Active opto-mechanical filter

Haixing Miao

Reporting joint research by:

Australian National University: Robert Ward

California Institute of Technology: Rana Adhikari, Yanbei Chen, Zach Korth

Louisiana State University: Thomas Corbitt

University of Birmingham: Andreas Freise, Haixing Miao, and Mengyao Wang

University of Western Australia: David Blair, Stefan Danilishin, Yiqiu Ma, and Chunnong Zhao

Outline

- **Background**
  - Different filtering schemes
  - Passive filter cavity
- **Active opto-mechanical filter**
  - Case 1: Tunable bandwidth filter cavity
  - Case 2: Phase (in)sensitive parametric amplifier
  - Case 3: Unstable filter with “phase advance”
- **Thermal noise issue**
  - Thermal noise requirement
  - Optical dilution idea

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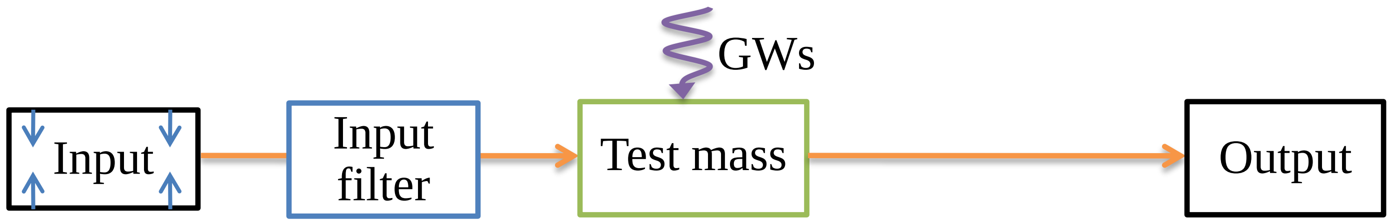
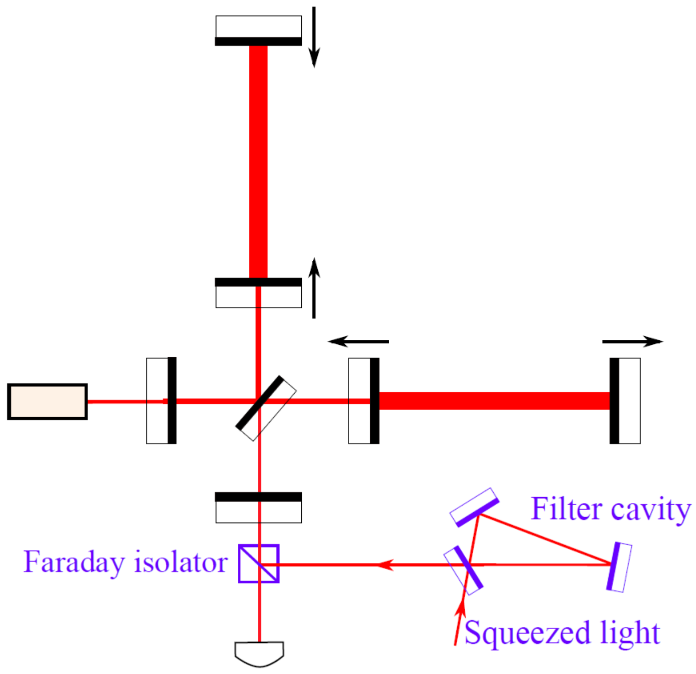
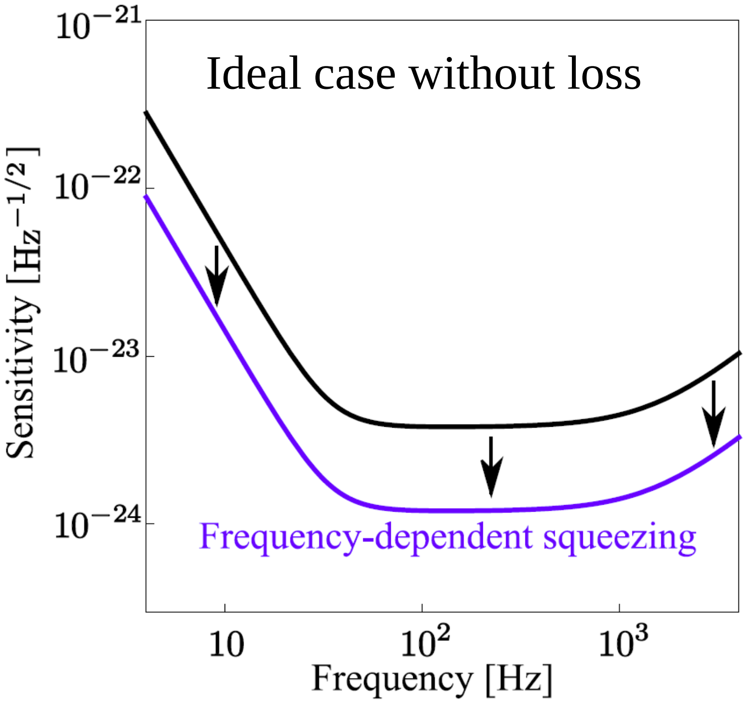
Frequency-dependent squeezing

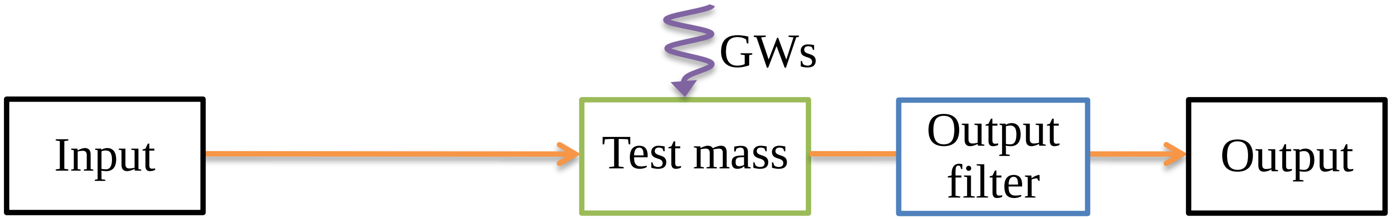


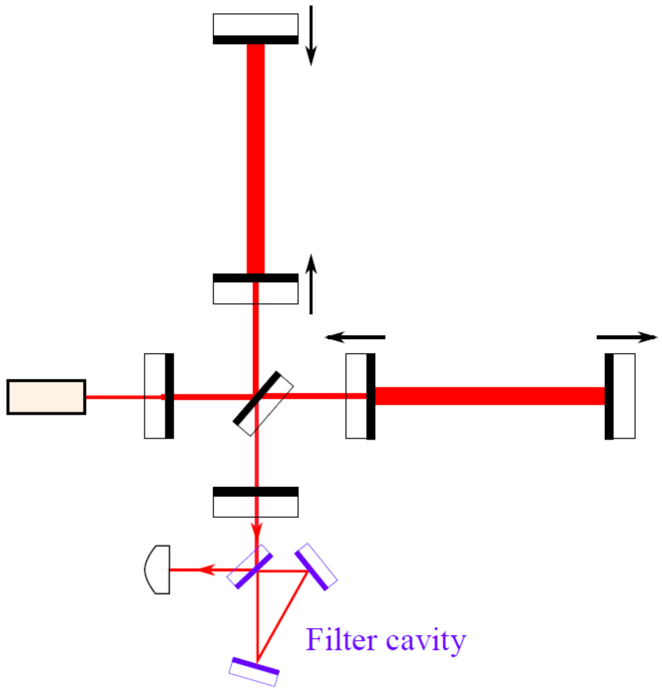


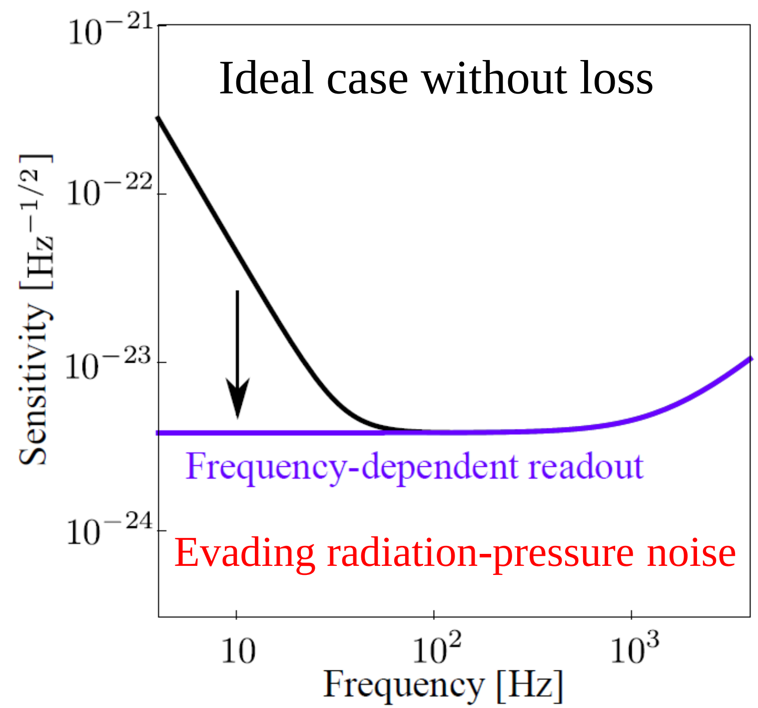


Reference: H. Kimble, Y. Levin, A. Matsko, K. Thorne, and S. Vyatchanin, PRD 65, 022002 (2001)

Frequency-dependent readout

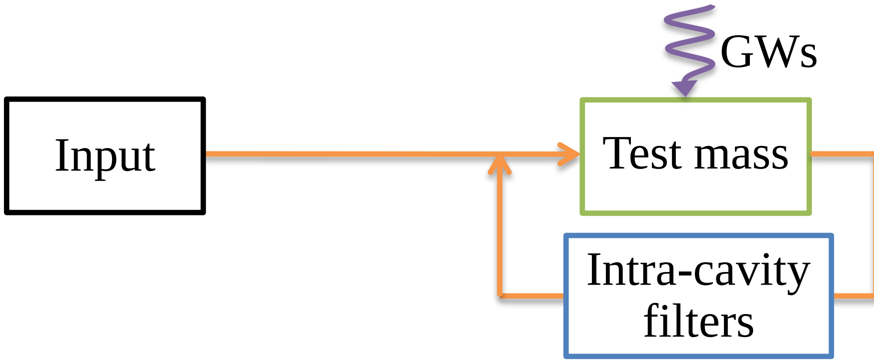




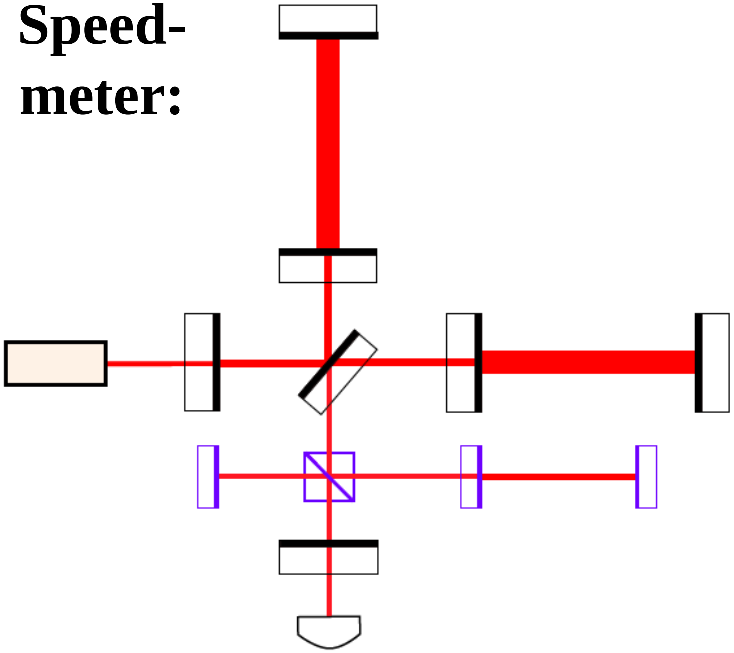


Reference: H. Kimble, Y. Levin, A. Matsko, K. Thorne, and S. Vyatchanin, PRD **65**, 022002 (2001).

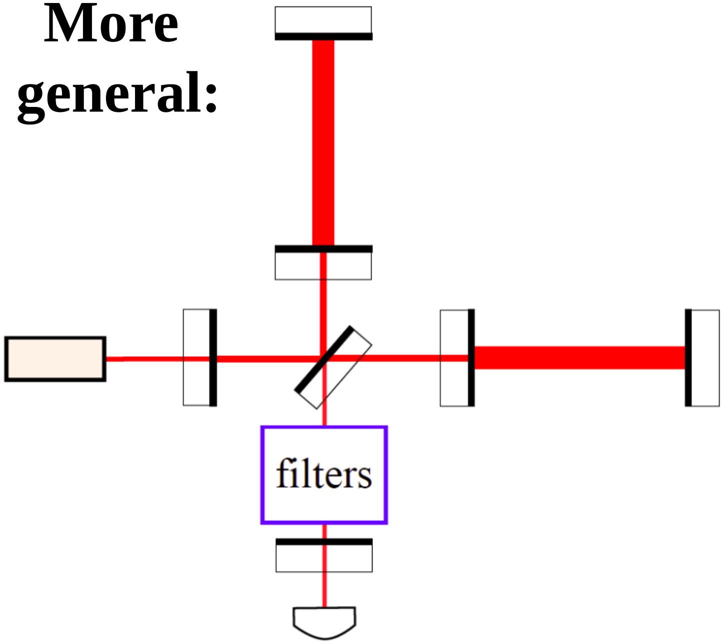
Intra-cavity filtering



Speed-meter:



More general:



Reference: M. Wang, H. Miao, A. Freise, and Y. Chen, PRD **89**, 062009 (2014)

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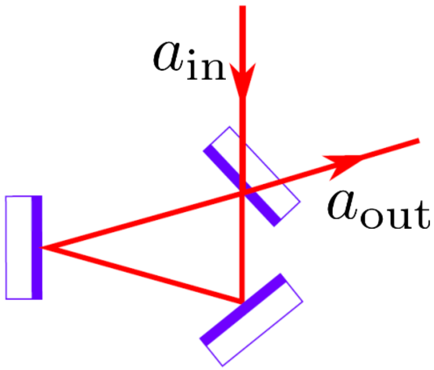
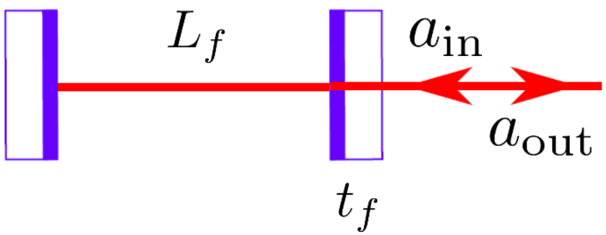
Passive filter cavity

Triangular cavity **Linear cavity**





Input-output relation (for sideband):

$$a_{\rm out}(\Omega) = \frac{i\gamma_f - (\Omega - \Delta_f)}{i\gamma_f + (\Omega - \Delta_f)} a_{\rm in}(\Omega)$$

Bandwidth: $\gamma_f \equiv \frac{t_f^2}{4(L_f/c)}$

Detuning: $\Delta_f = \omega_{\rm cav} - \omega_0$

Order of magnitude:

$$\gamma_f/(2\pi) = 50{\rm Hz}\left(\frac{150{\rm m}}{L_f}\right)\left(\frac{10^4}{\rm Finesse}\right) = 50{\rm Hz}\left(\frac{15{\rm m}}{L_f}\right)\left(\frac{10^5}{\rm Finesse}\right)$$

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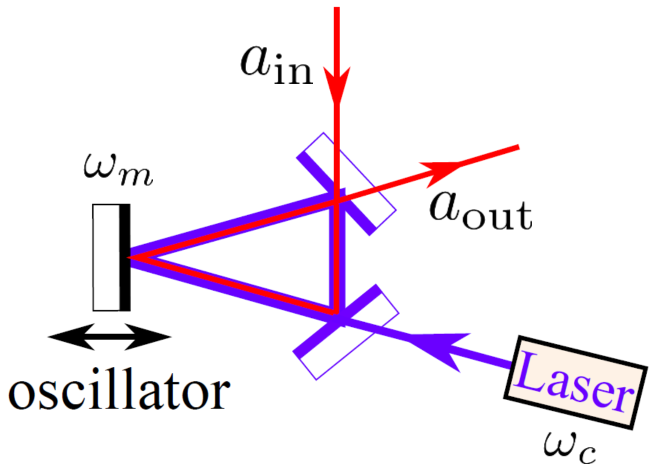
Active opto-mechanical filter

Introducing control laser & mechanical oscillator:



$$E(t) = \underbrace{E_c e^{-i\omega_c t}}_{\text{Control laser}} + \underbrace{a(t) e^{-i\omega_0 t}}_{\text{Signal sidebands}} + \text{c.c}$$

$$x(t) = \underbrace{X(t) e^{-i\omega_m t} + X^*(t) e^{i\omega_m t}}_{\text{Mechanical motion}}$$

Radiation pressure coupling:

$$H_{\rm int}(t) = \frac{P(t)}{c} x(t)$$

$$\propto E^2(t) x(t)$$

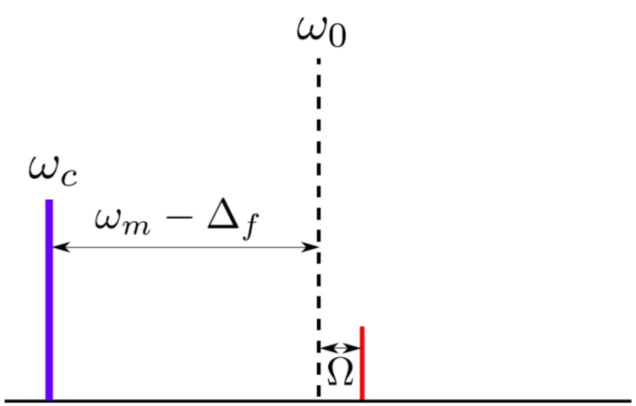
$\Rightarrow$

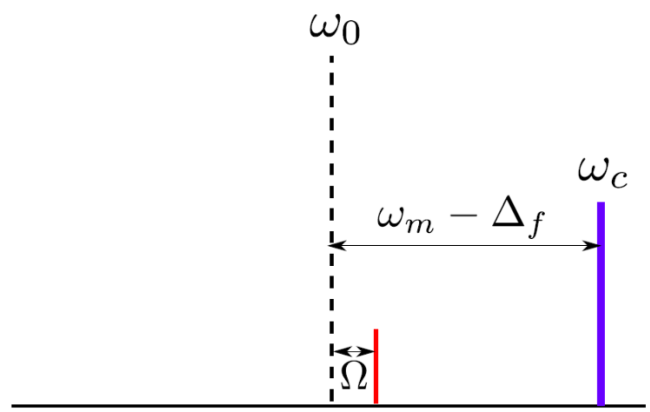
$$E_c^* a(t) X^*(t) e^{i(\omega_c - \omega_0 + \omega_m)t} + \text{c.c}$$

$$E_c^* a(t) X(t) e^{-i(\omega_c - \omega_0 - \omega_m)t} + \text{c.c}$$

Analogous to "three-wave mixing"

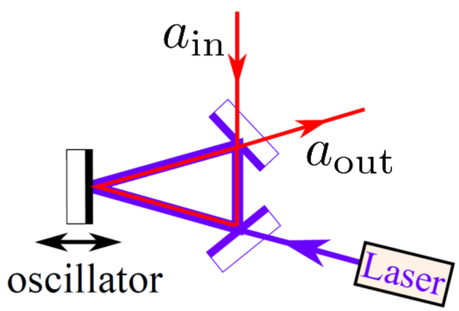
Active opto-mechanical filter





$$E_c^* a(t) X^*(t) e^{-i\Delta_f t} + \mathrm{c.c} \qquad E_c^* a(t) X(t) e^{-i\Delta_f t} + \mathrm{c.c}$$

Input-output relation ($\omega_m \gg \gamma_f$):



$$a_{\rm out}(\Omega) \approx \frac{i(\pm\gamma_{\rm opt} - \gamma_m) - (\Omega - \Delta_f)}{i(\pm\gamma_{\rm opt} + \gamma_m) + (\Omega - \Delta_f)} a_{\rm in}(\Omega)$$

$$\gamma_{\rm opt} \equiv \frac{4 P_c \omega_c}{m \omega_m c^2 t_f^2} \qquad \gamma_m \equiv \frac{\omega_m}{Q_m}$$

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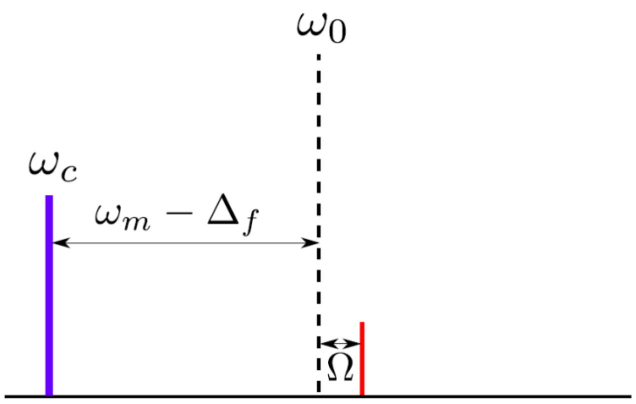
Tunable bandwidth filter cavity



Input-output relation ($\gamma_{\rm opt} \gg \gamma_m$):

$$a_{\rm out}(\Omega) \approx \frac{i\gamma_{\rm opt} - (\Omega - \Delta_f)}{i\gamma_{\rm opt} + (\Omega - \Delta_f)} a_{\rm in}(\Omega)$$

Equivalent cavity bandwidth:

$$\gamma_{\rm opt} = \frac{4P_c\omega_c}{m\omega_m c^2 t_f^2}$$

Order of magnitude estimate:

$$\gamma_{\rm opt}/(2\pi) = 50{\rm Hz}\left(\frac{P_c}{15W}\right)\left(\frac{1{\rm mg}}{m}\right)\left(\frac{1{\rm MHz}}{\omega_m/(2\pi)}\right)\left(\frac{\rm Finesse}{10^4}\right)$$

Length can be tens of centimeter, as long as $\omega_m \gg \gamma_f$.

Reference: Y. Ma, S. Danilishin, C. Zhao, H. Miao, W. Korth, Y. Chen, R. Ward, and D. Blair, PRL **113**, 151102 (2014)

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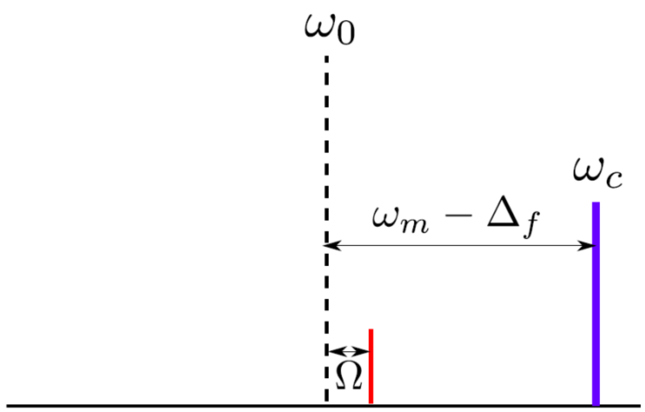
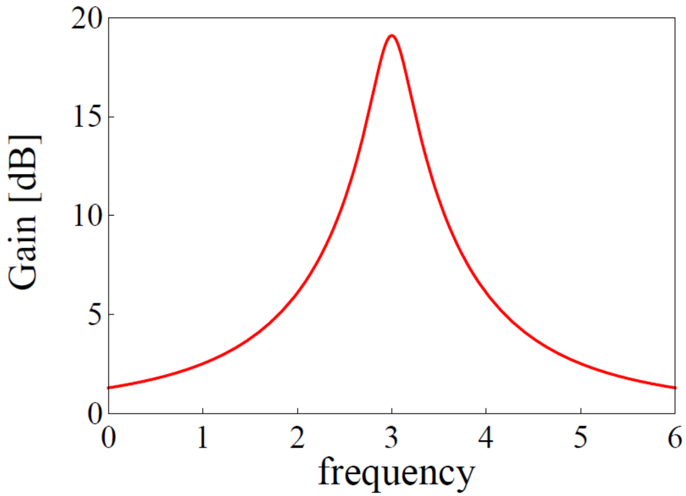
Phase (in)sensitive amplifier





Phase insensitive parametric amplifier:

$$a_{\text{out}}(\Omega) \approx \frac{i(-\gamma_{\text{opt}} - \gamma_m) - (\Omega - \Delta_f)}{i(-\gamma_{\text{opt}} + \gamma_m) + (\Omega - \Delta_f)} a_{\text{in}}(\Omega) \qquad \left| \frac{a_{\text{out}}(\Omega)}{a_{\text{in}}(\Omega)} \right| > 1$$

$\omega_c \approx \omega_0$ ⇨ **Phase sensitive amplifier (ponderomotive squeezing)**

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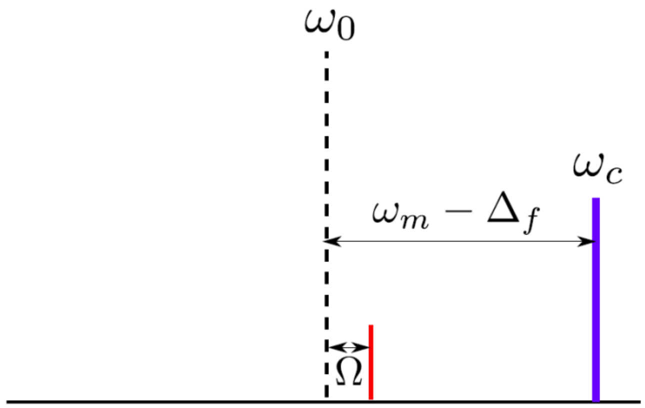
Unstable filter with phase advance

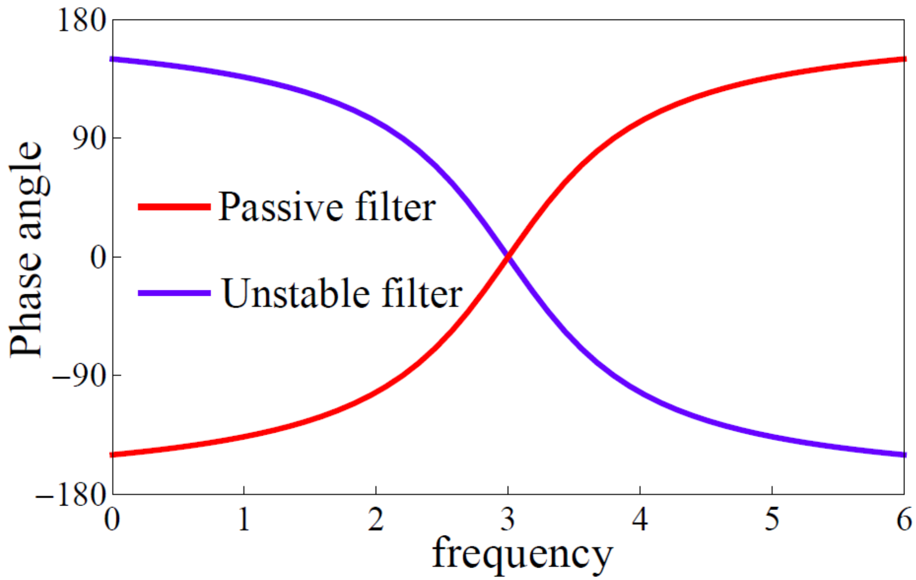


Input-output relation ($\gamma_{\rm opt} \gg \gamma_m$):

$$a_{\rm out}(\Omega) \approx \frac{-i\gamma_{\rm opt} - (\Omega - \Delta_f)}{-i\gamma_{\rm opt} + (\Omega - \Delta_f)} a_{\rm in}(\Omega)$$

In contrast, for a passive cavity

$$a_{\rm out}(\Omega) = \frac{i\gamma_f - (\Omega - \Delta_f)}{i\gamma_f + (\Omega - \Delta_f)} a_{\rm in}(\Omega)$$



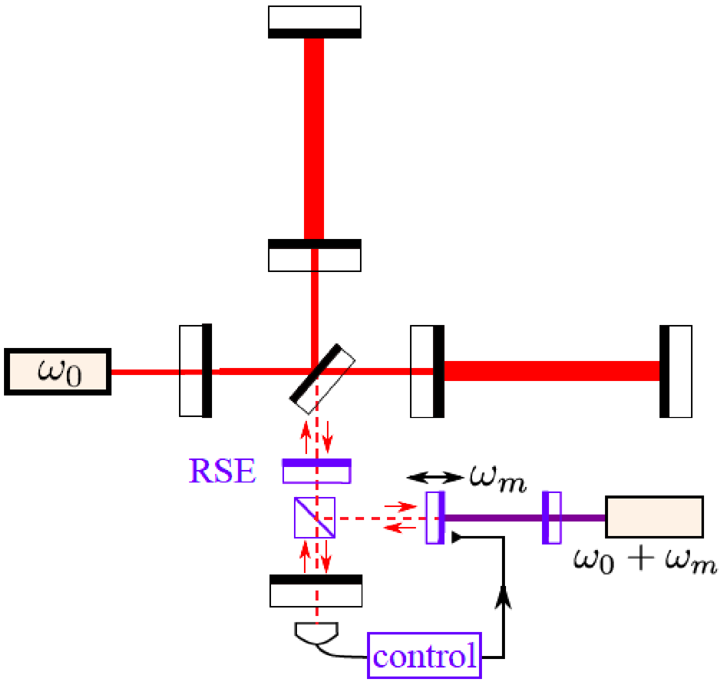
Phase advance
instead of phase lag

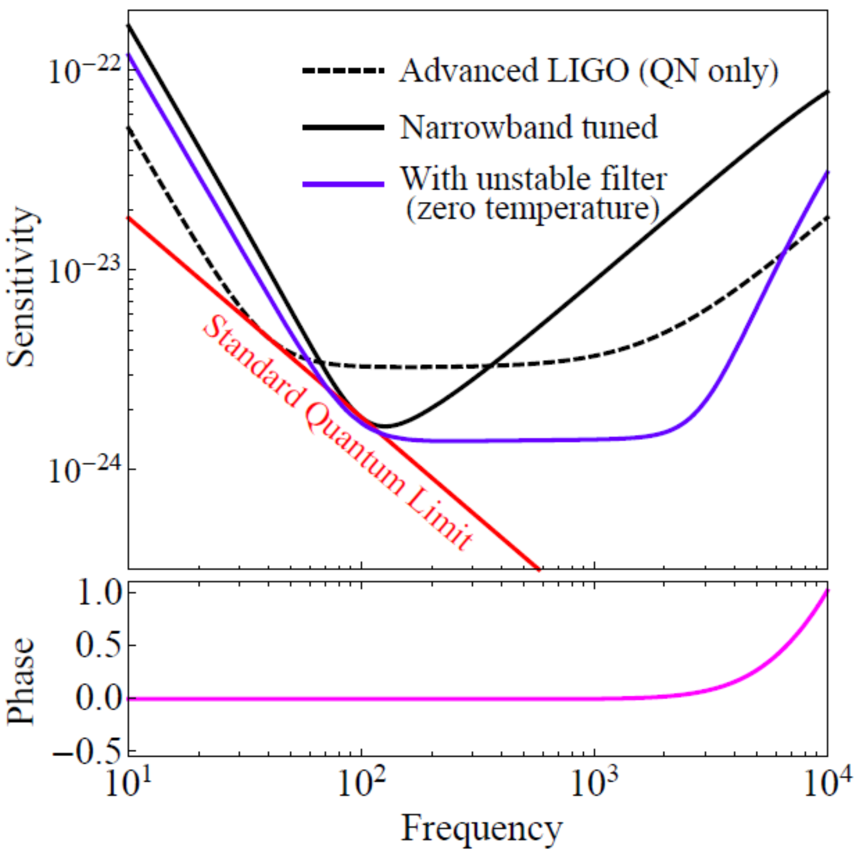
⇩

Unstable (acausal)

Unstable filter with phase advance

Cancelling arm cavity propagation phase delay:



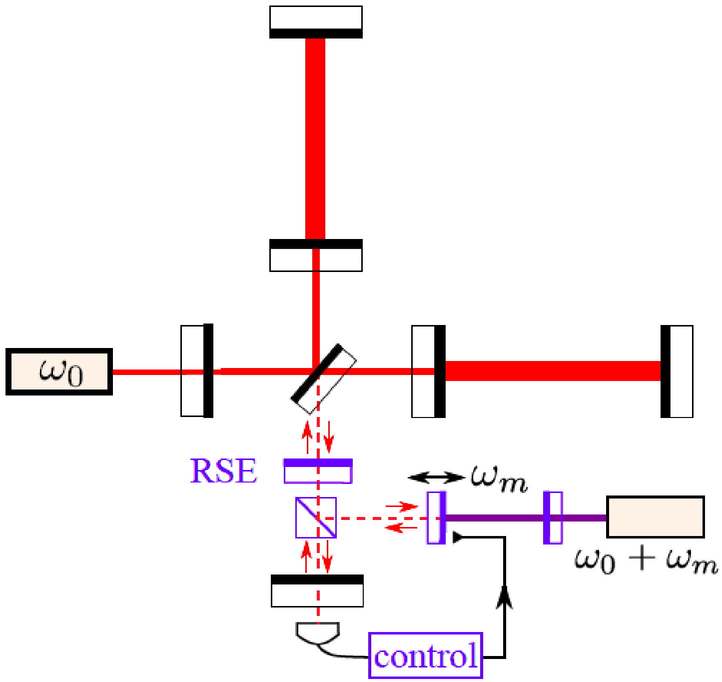


Using feedback to stabilize the whole system

Reference: H. Miao, Y. Ma, C. Zhao and Y. Chen (in preparation).

Unstable filter with phase advance

Cancelling arm cavity propagation phase delay:



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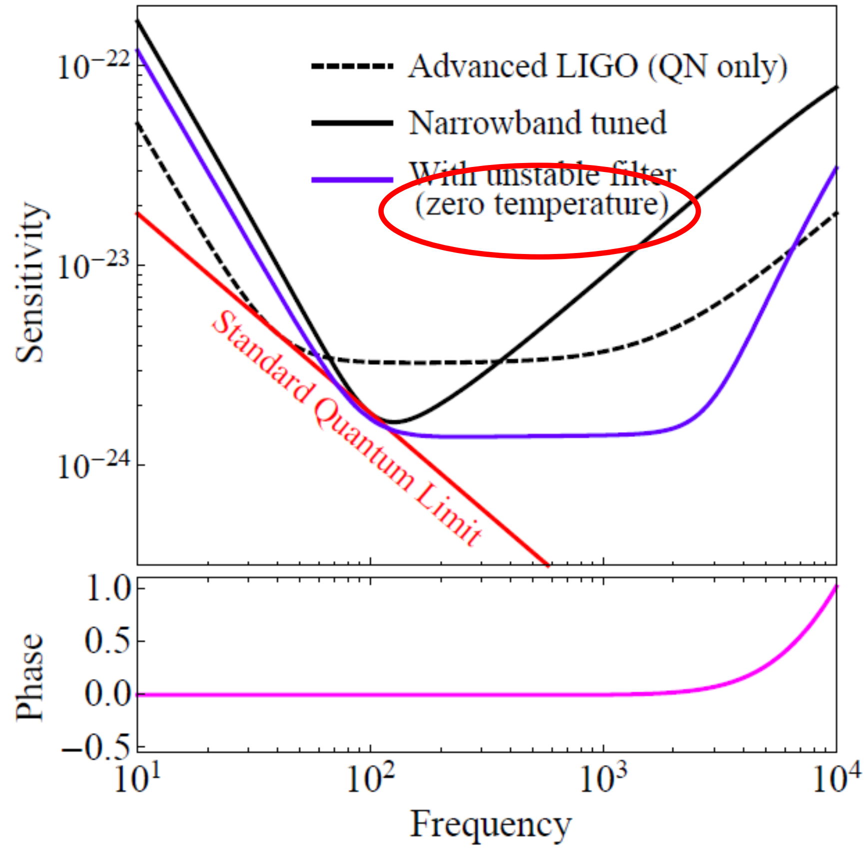

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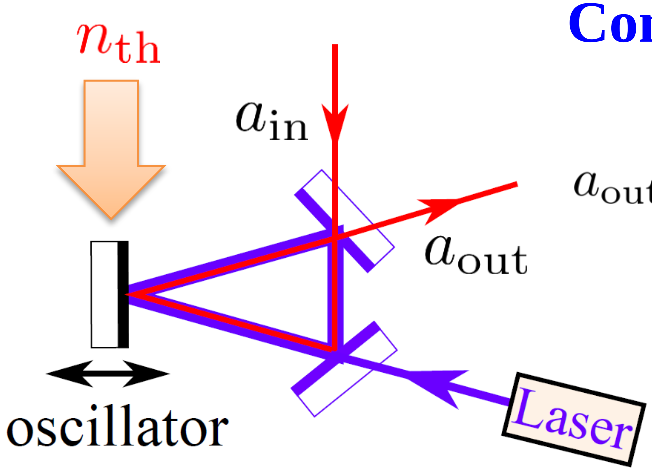
Thermal noise requirement



Complete input-output relation:

$$a_{\rm out}(\Omega) = \frac{i(\pm\gamma_{\rm opt} - \gamma_m) - (\Omega - \Delta_f)}{i(\pm\gamma_{\rm opt} + \gamma_m) + (\Omega - \Delta_f)} a_{\rm in}(\Omega) + \frac{2\sqrt{\gamma_m \gamma_{\rm opt}}}{i(\gamma_{\rm opt} + \gamma_m) + (\Omega - \Delta_f)} n_{\rm th}(\Omega)$$

For thermal noise to be small (at the quantum level):

$$\frac{k_B T}{\hbar \gamma_{\rm opt} Q_m} < 1$$

Order of magnitude estimate:

$$T < 2{\rm mK} \left(\frac{Q_m}{10^6}\right) \left(\frac{\gamma_{\rm opt}/(2\pi)}{50{\rm Hz}}\right)$$

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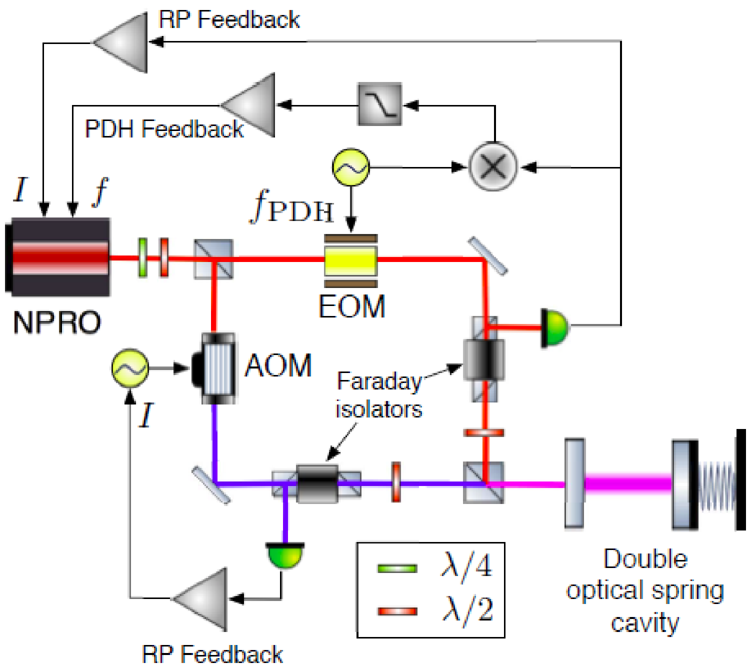
Optical dilution idea



Optical spring effect:

$$\omega_m \rightarrow \sqrt{\omega_{\mathrm{opt}}^2 + \omega_m^2}$$

$$\frac{\omega_{\mathrm{opt}}}{\omega_m} \approx 10^2 \sim 10^3$$

Suppressing radiation pressure noise via feedback:

$$Q_m^{\mathrm{new}} \approx \frac{\omega_{\mathrm{opt}}}{\gamma_m} \gg Q_m$$

New requirement using optical dilution idea:

$$T < 2\mathrm{K} \left(\frac{Q_m^{\mathrm{new}}}{10^9}\right) \left(\frac{\gamma_{\mathrm{opt}}/(2\pi)}{50\mathrm{Hz}}\right)$$ **Still challenging**

Reference: Z. Korth, H. Miao, T. Corbitt, G. Cole, Y. Chen, and R. Adhikari PRA **88**, 033805 (2013)

The end

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