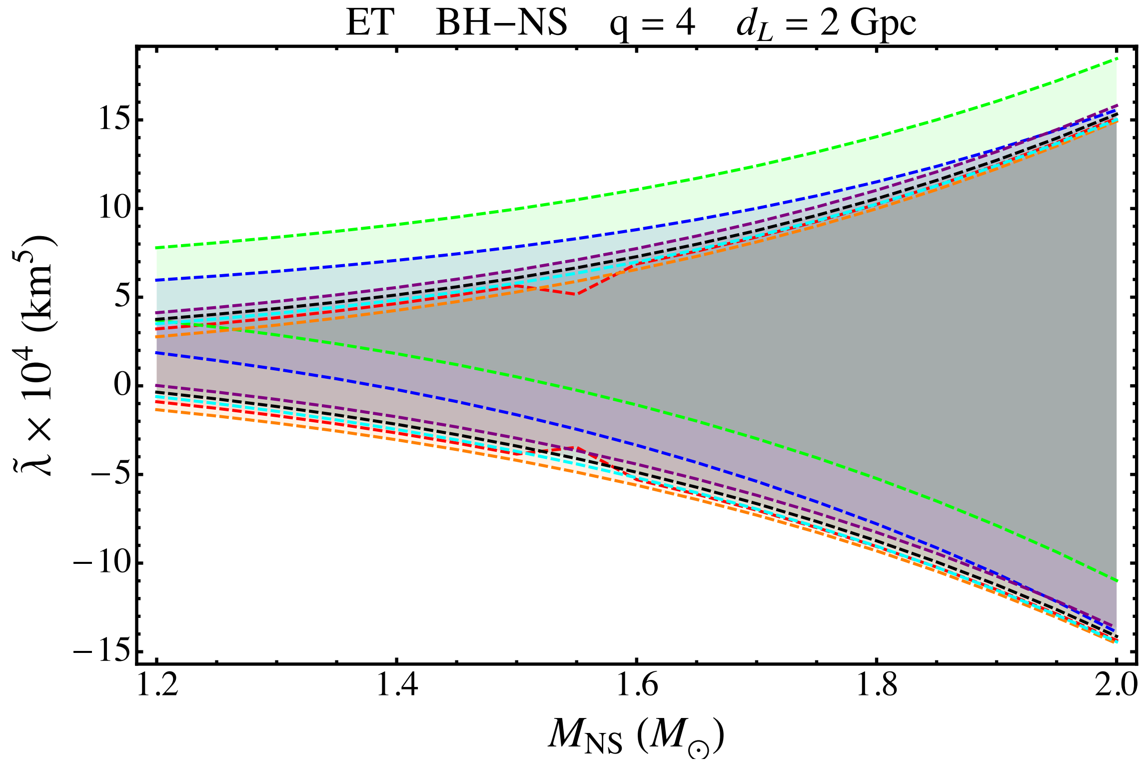
ET can at identify the class to which the NS equations of state belongs

The Einstein Telescope: BH-NS









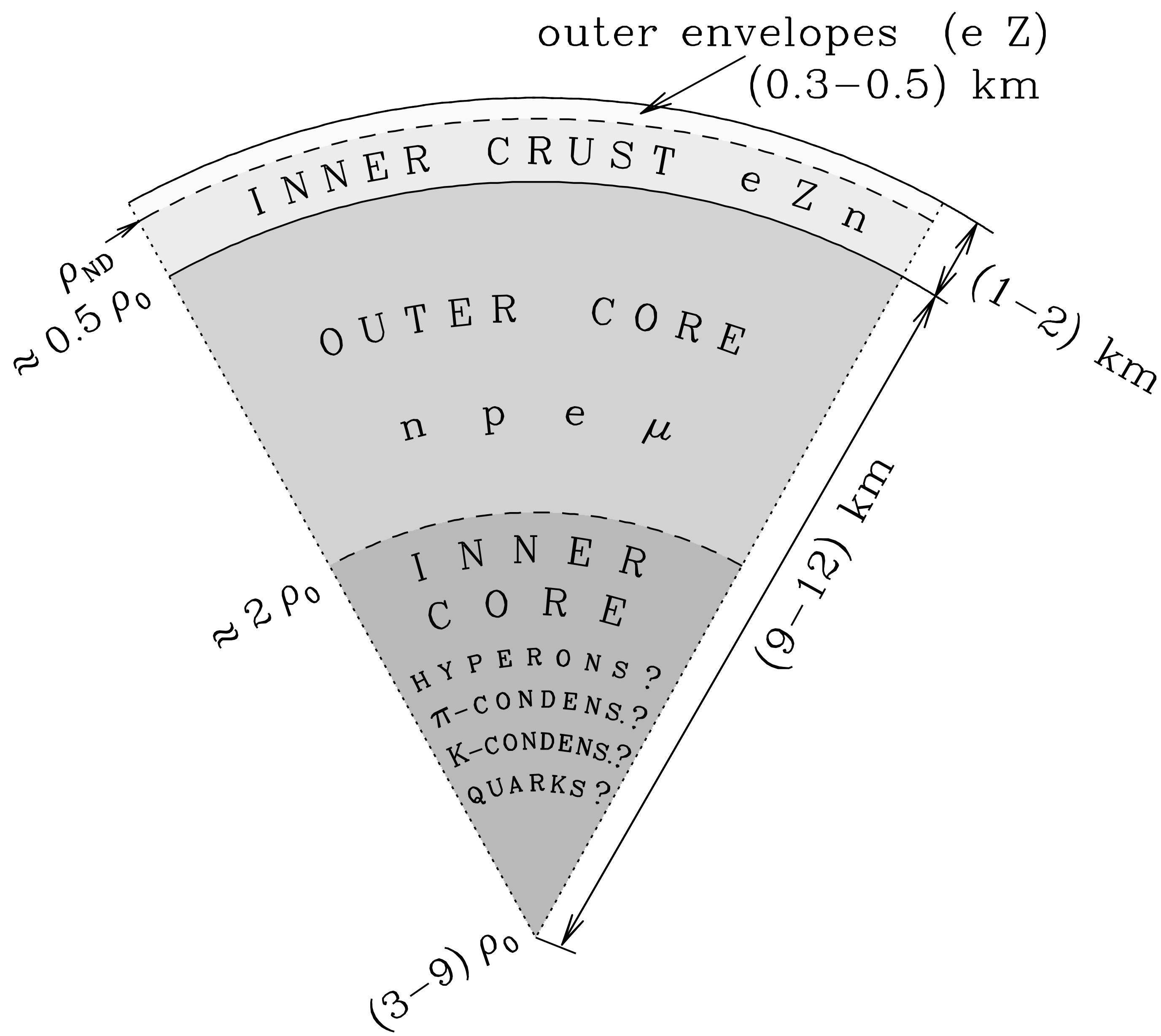
Summary

- Using $f_{\rm cut}$ to gain information on the NS structure
  - If the goal is to measure $\mathcal{C}$, the cut-off frequency is ineffective
  - Focusing on λ the error reduces up to 30% for more distant sources

- Comparing λ and $\mathcal{C}$ as EOS indicators
  - The tidal deformability is much better than the compactness
  - Adv detectors can set constraints on the EOS for $M_{\rm NS} \lesssim 1.5 M_\odot$ and $d_{\rm L} \lesssim 100$ Mpc (NS-NS)
  - ET can distinguish the EOS up to the max observed NS mass and for larger distances (NS-NS)
  - It seems unlike that BH-NS binaries can provide information on the NS structure for realistic mass ratio

Backup slides

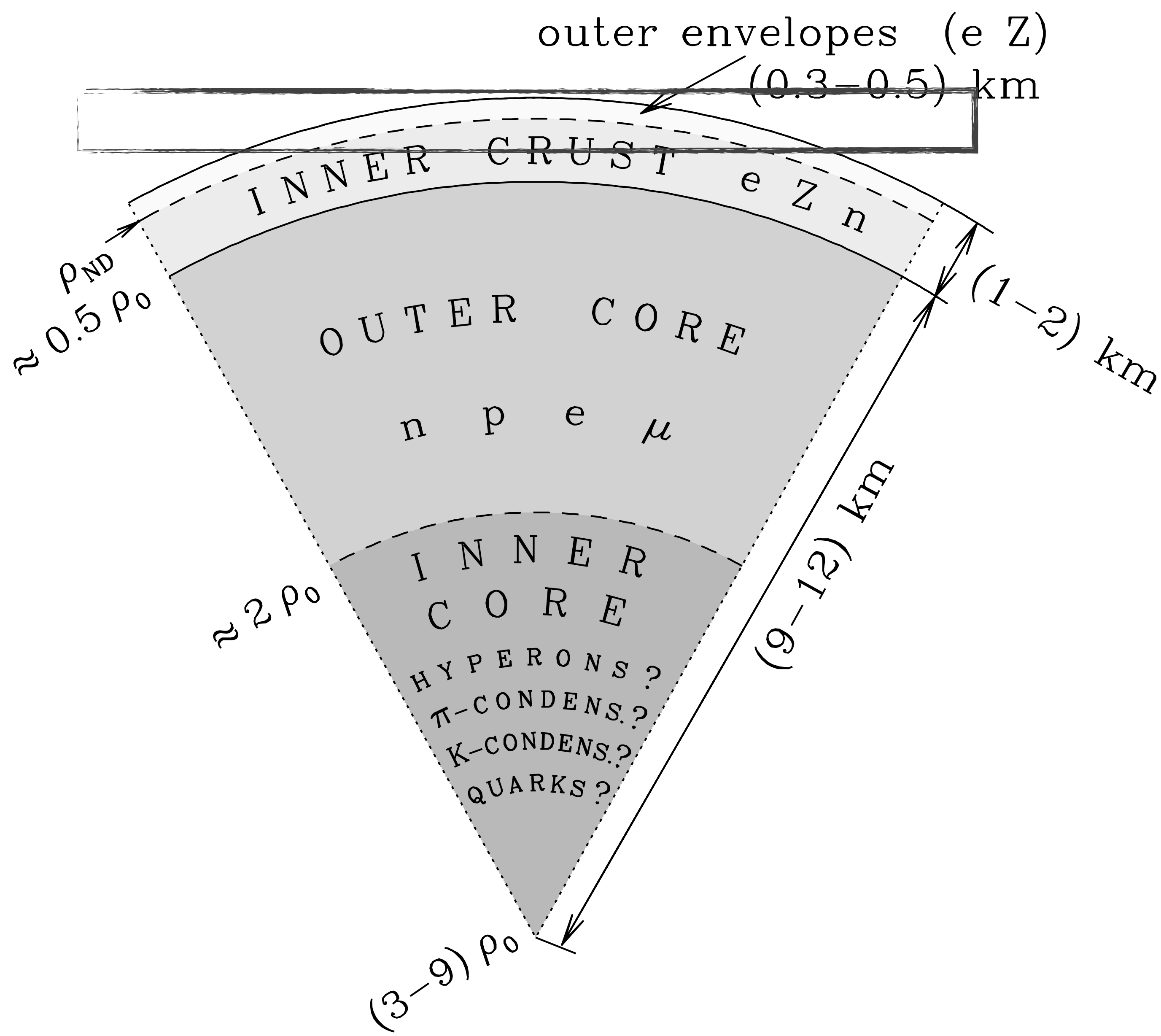
The NS structure



$$\rho_0 \simeq 2.67 \times 10^{14} \mathrm{g\ cm}^{-3}$$

(credits M. Fortin)

The NS structure

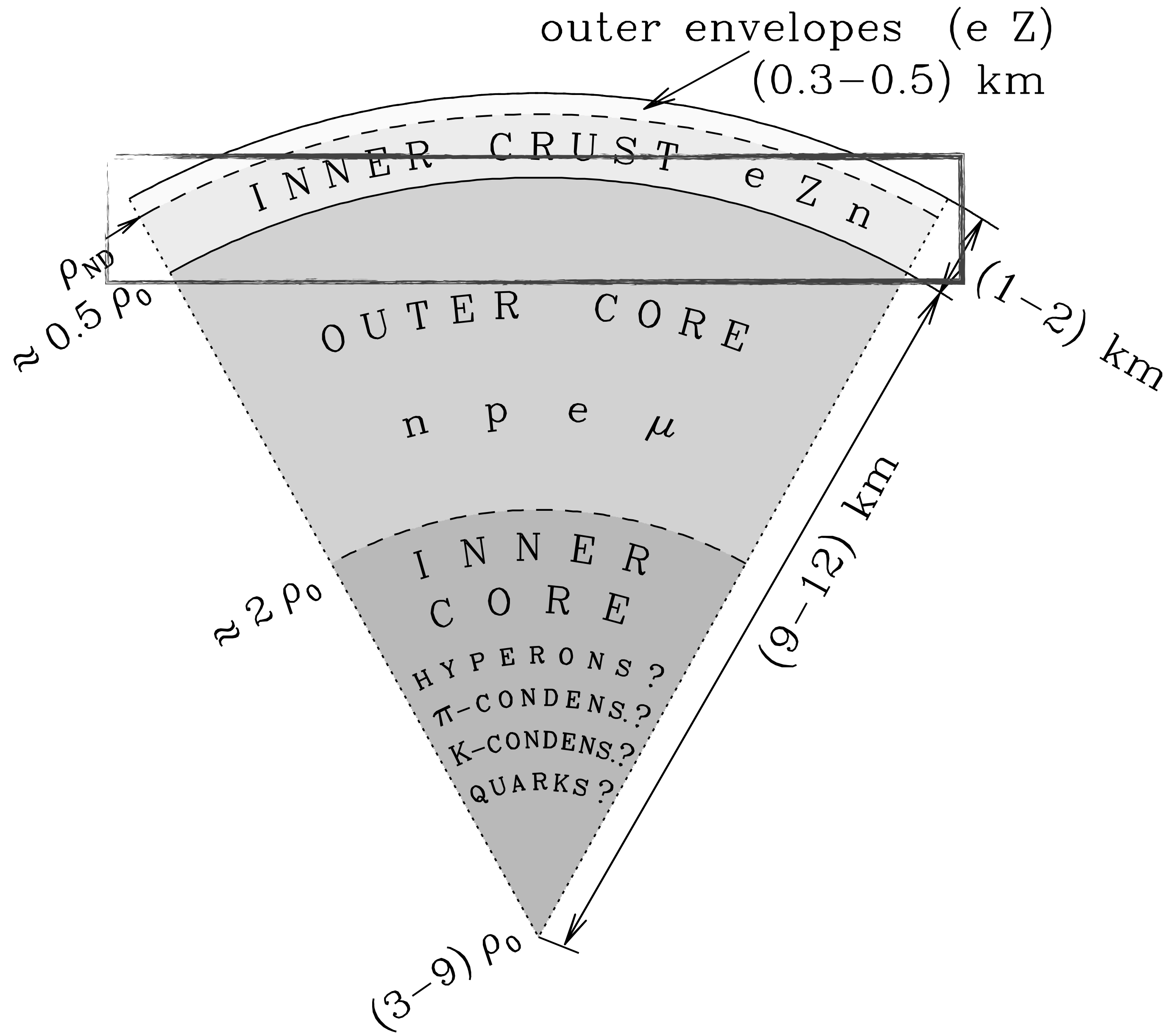


Nuclei lattice within e^- gas

$$\rho_0 \simeq 2.67 \times 10^{14} \mathrm{g\ cm}^{-3}$$

(credits M. Fortin)

The NS structure



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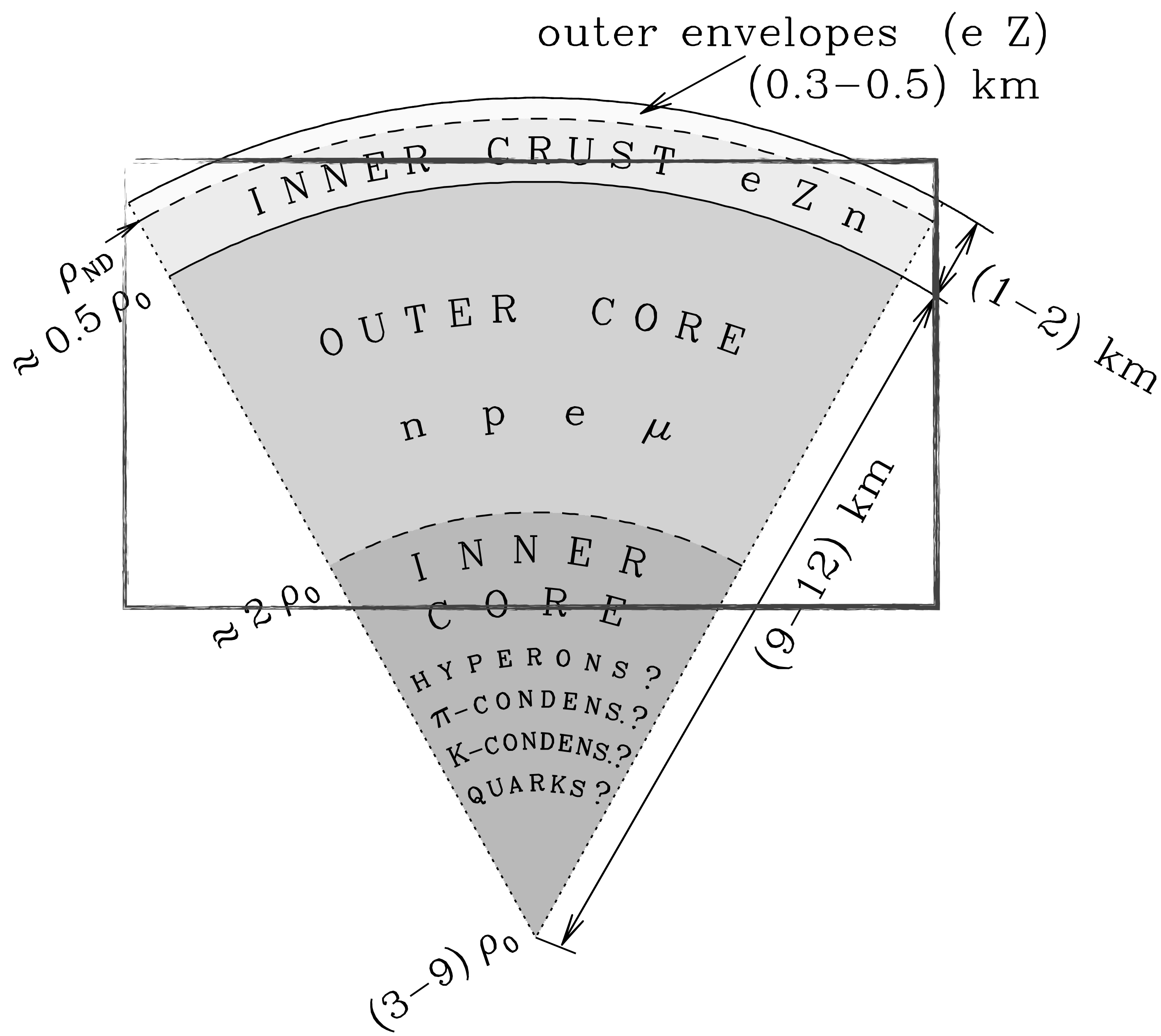
Pasta phases

(credits M. Fortin)

The NS structure



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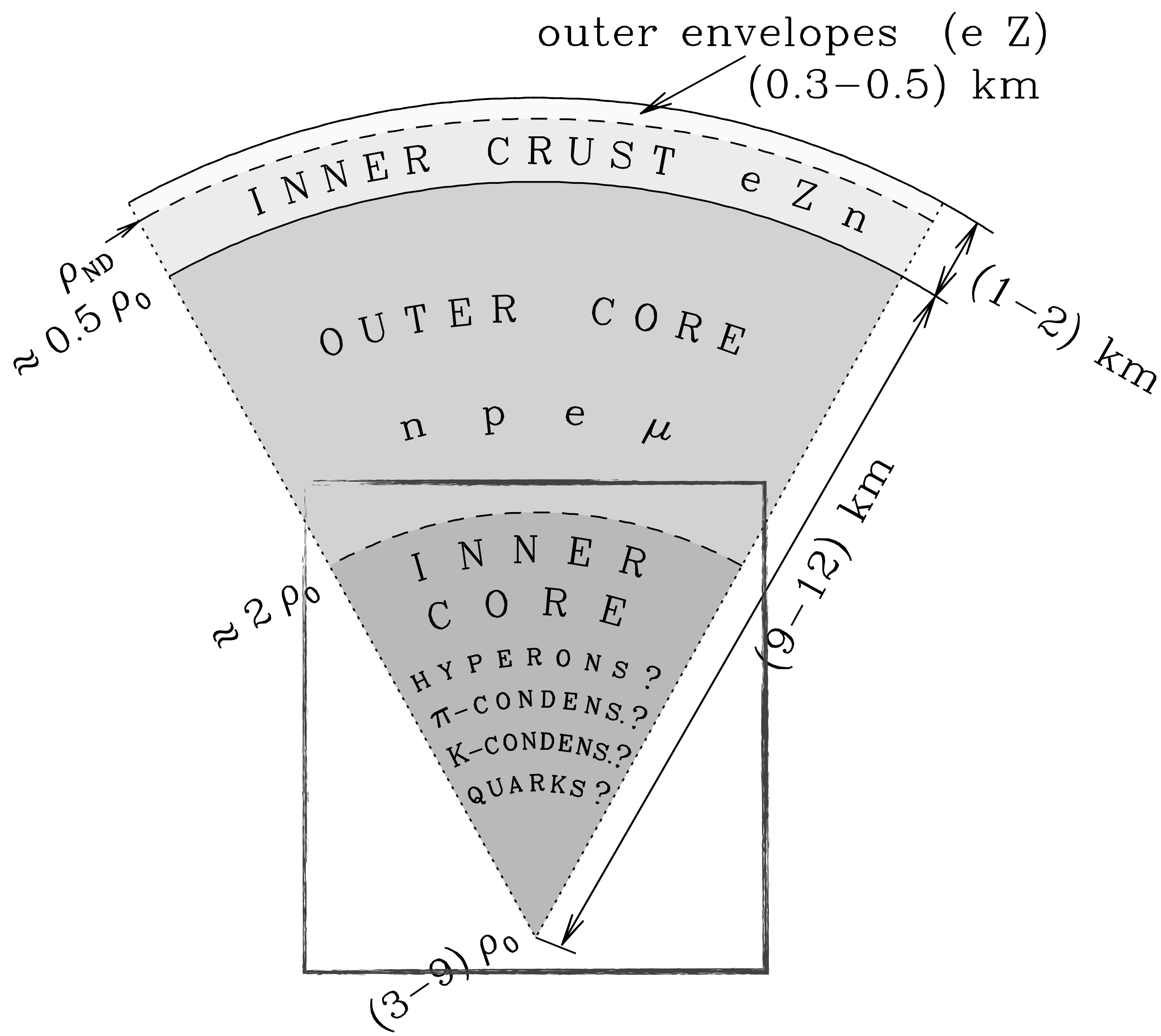
Nucleonic matter in $\beta-$equilibrium

(credits M. Fortin)

The NS structure



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Phase transitions, quark-gluon plasma ...

(credits M. Fortin)

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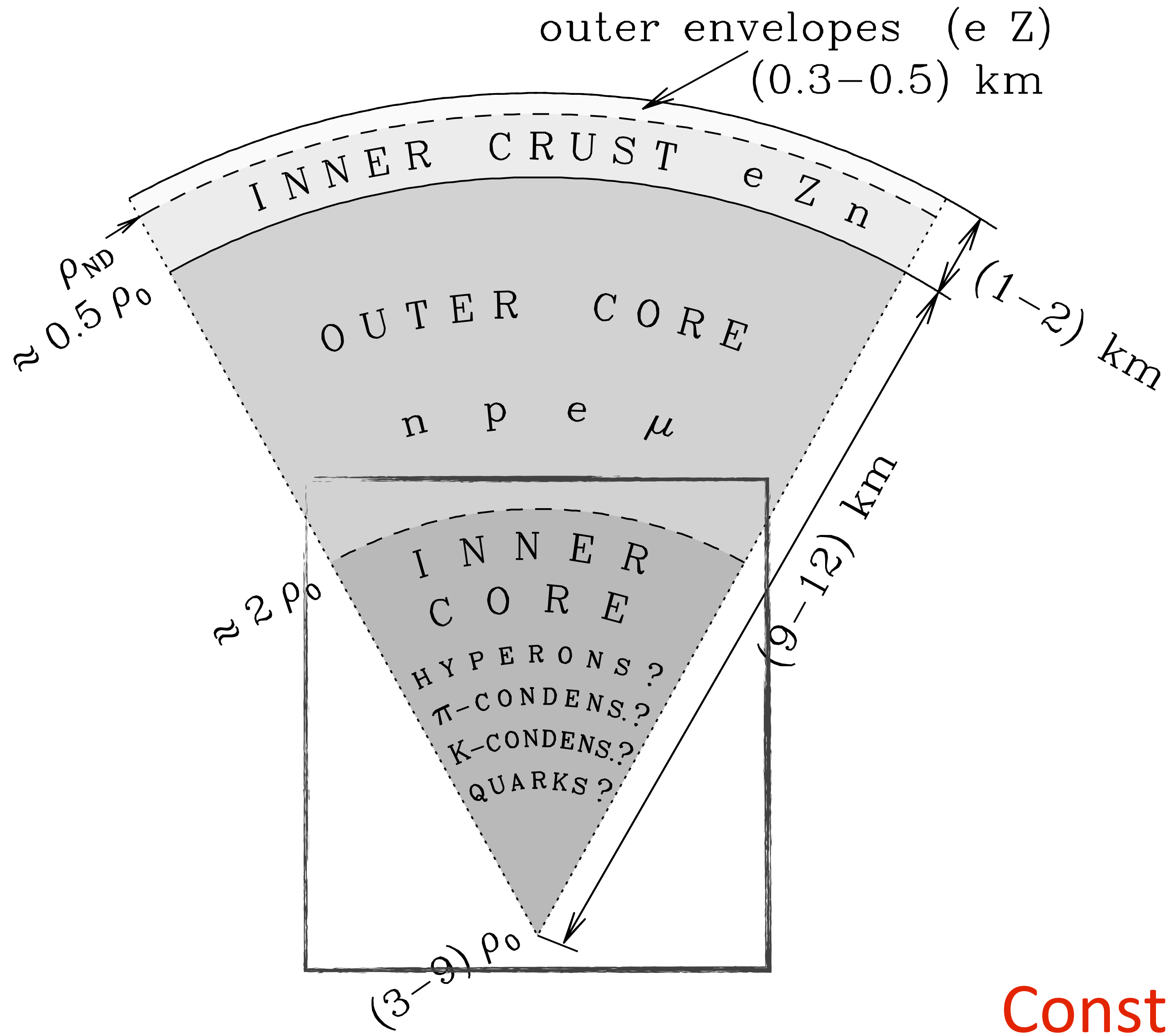

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Phase transitions, quark-gluon plasma ...

Constraints on strong interactions at super-nuclear density

(credits M. Fortin)

Tidal effects and the EOS

MCMC, Fisher Matrix, have shown that λ can be measured by Adv Detectors

Del Pozzo et al., PRL 111, 071101 (2013)
Damour et al., PRD 85, 123007 (2012)

GWs from BH-NS binaries may constrain $R_{\rm NS}$ for AdVirgo/LIGO with accuracy of 10% - 40%

B.Lackey et al., PRD 85, 044061 (2011)
J.Read et al. (2013)

- ☑ An order of magnitude better in the accuracy for the Einstein Telescope

Already competitive with observations of X-ray bursters and low-mass X-ray binaries, allowing for measurement of $\sigma_{R_{\rm NS}}/R_{\rm NS} \gtrsim 10\%$

F. Ozel, Rept. Prog. Phys. 76, 016901 (2013)

I λove Q

Why do I-love-Q ?

- ❑ I, Q, λ depend the most on the NS structure near the crust, where realistic EOS agree
- ❑ No-hair and strong equivalence principle → Effacement principle in GR

When do I-love-Q ?

- ❑ Measurement of one member of the trio provides information about the other two
  - ☑ Constrain the NS exterior properties
- ❑ On a GW front they break the degeneracy between quadrupole moment and spins
- ❑ Test GR in strong field regime theory/EOS independent

I λove Q