Direct measurement of optical coating thermal noise on a large frequency range

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ET Symposium – Lyon – november 19th, 2014

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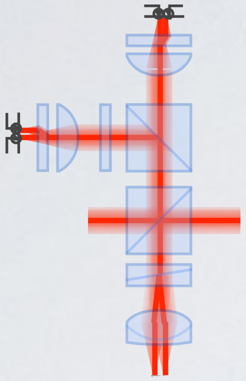
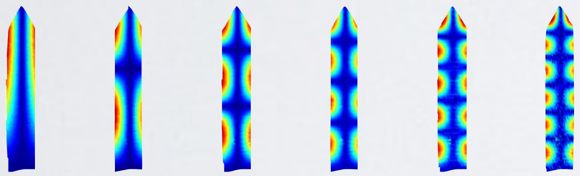
Outline



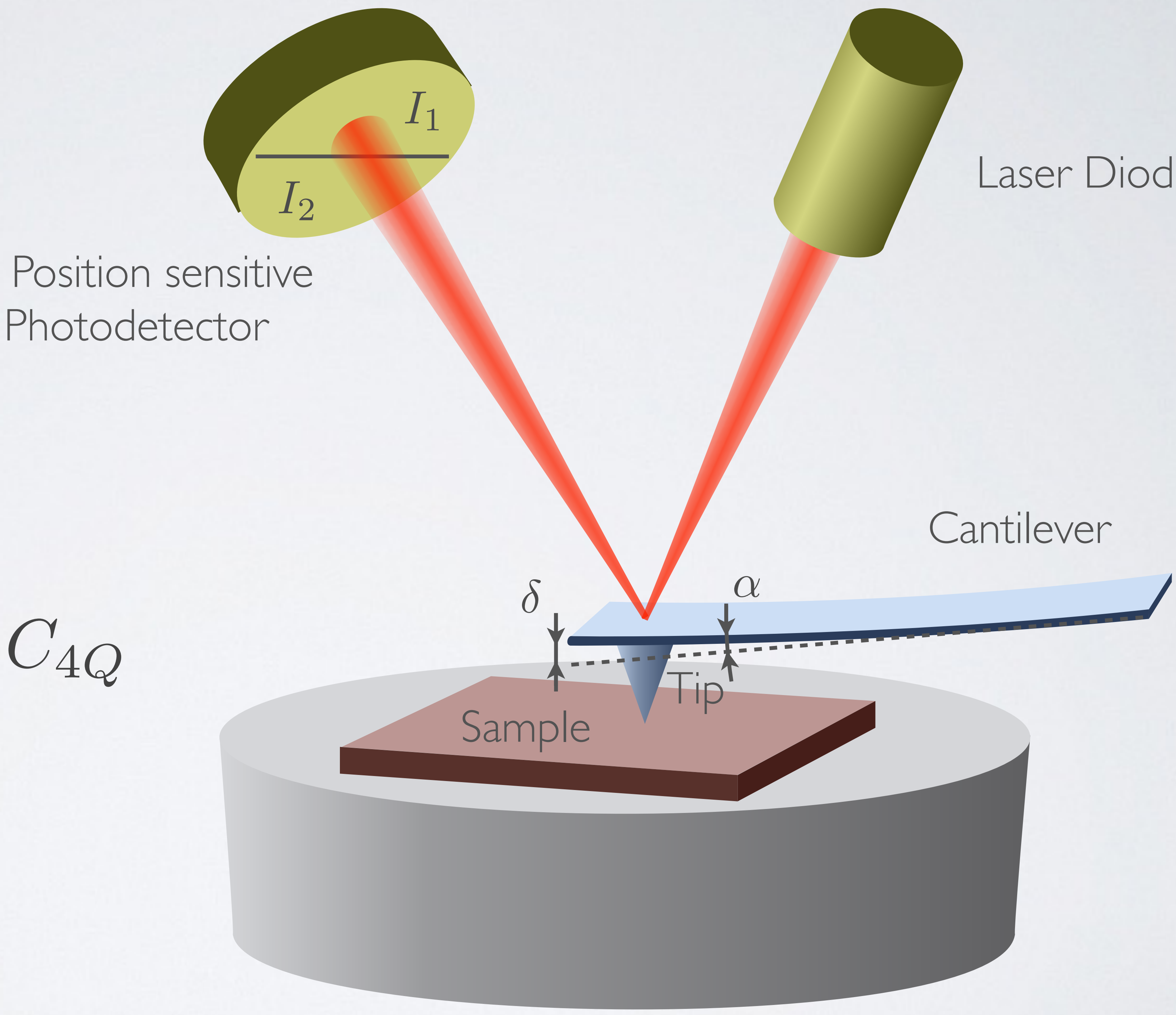
- Quadrature phase interferometry
  - Experimental setup
  - Measurement of thermal noise



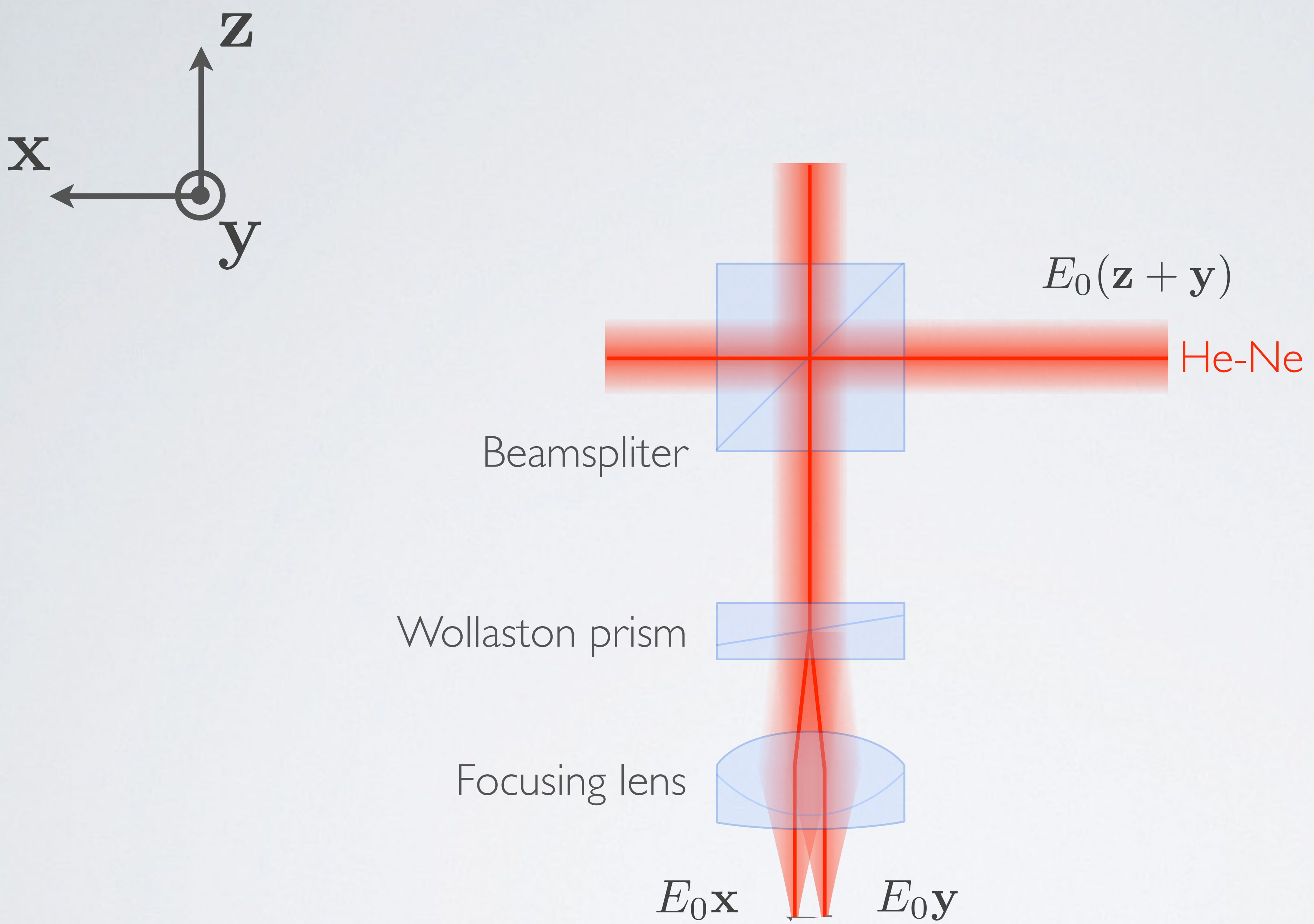
- Micro-cantilever response from thermal noise
  - Full measurement of response with Kramers-Kronig relations
  - Viscoelasticity of coating layer

$$C_{4Q} = \frac{I_1 - I_2}{I_1 + I_2}$$

$$\delta = \left(\frac{\partial \delta}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial C}\right) C_{4Q}$$

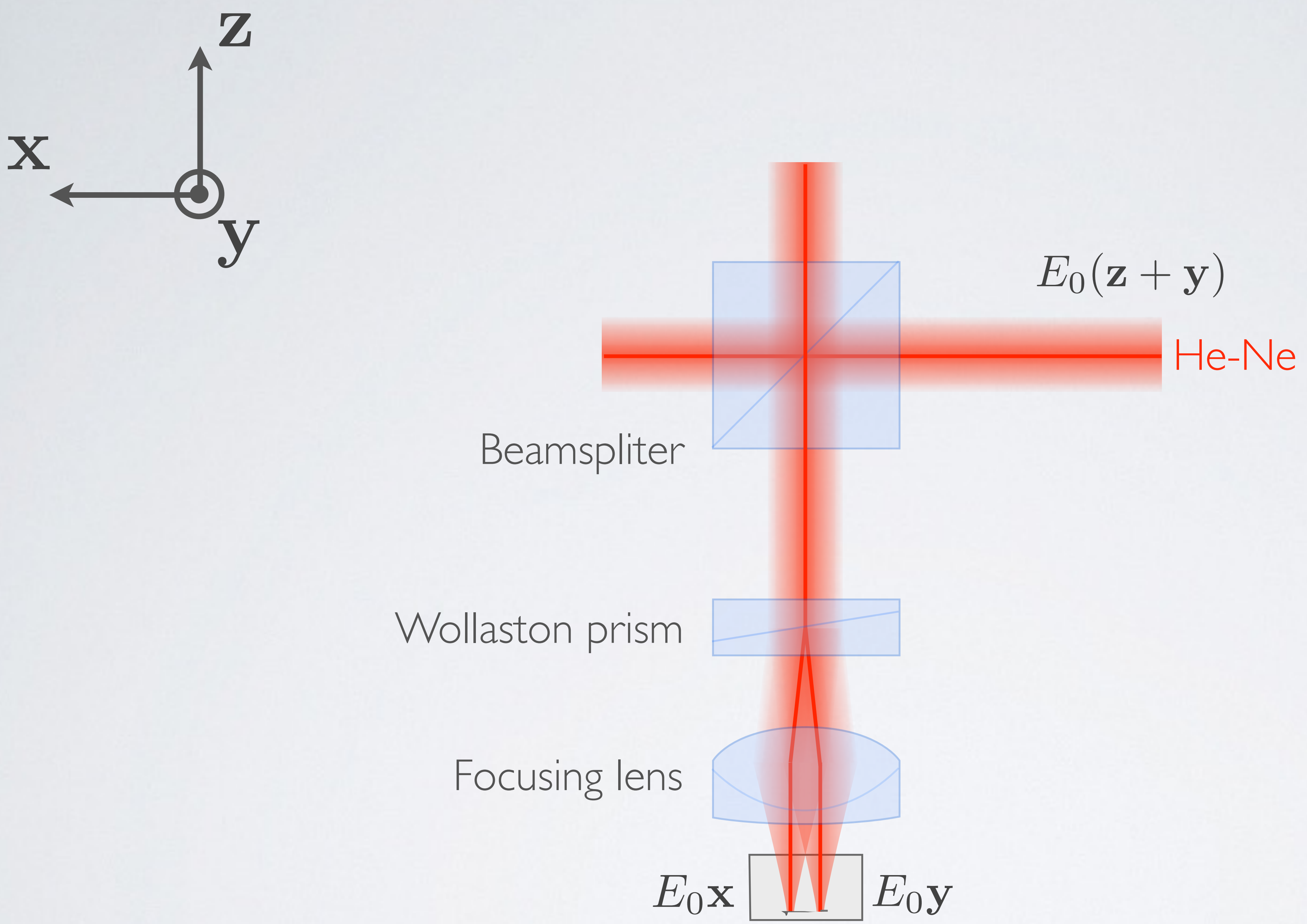


Interferometer: measurement area



C. Schonenberger and S. F. Alvarado, *Review of Scientific Instruments* **60**, 3131–3134, 1989.

Interferometer: measurement area



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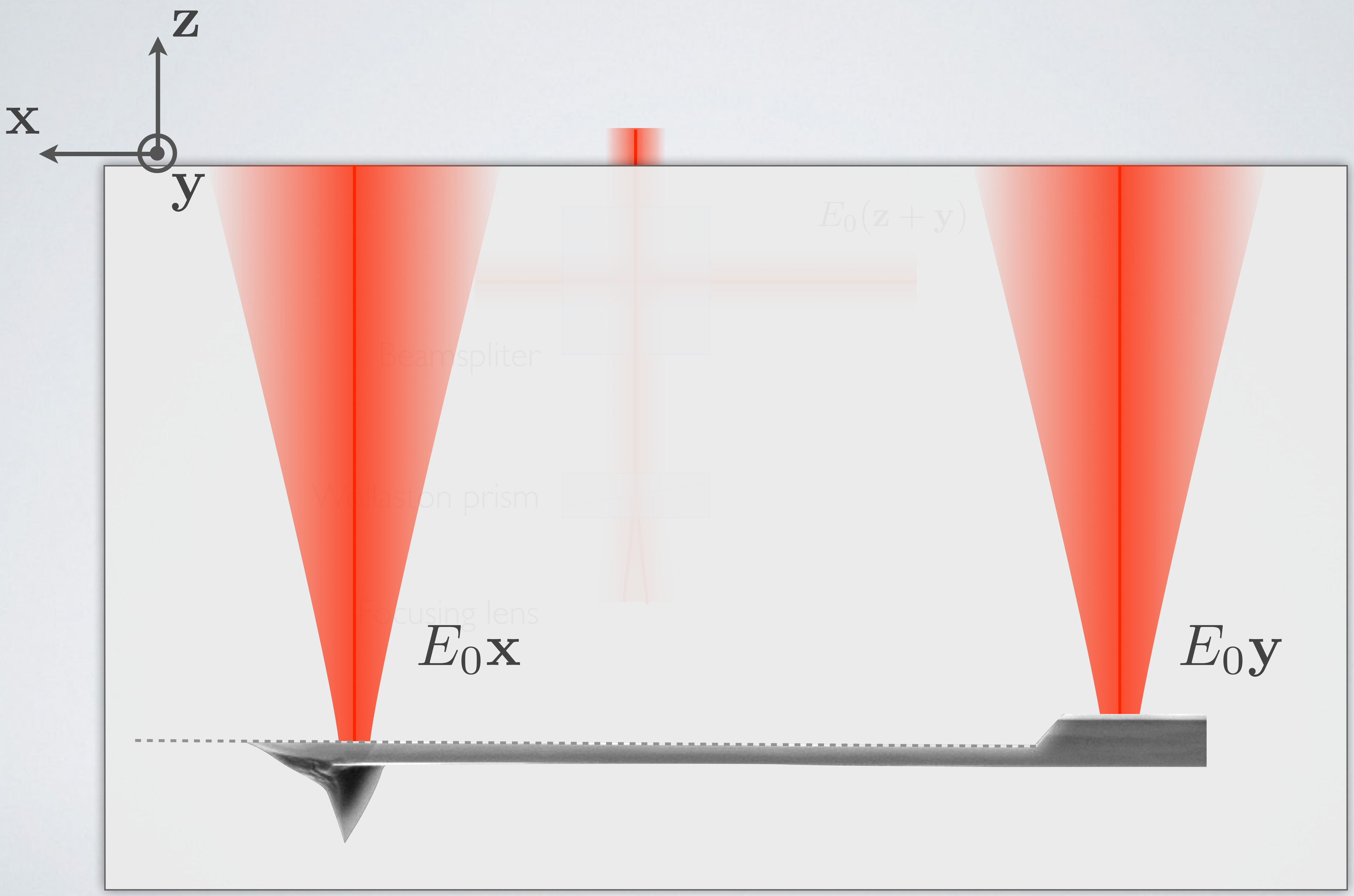
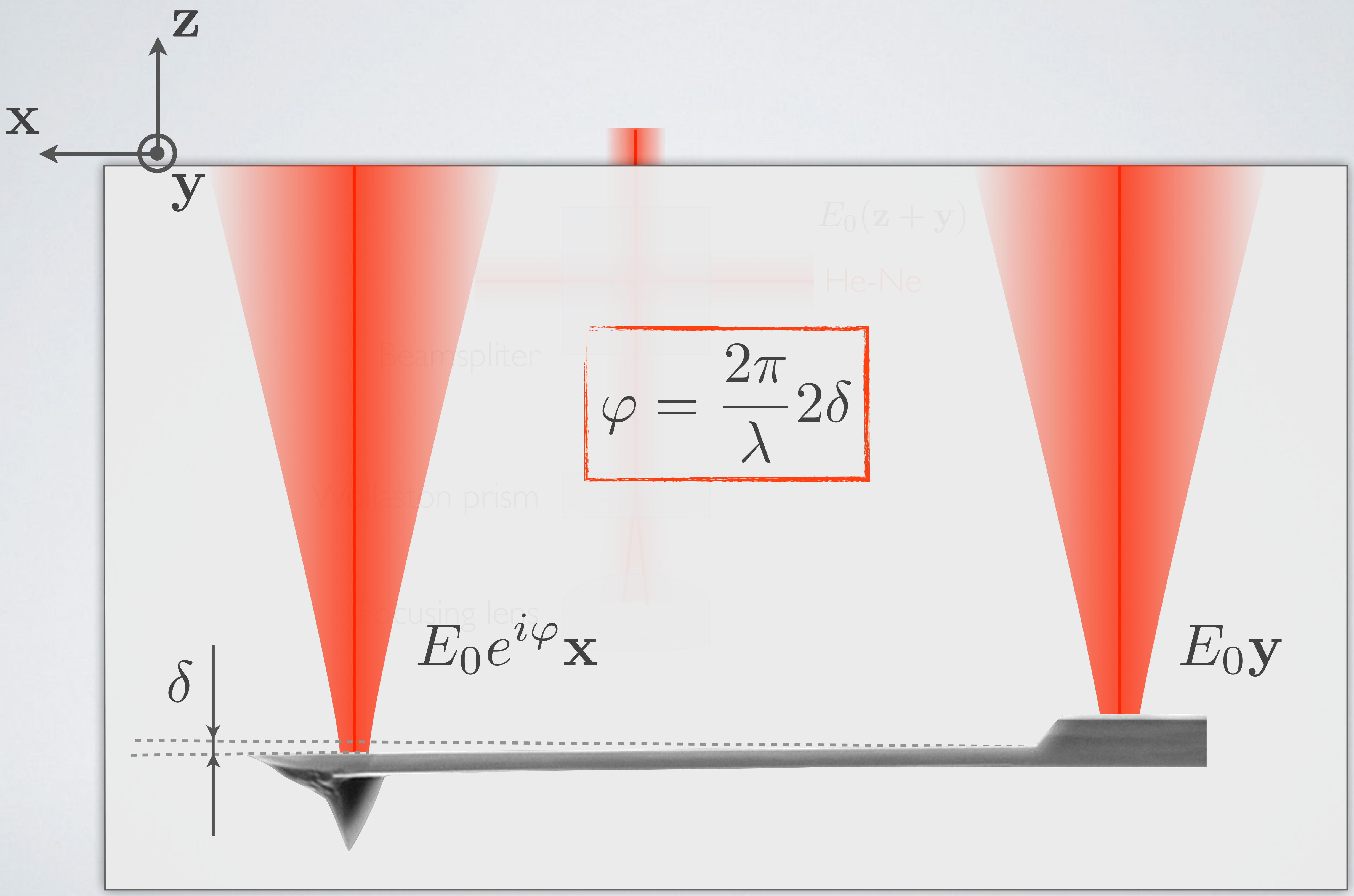
Interferometer: measurement area



C. Schonenberger and S. F. Alvarado, *Review of Scientific Instruments* **60**, 3131–3134, 1989.

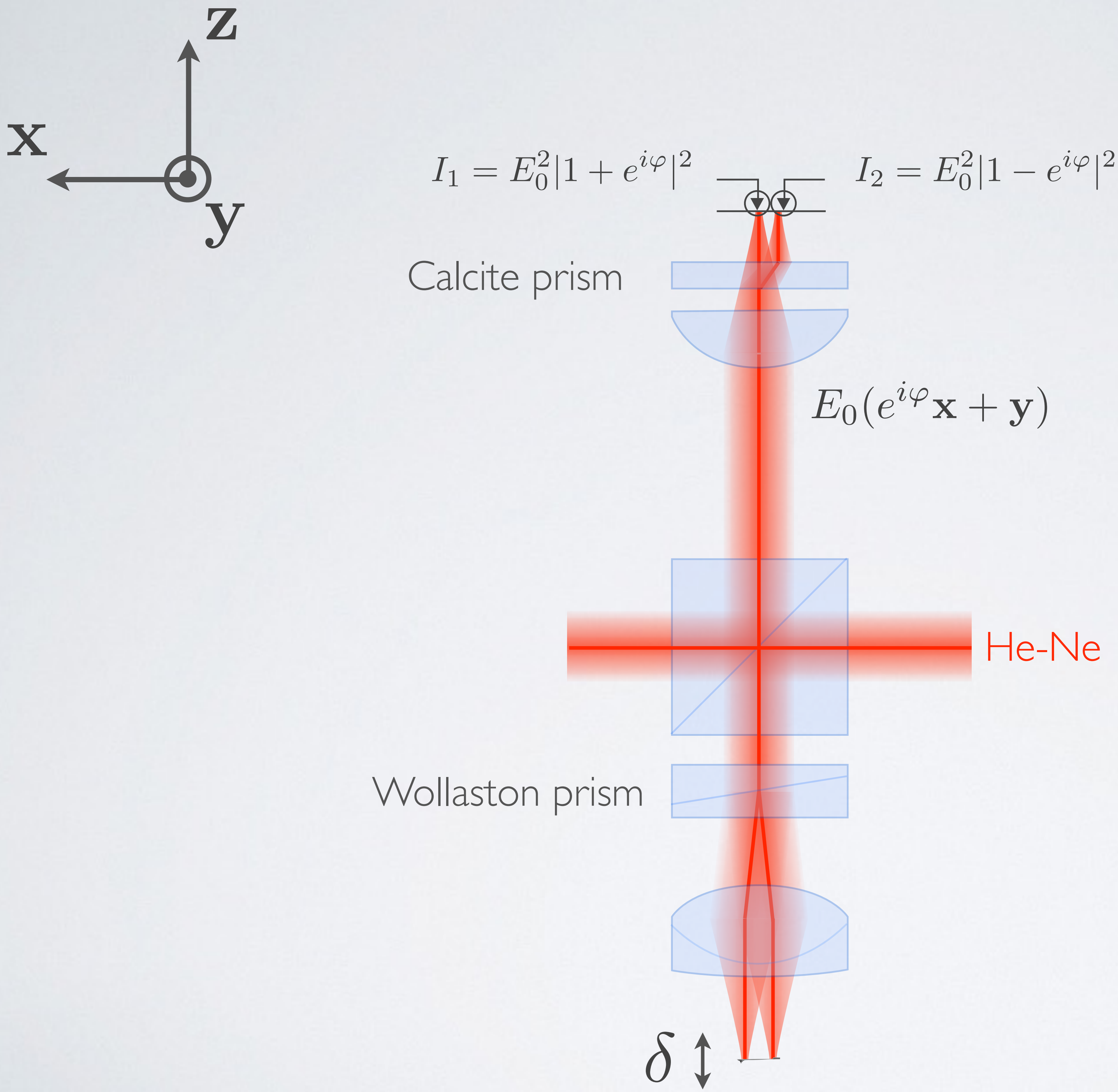
Interferometer: measurement area



$$\varphi = \frac{2\pi}{\lambda} 2\delta$$

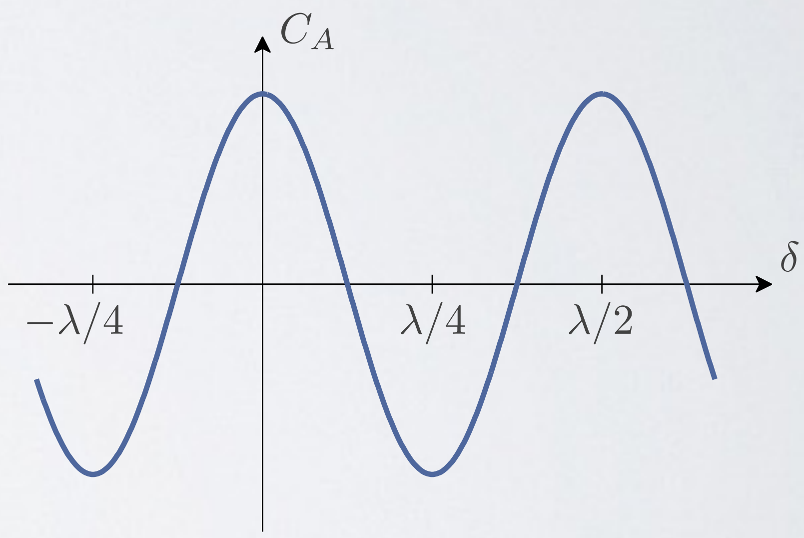
C. Schonenberger and S. F. Alvarado, *Review of Scientific Instruments* **60**, 3131–3134, 1989.

Interferometer: analysis area



Photodiodes A

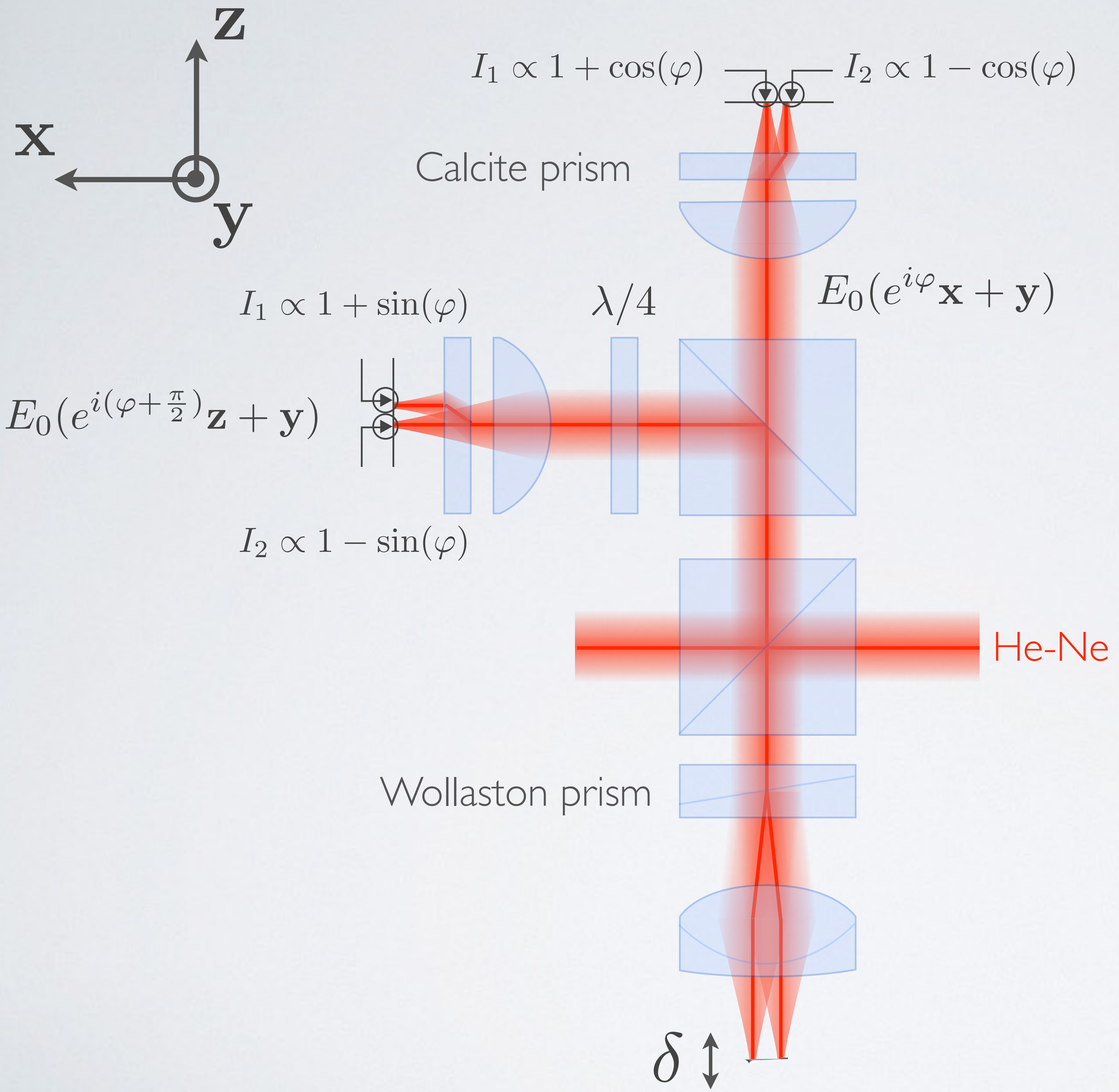
$$C_A = \frac{I_1 - I_2}{I_1 + I_2} = \cos(\varphi)$$



$$\varphi = \frac{2\pi}{\lambda} 2\delta$$

C. Schonenberger and S. F. Alvarado, *Review of Scientific Instruments* **60**, 3131–3134, 1989.

Interferometer: analysis area

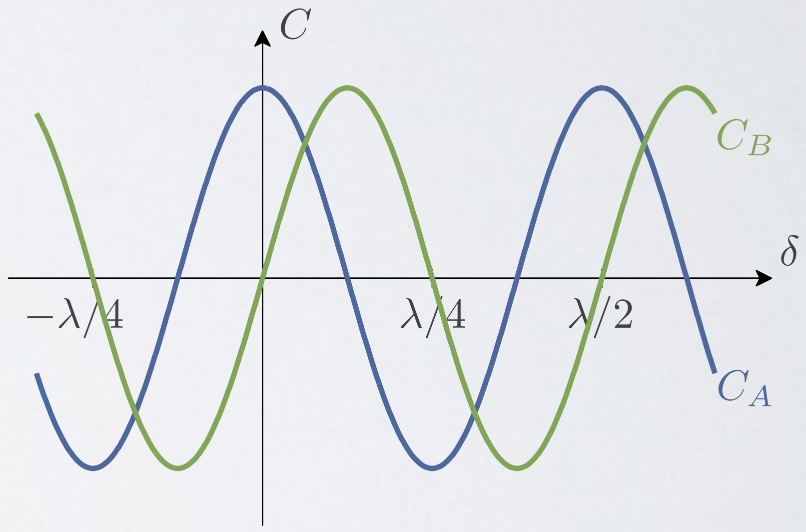


Photodiodes A

$$C_A = \cos(\varphi)$$

Photodiodes B

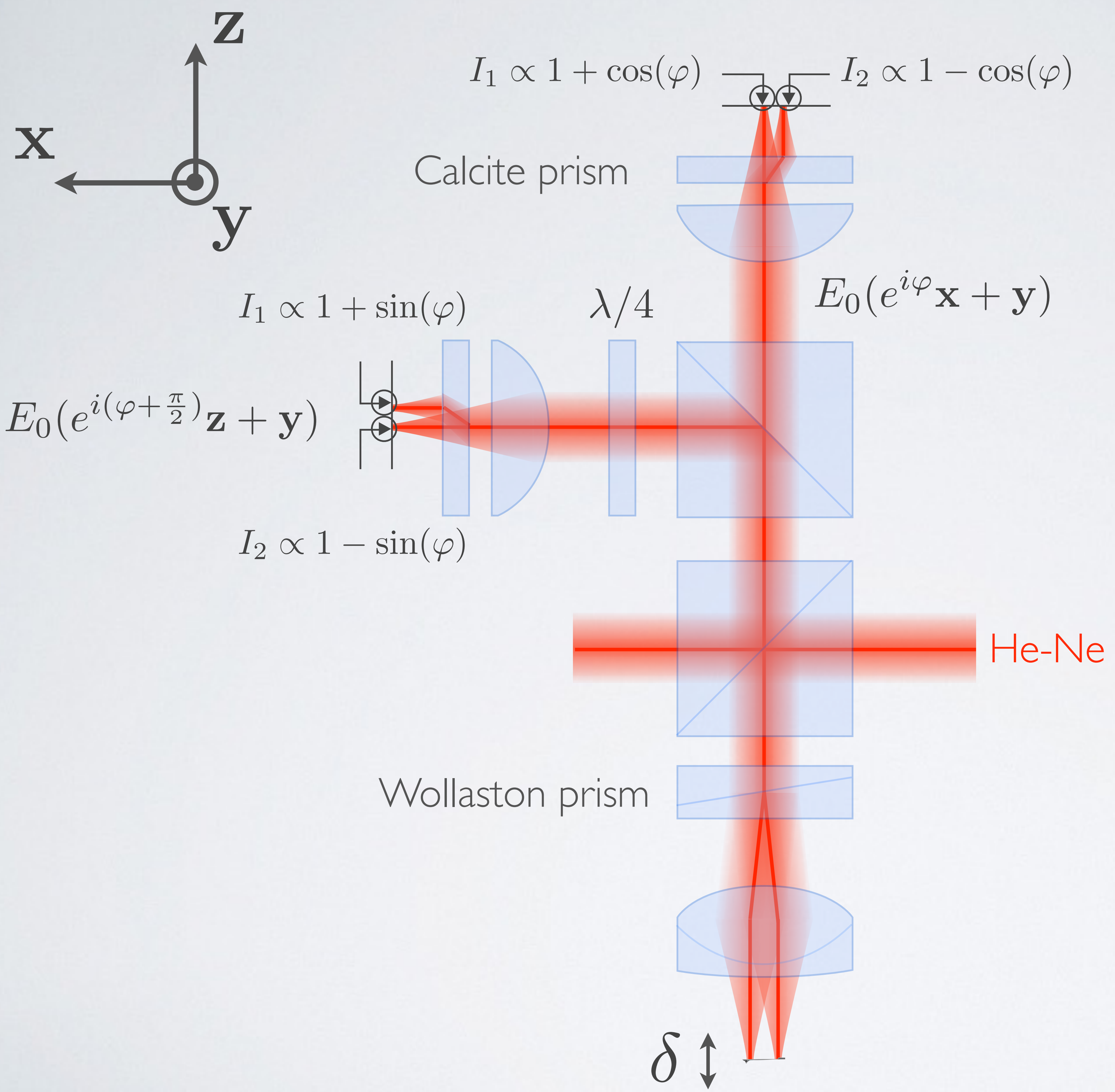
$$C_B = \sin(\varphi)$$



$$\varphi = \frac{2\pi}{\lambda} 2\delta$$

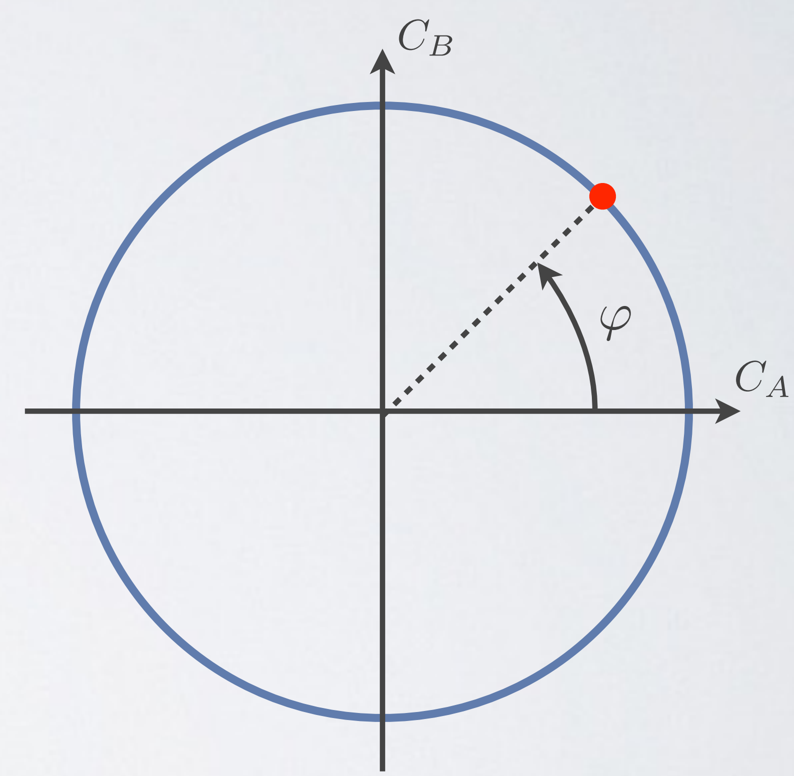
P. Paolino, F. Aguilar, L. Bellon, *Review of Scientific Instruments* **84**, 095001, 2013.

Interferometer: analysis area



Photodiodes A & B

$$C^\star = C_A + iC_B = e^{i\varphi}$$



$$\varphi = \frac{2\pi}{\lambda} 2\delta$$

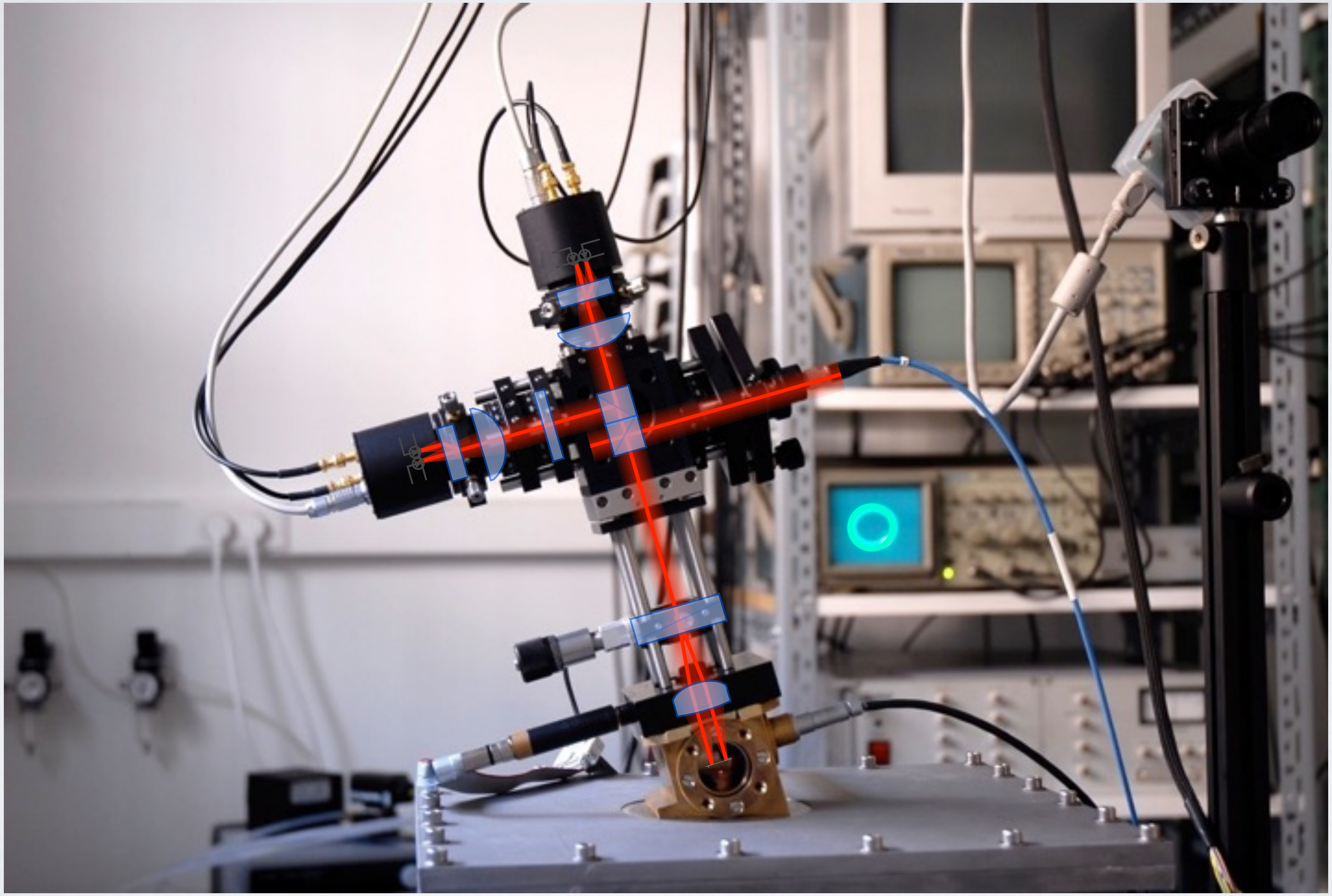
P. Paolino, F. Aguilar, L. Bellon, *Review of Scientific Instruments* **84**, 095001, 2013.

Interferometer: realisation

F. Vittoz
Atelier mécanique

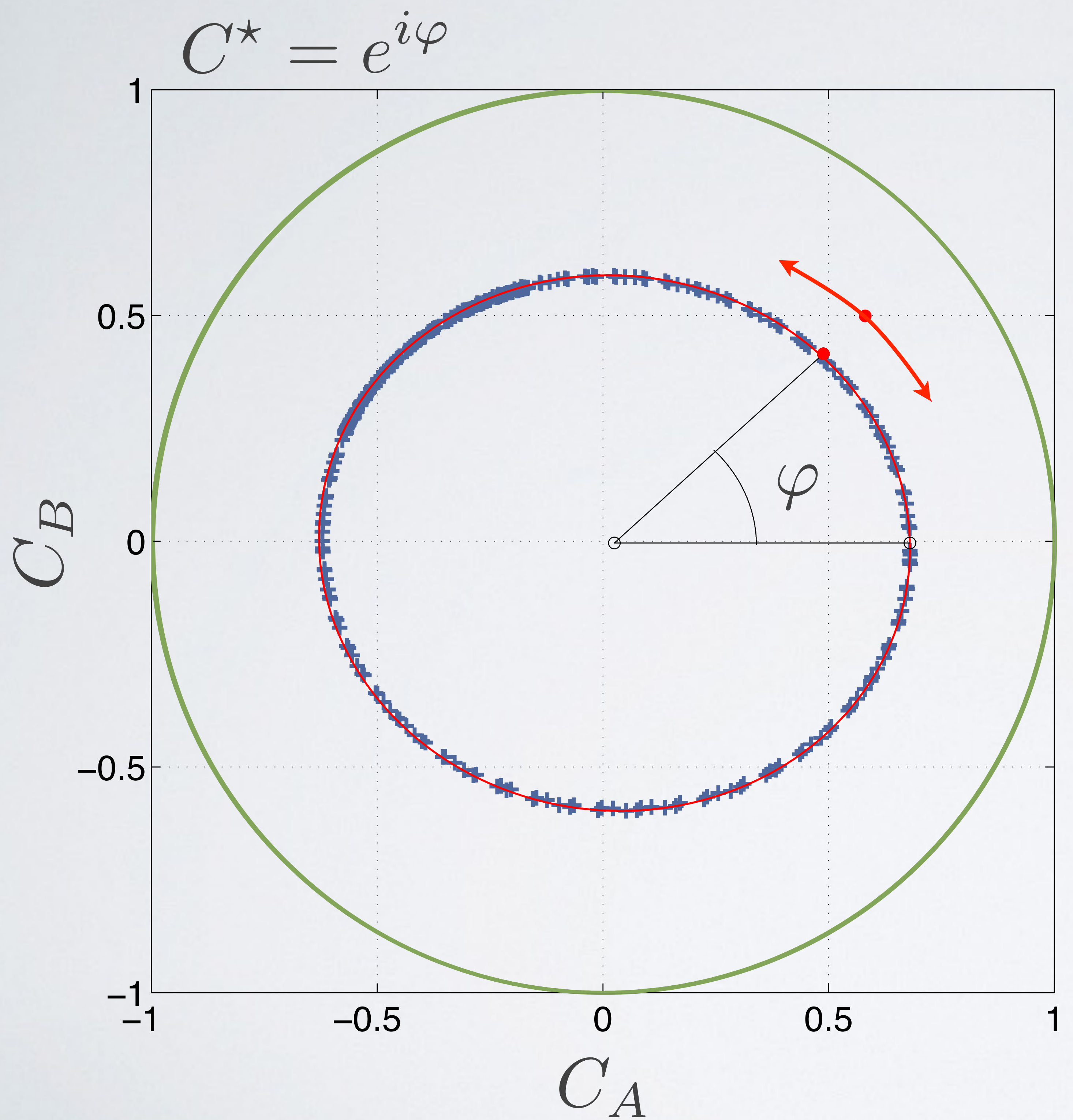
F. Ropars
Atelier électronique

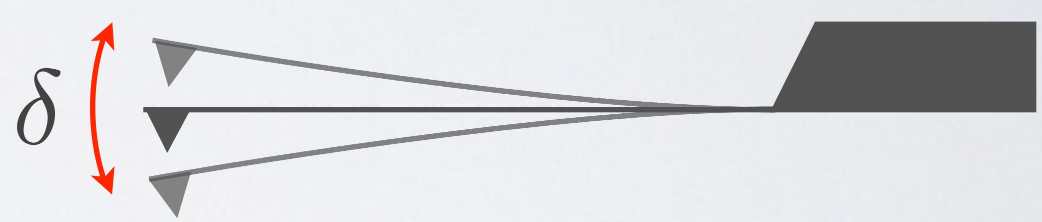
P. Paolino, F. Aguilar
Doctorants



P. Paolino, F. Aguilar, L. Bellon, *Review of Scientific Instruments* **84**, 095001, 2013.

Interferometer: calibration

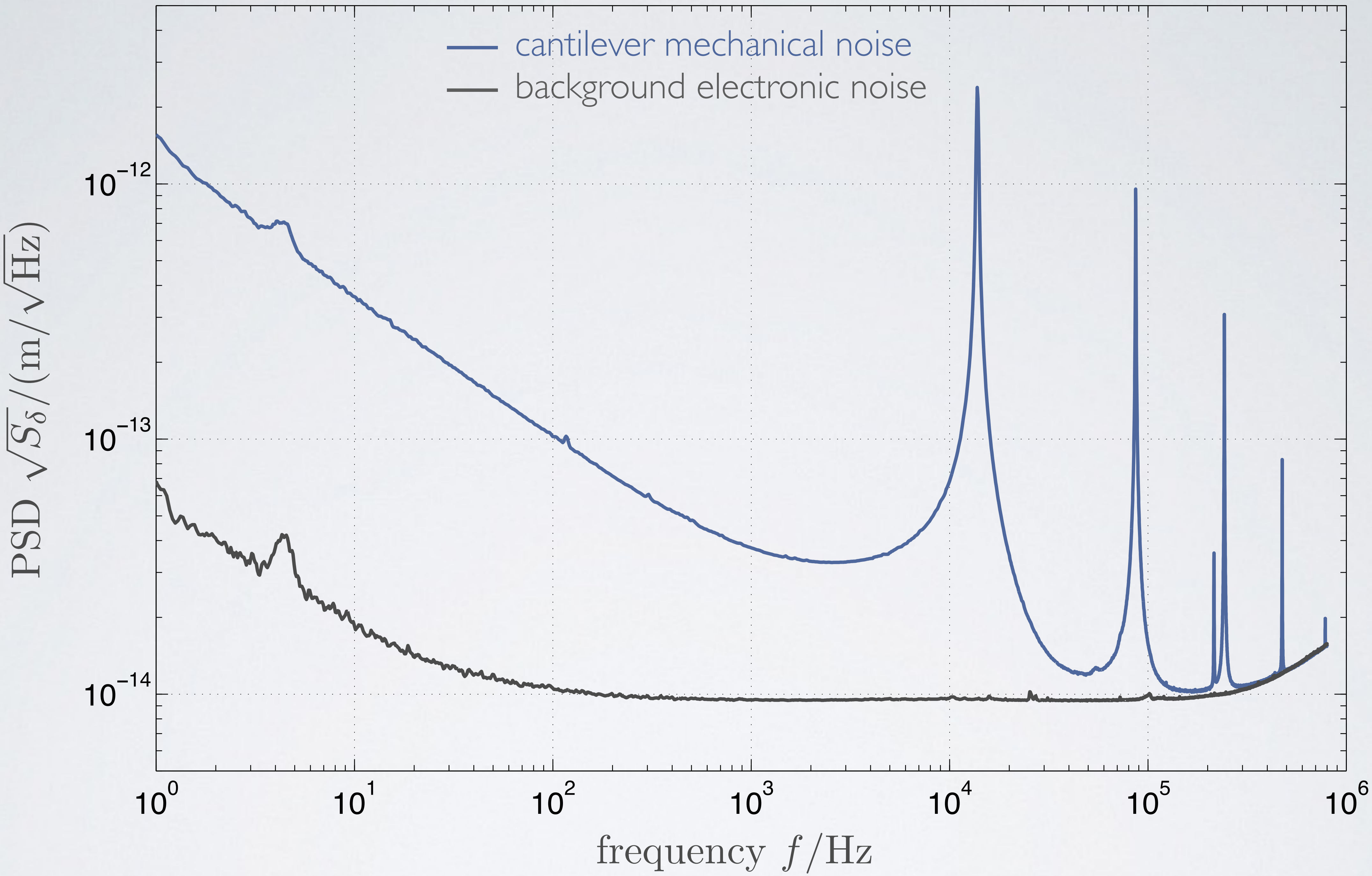




$$\varphi = \frac{2\pi}{\lambda} 2\delta$$

P. Paolino, F. Aguilar, L. Bellon, *Review of Scientific Instruments* **84**, 095001, 2013.

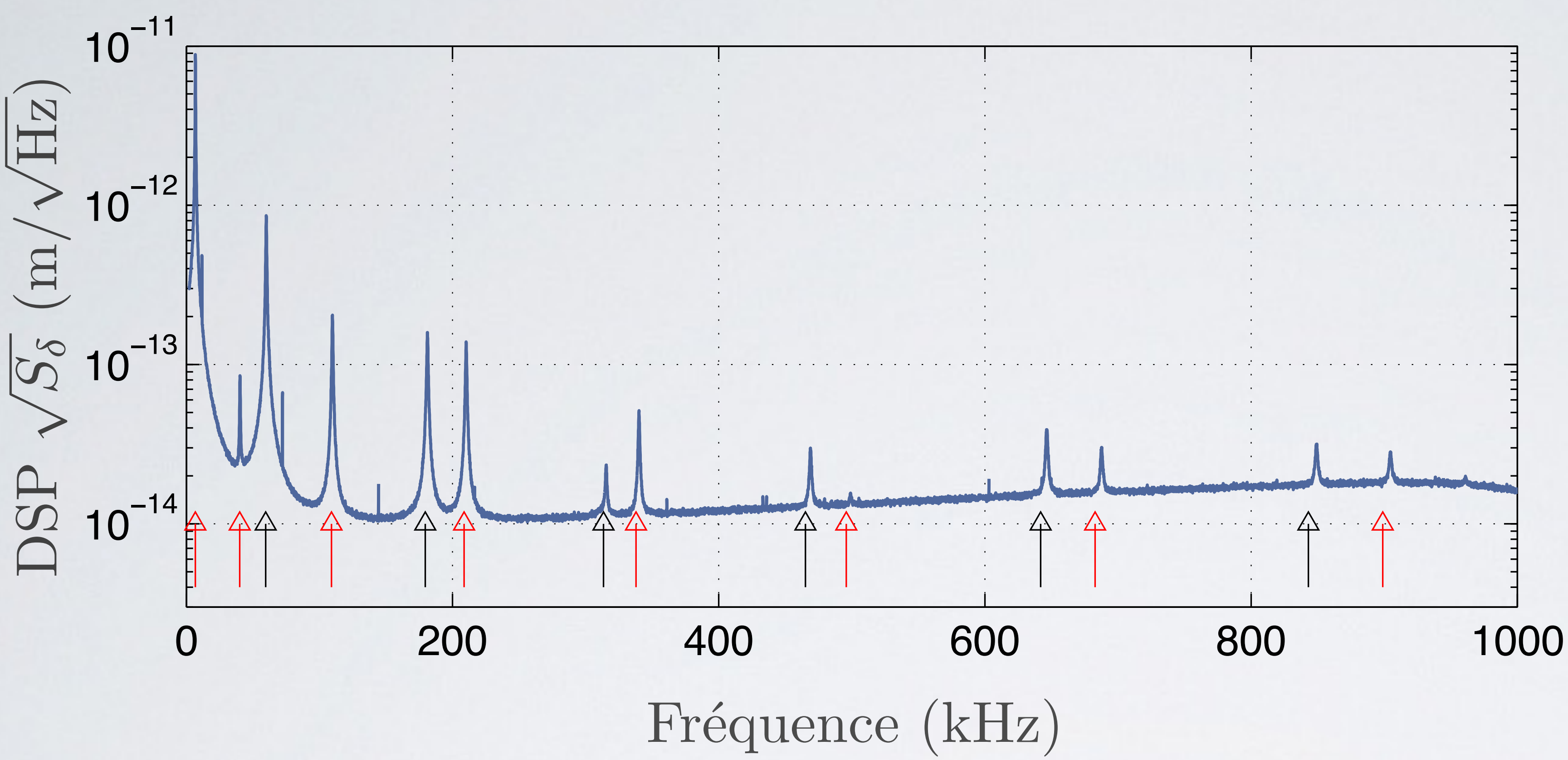
Noise measurement

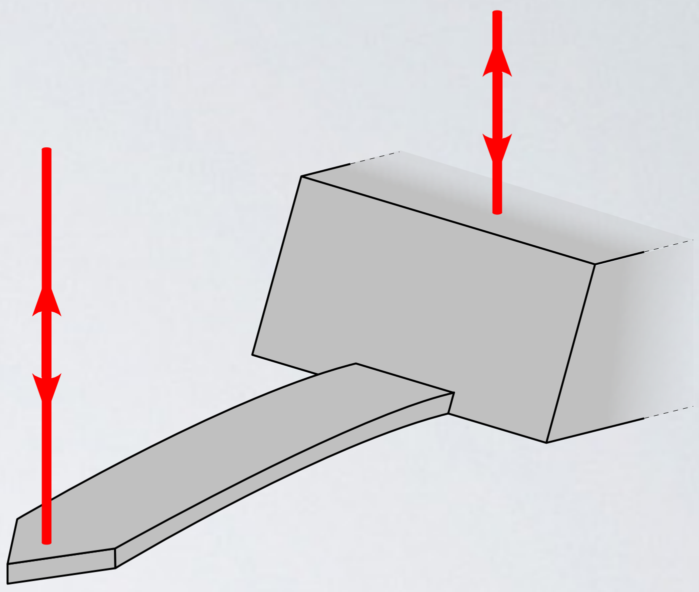


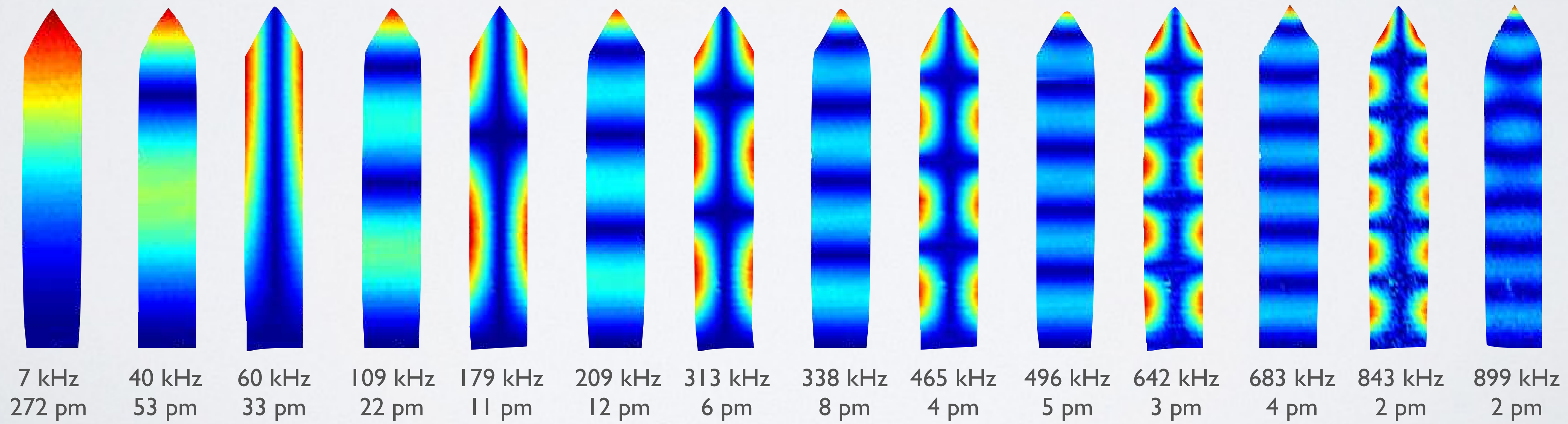
$$S_\delta(f) = \lim_{T\to\infty} \left\langle \frac{1}{2T} \left| \int_{-T}^{T} \delta(t) e^{i\omega t} dt \right|^2 \right\rangle$$

$$\langle \delta(t)^2 \rangle = \int_{\Delta f} S_\delta(f) df$$

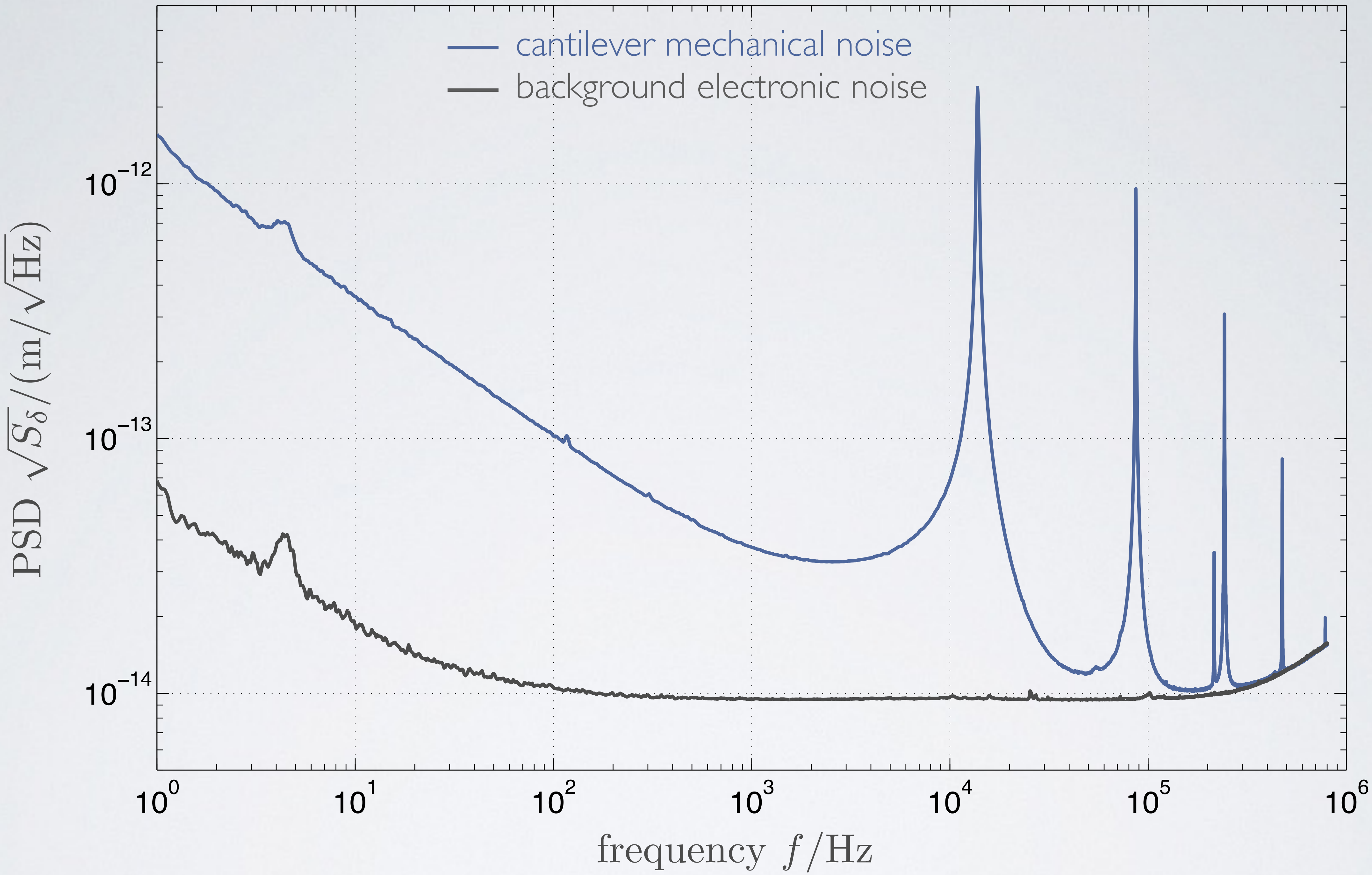
Mapping of thermal fluctuations





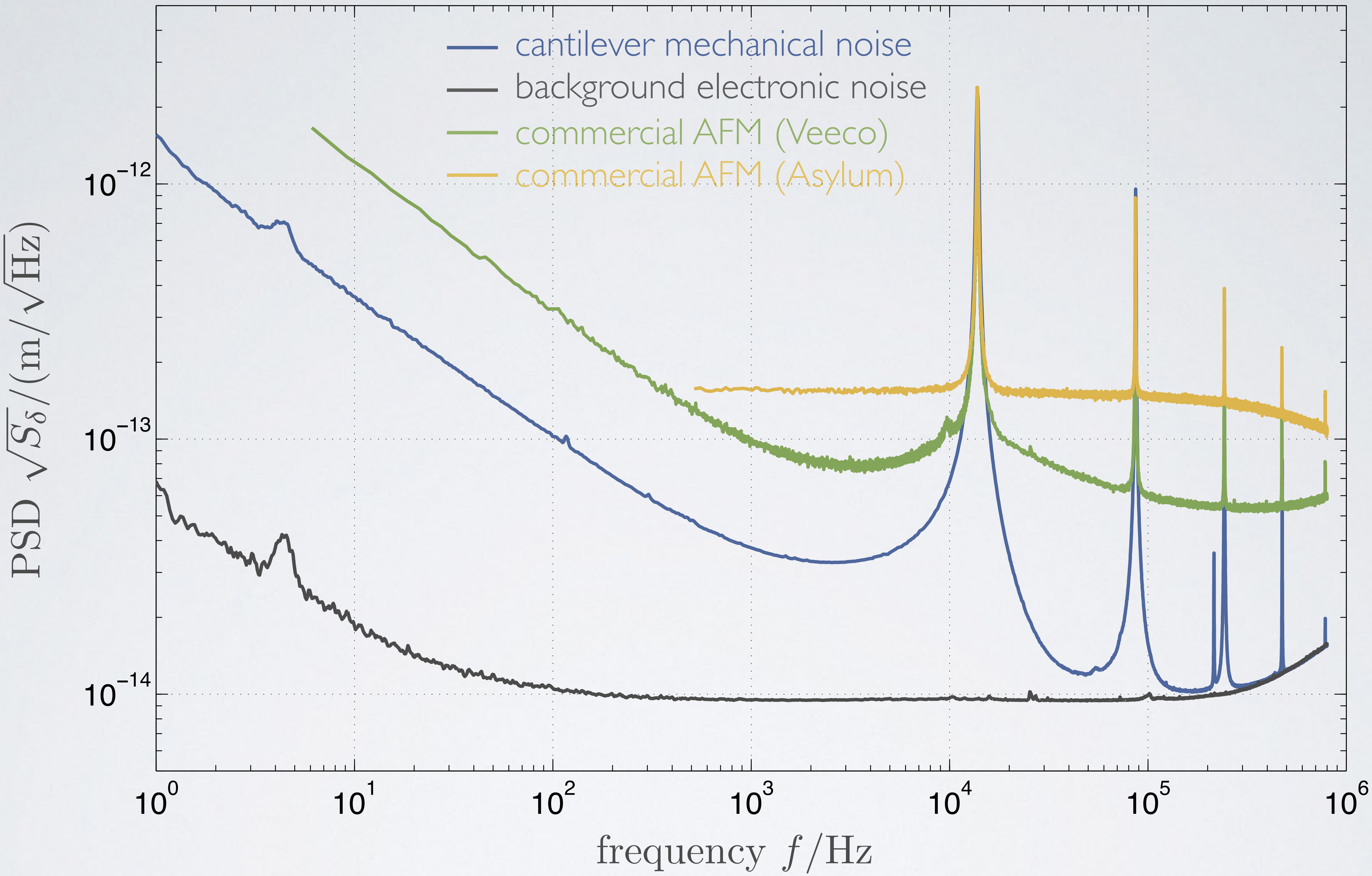


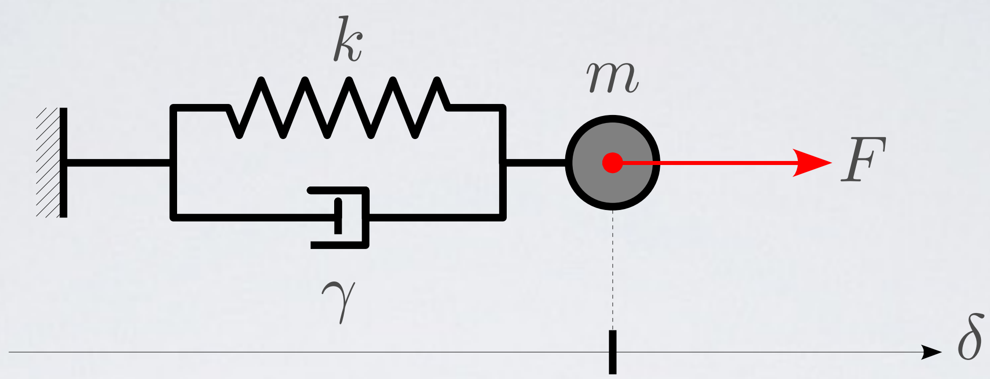
Noise measurement



light power : 100 µW - spot size : 5µm

Noise measurement





$$m\ddot{\delta}(t) = -k\delta(t) - \gamma\dot{\delta}(t) + F(t)$$

Response function

$$G_{\mathrm{SHO}}(\omega) = \frac{F(\omega)}{\delta(\omega)} = k - m\omega^2 + i\gamma\omega \longrightarrow G_{\mathrm{Sader}}(\omega) = k - m_{\mathrm{eff}}(\omega)\omega^2 + i\gamma_{\mathrm{eff}}(\omega)\omega$$

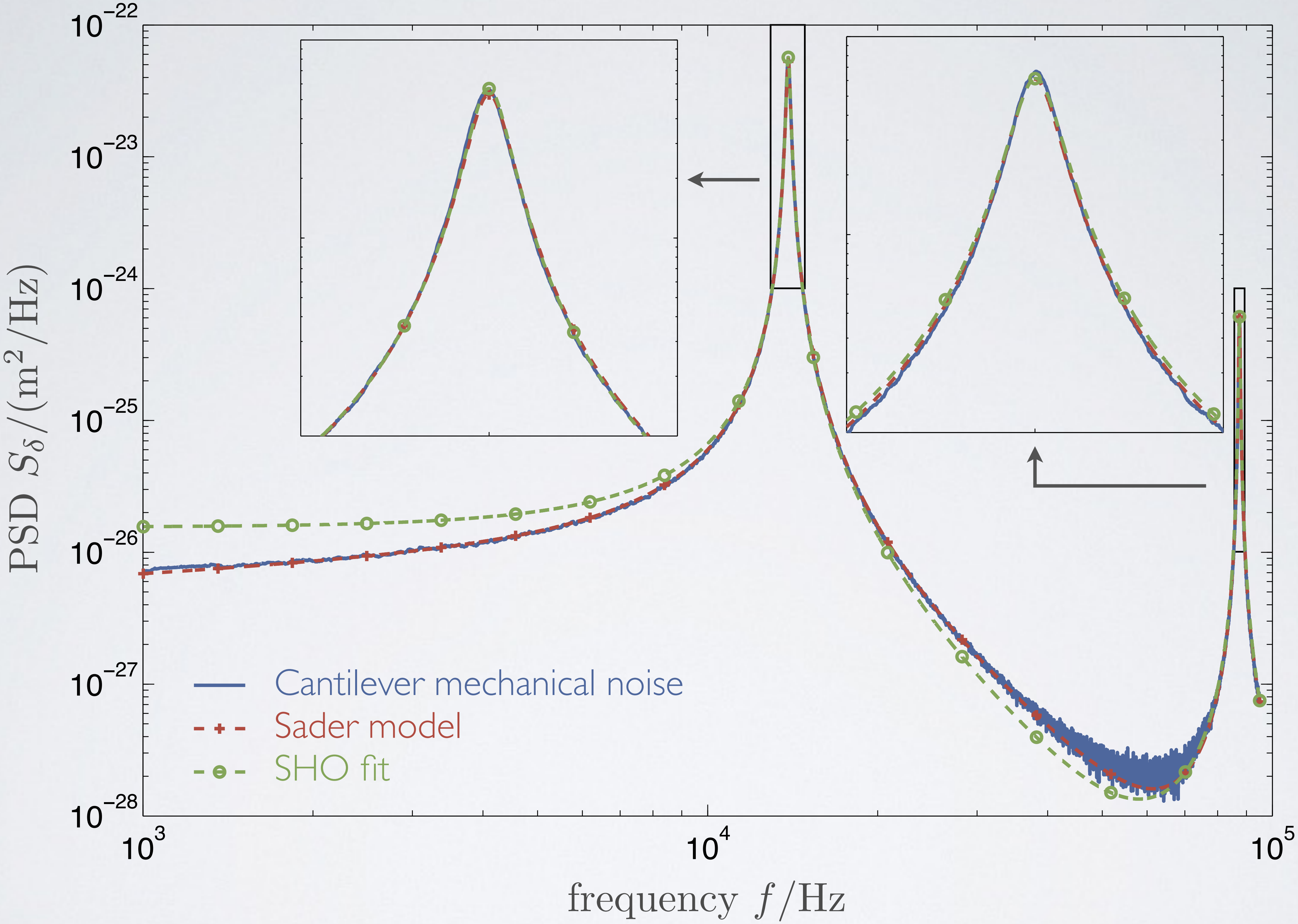
SHO model

Hydrodynamic loading model

Fluctuation-dissipation theorem

$$S_\delta(\omega) = -\frac{2k_BT}{\pi\omega}\mathrm{Im}\left[\frac{1}{G_{\mathrm{Sader}}(\omega)}\right] = \frac{2k_BT}{\pi}\frac{\gamma_{\mathrm{eff}}}{(k - m_{\mathrm{eff}}\omega^2)^2 + (\gamma_{\mathrm{eff}}\omega)^2}$$

Modeling noise : SHO and Sader model



From noise to response measurement

Mechanical response

model response

$$G_{\mathrm{Sader}}(\omega) = k - m_{\mathrm{eff}}\omega^2 + i\gamma_{\mathrm{eff}}\omega$$

FDT →

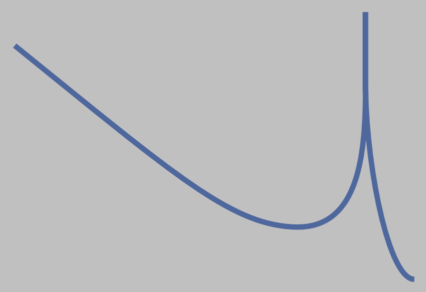
Thermal noise spectrum

theoretical spectrum

$$S_\delta(\omega) \propto \mathrm{Im}\left[\frac{1}{G_{\mathrm{Sader}}(\omega)}\right]$$

↕ *Compare*

measured spectrum



From noise to response measurement

Mechanical response

model response

$$G_{\text{Sader}}(\omega) = k - m_{\text{eff}}\omega^2 + i\gamma_{\text{eff}}\omega$$

measured response

$$\text{Im}\left[\frac{1}{G(\omega)}\right]$$

FDT →

← **FDT^{-1}**

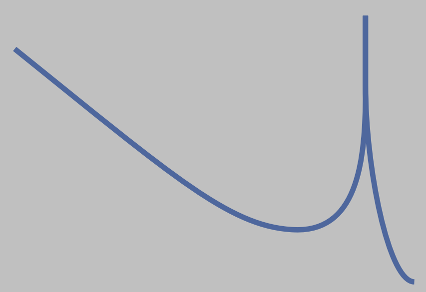
Thermal noise spectrum

theoretical spectrum

$$S_\delta(\omega) \propto \text{Im}\left[\frac{1}{G_{\text{Sader}}(\omega)}\right]$$

Compare

measured spectrum



From noise to response measurement

Mechanical response

model response

$$G_{\text{Sader}}(\omega) = k - m_{\text{eff}}\omega^2 + i\gamma_{\text{eff}}\omega$$

FDT →

measured response

$$G(\omega) \xleftarrow{\textbf{KK}} \text{Im}\left[\frac{1}{G(\omega)}\right]$$

← **FDT^{-1}**

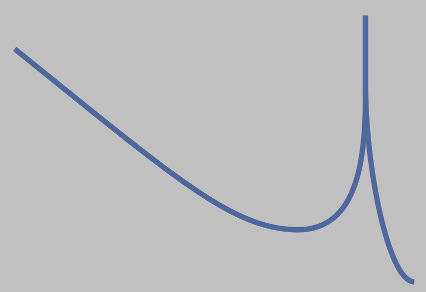
Thermal noise spectrum

theoretical spectrum

$$S_\delta(\omega) \propto \text{Im}\left[\frac{1}{G_{\text{Sader}}(\omega)}\right]$$

Compare

measured spectrum



From noise to response measurement

Mechanical response

model response

$$G_{\text{Sader}}(\omega) = k - m_{\text{eff}}\omega^2 + i\gamma_{\text{eff}}\omega$$

FDT →

Compare

measured response

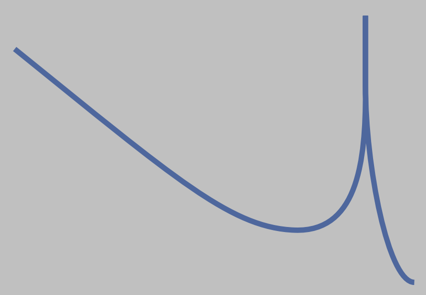
$$G(\omega) \xleftarrow{\textbf{KK}} \text{Im}\left[\frac{1}{G(\omega)}\right]$$

Thermal noise spectrum

theoretical spectrum

$$S_\delta(\omega) \propto \text{Im}\left[\frac{1}{G_{\text{Sader}}(\omega)}\right]$$

measured spectrum

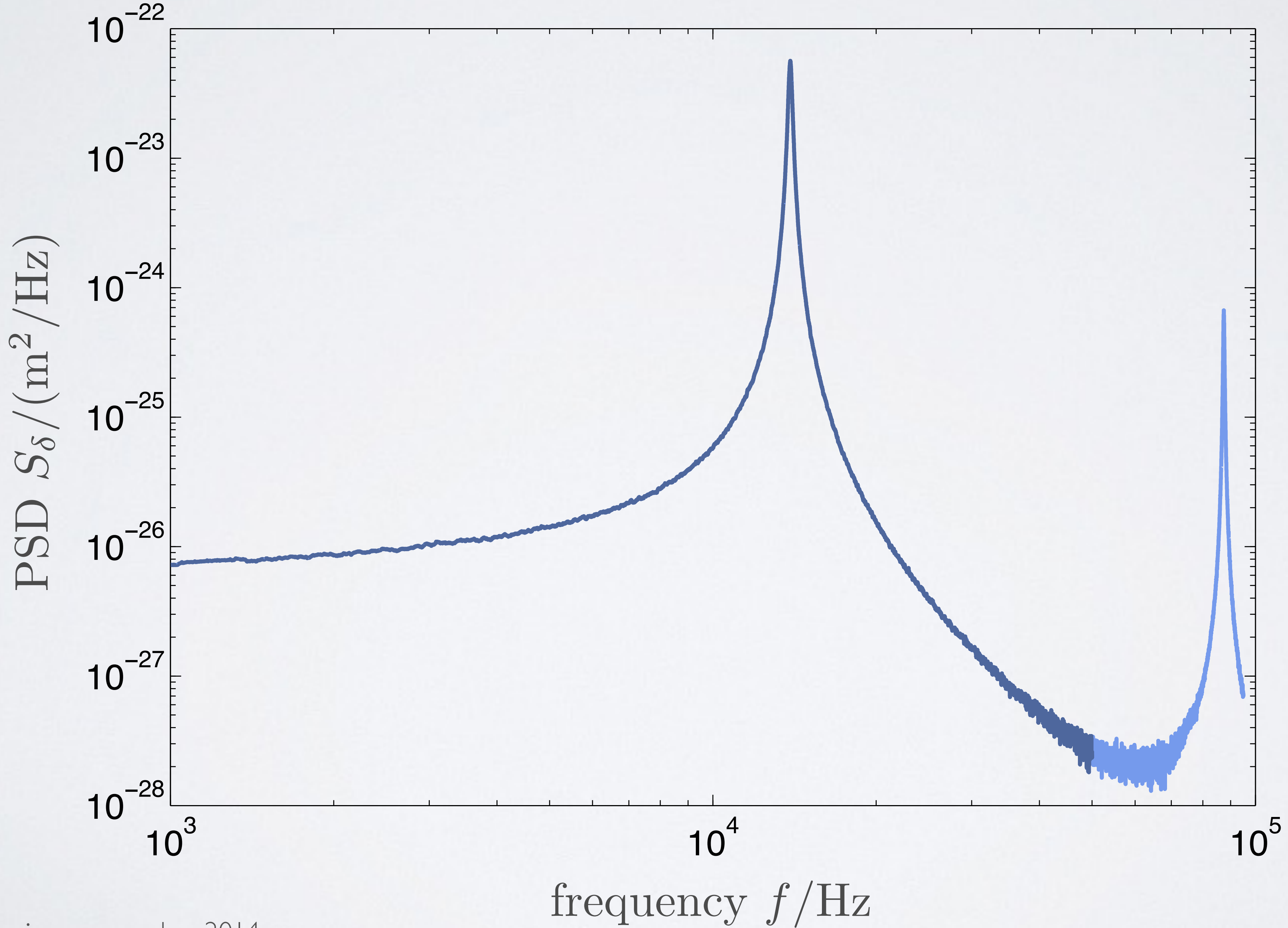


← **FDT^{-1}**

From noise to response measurement

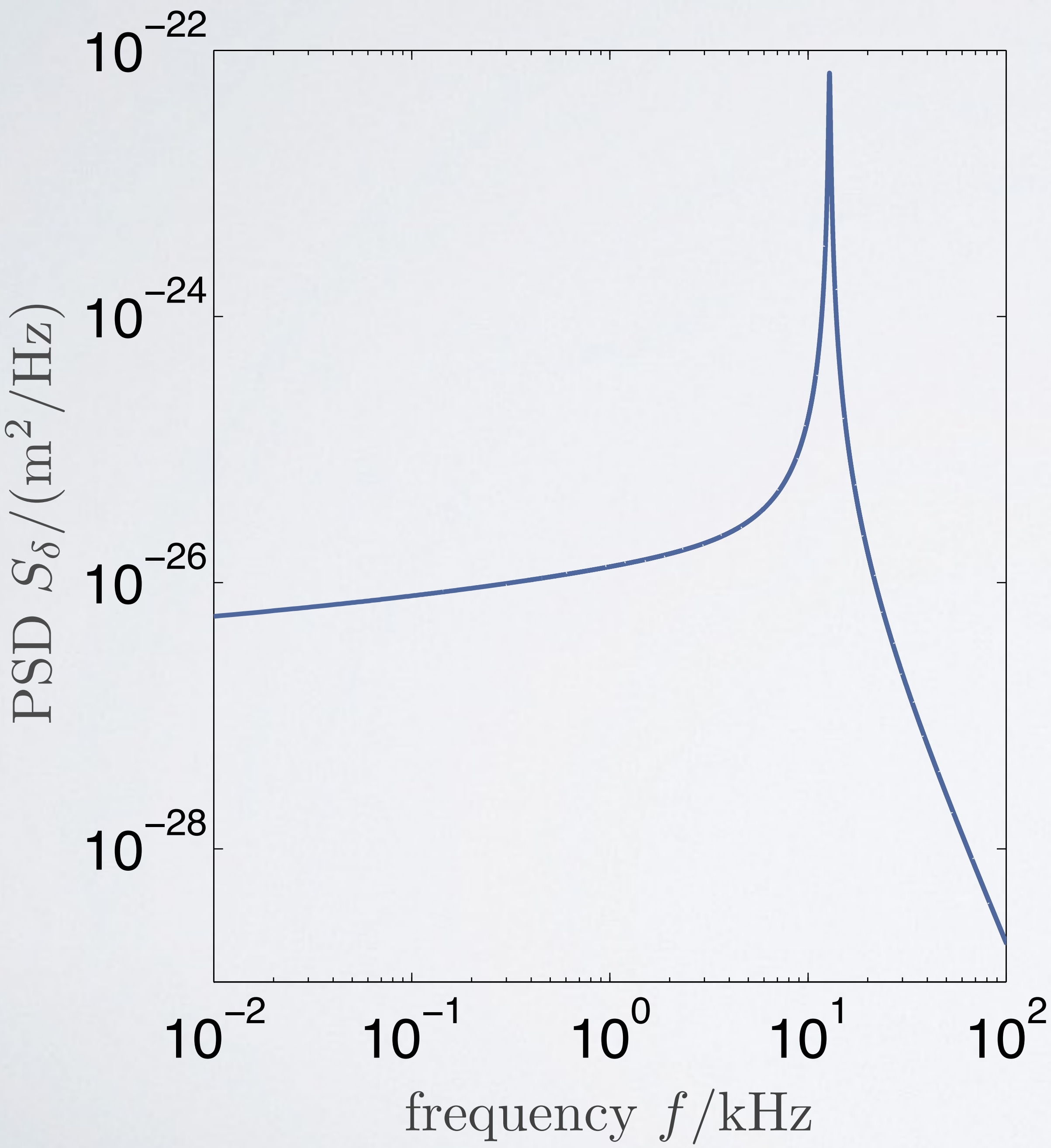
FDT : $\quad \mathrm{Im}\left[\frac{1}{G(\omega)}\right] = -\frac{\omega}{4k_BT}S_\delta(\omega)$

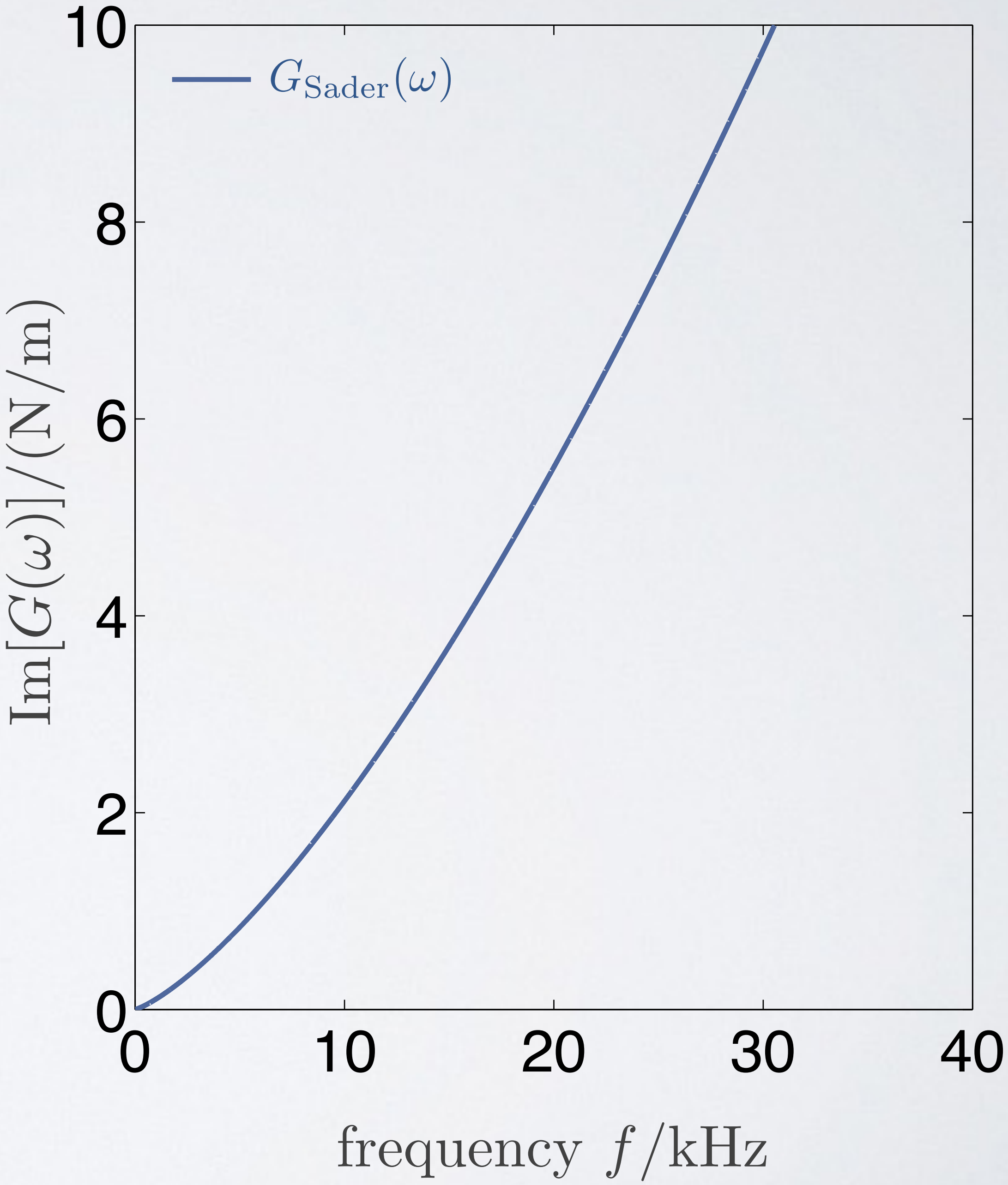
Kramers-Kronig : $\quad \mathrm{Re}\left[\frac{1}{G(\omega)}\right] = \frac{2}{\pi}\mathcal{PP}\int_0^\infty \frac{\Omega}{\Omega^2-\omega^2}\mathrm{Im}\left[\frac{1}{G(\omega)}\right]d\Omega$



Synthetic signal : Sader model

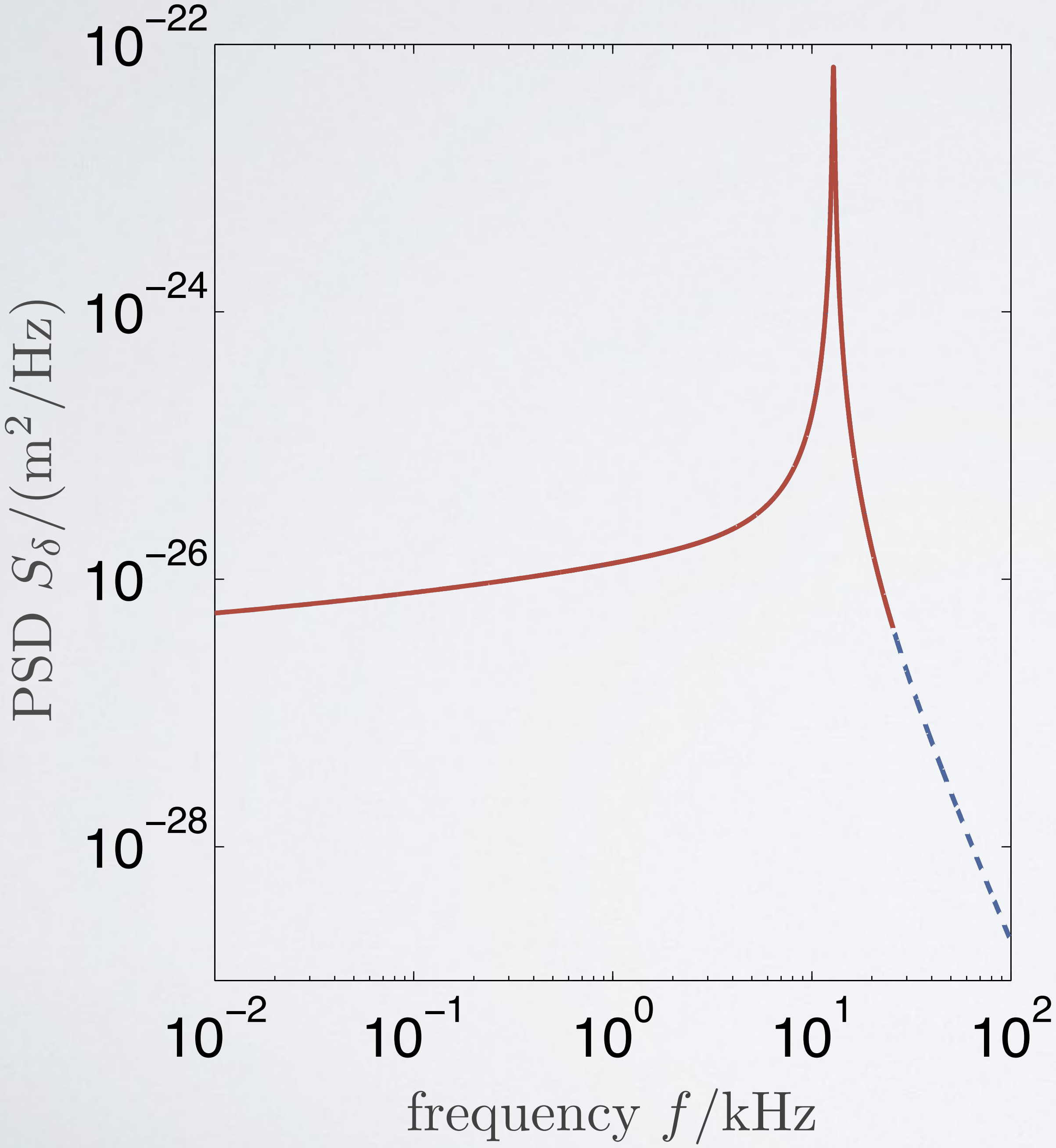
$$S_\delta(\omega) \propto \mathrm{Im}\left[\frac{1}{G(\omega)}\right] \xleftarrow{\textbf{FDT}} G_{\mathrm{Sader}}(\omega)$$

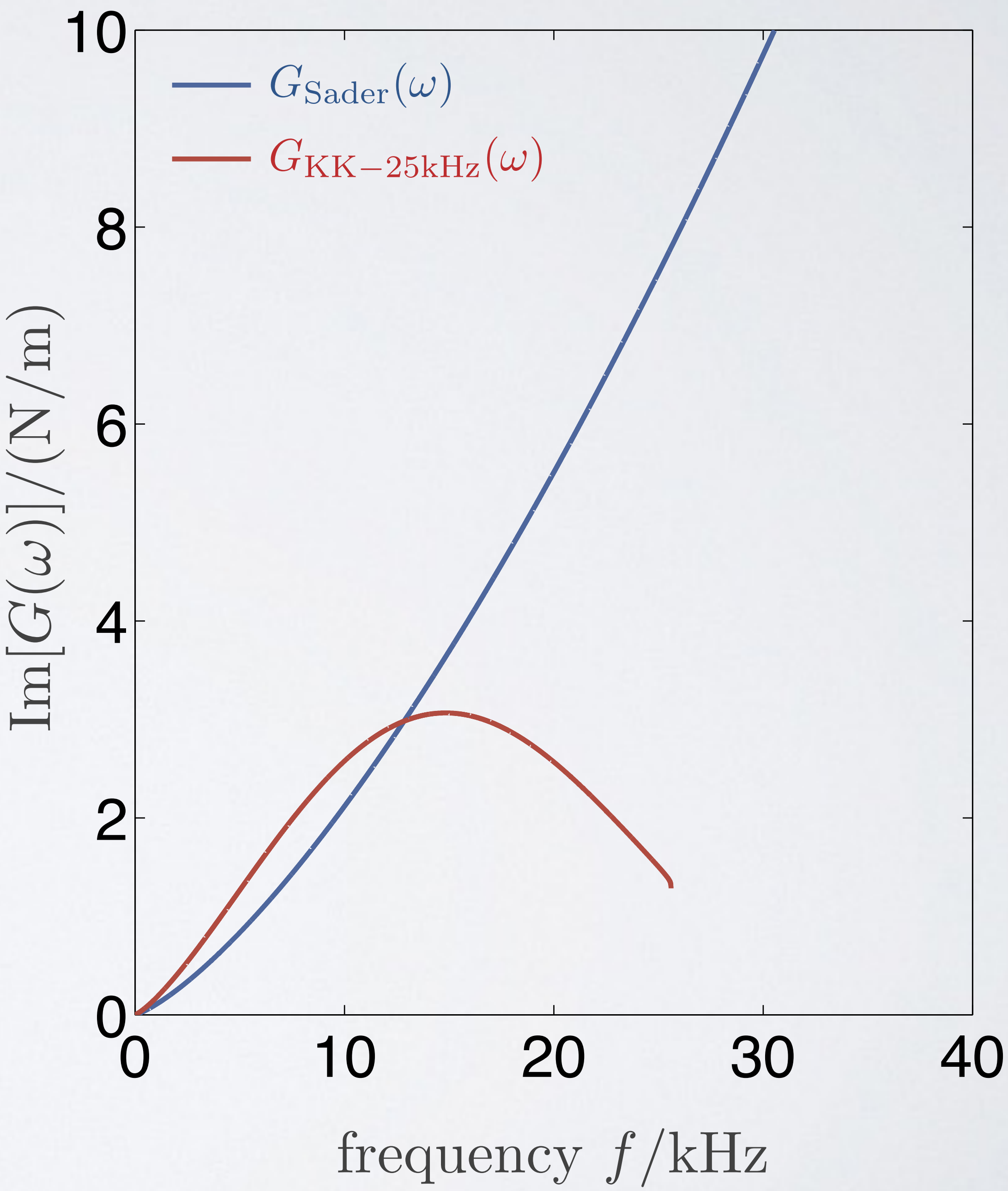




Synthetic signal : Sader model

$$S_\delta(\omega) \propto \mathrm{Im}\left[\frac{1}{G(\omega)}\right] \xrightarrow{\text{KK}} \mathrm{Re}\left[\frac{1}{G(\omega)}\right] = \frac{2}{\pi}\mathcal{PP}\int_0^\infty \frac{\Omega}{\Omega^2-\omega^2}\mathrm{Im}\left[\frac{1}{G(\omega)}\right] d\Omega$$

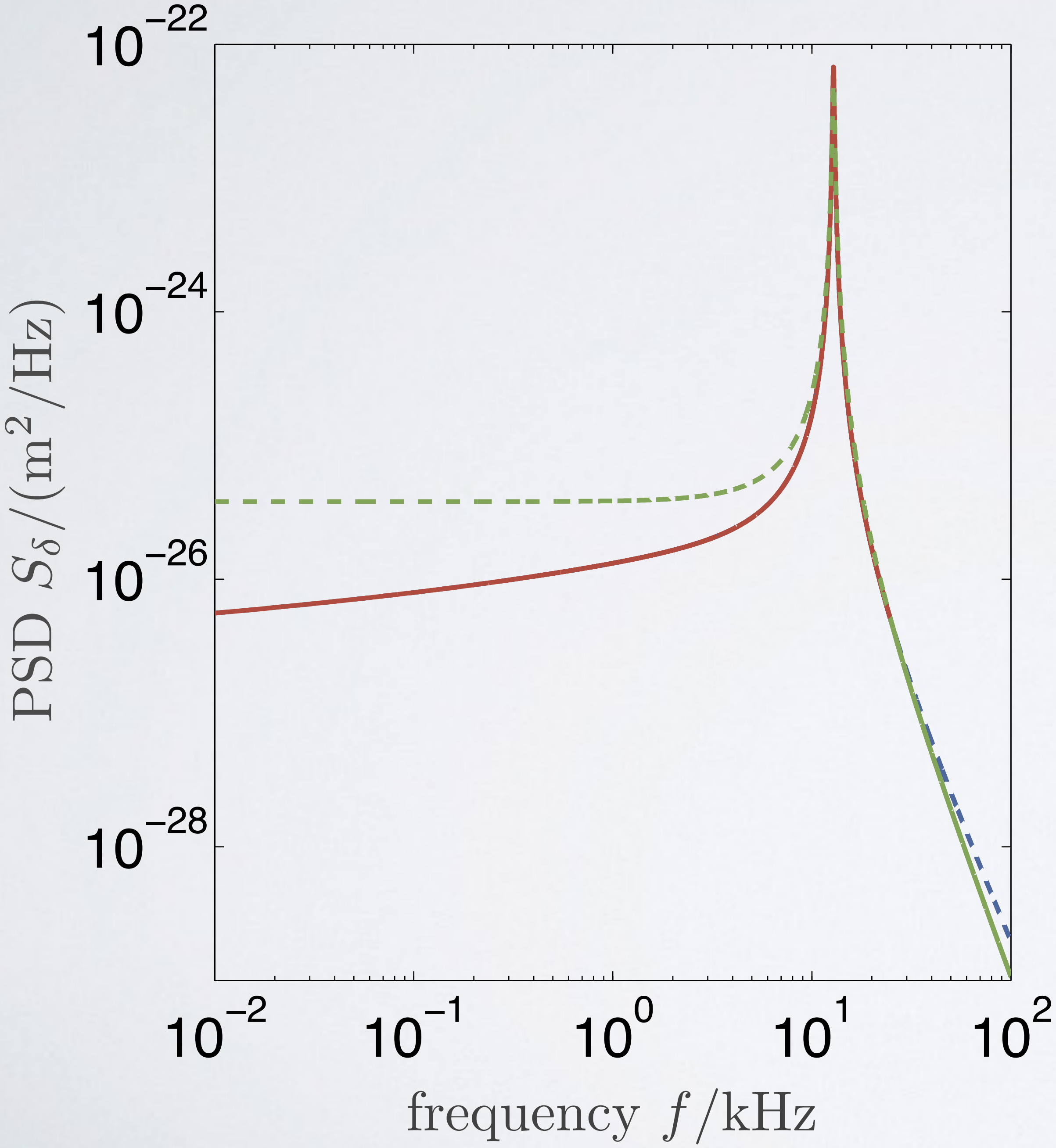


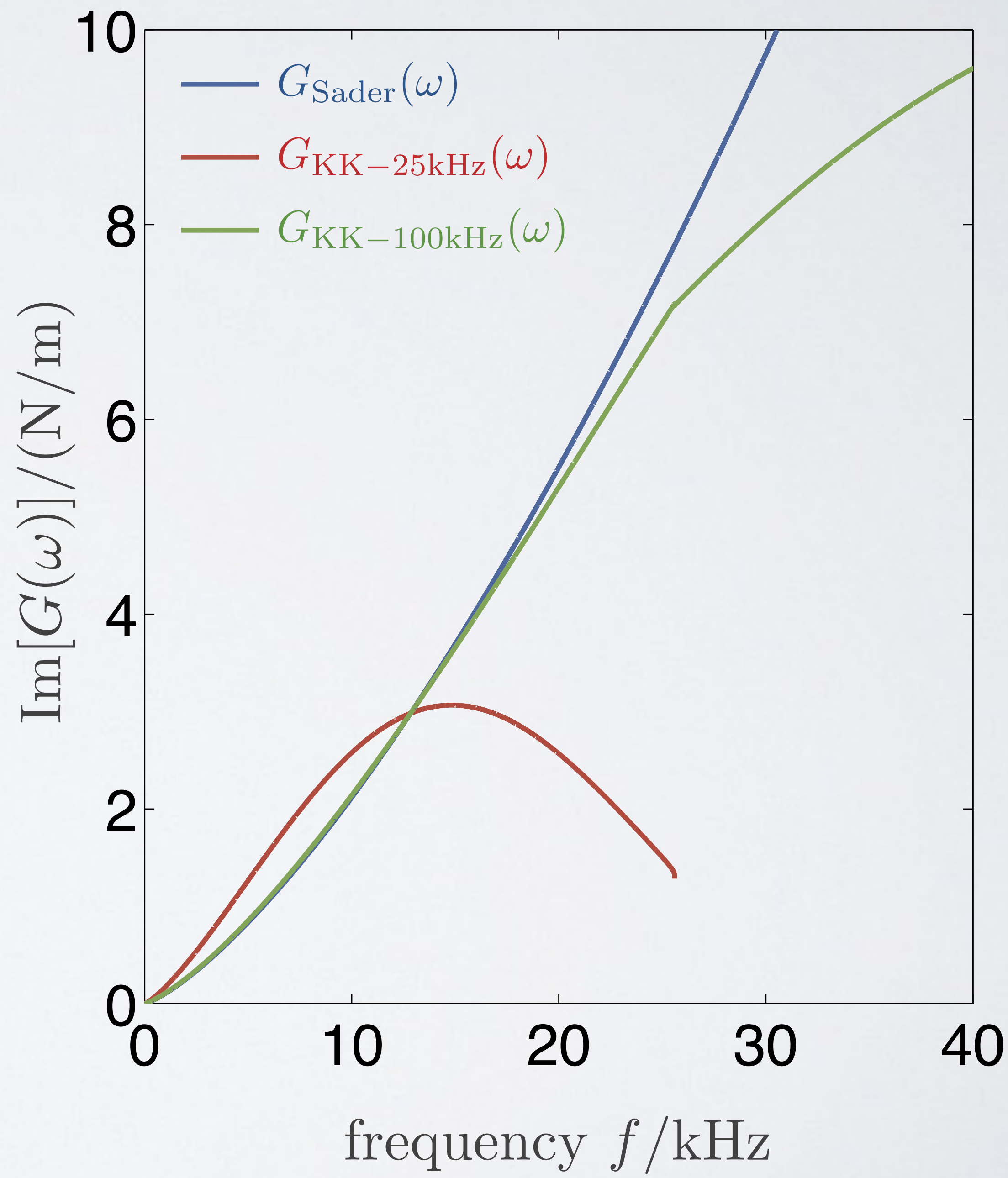


Reconstruction of the full response

Synthetic signal : Sader model

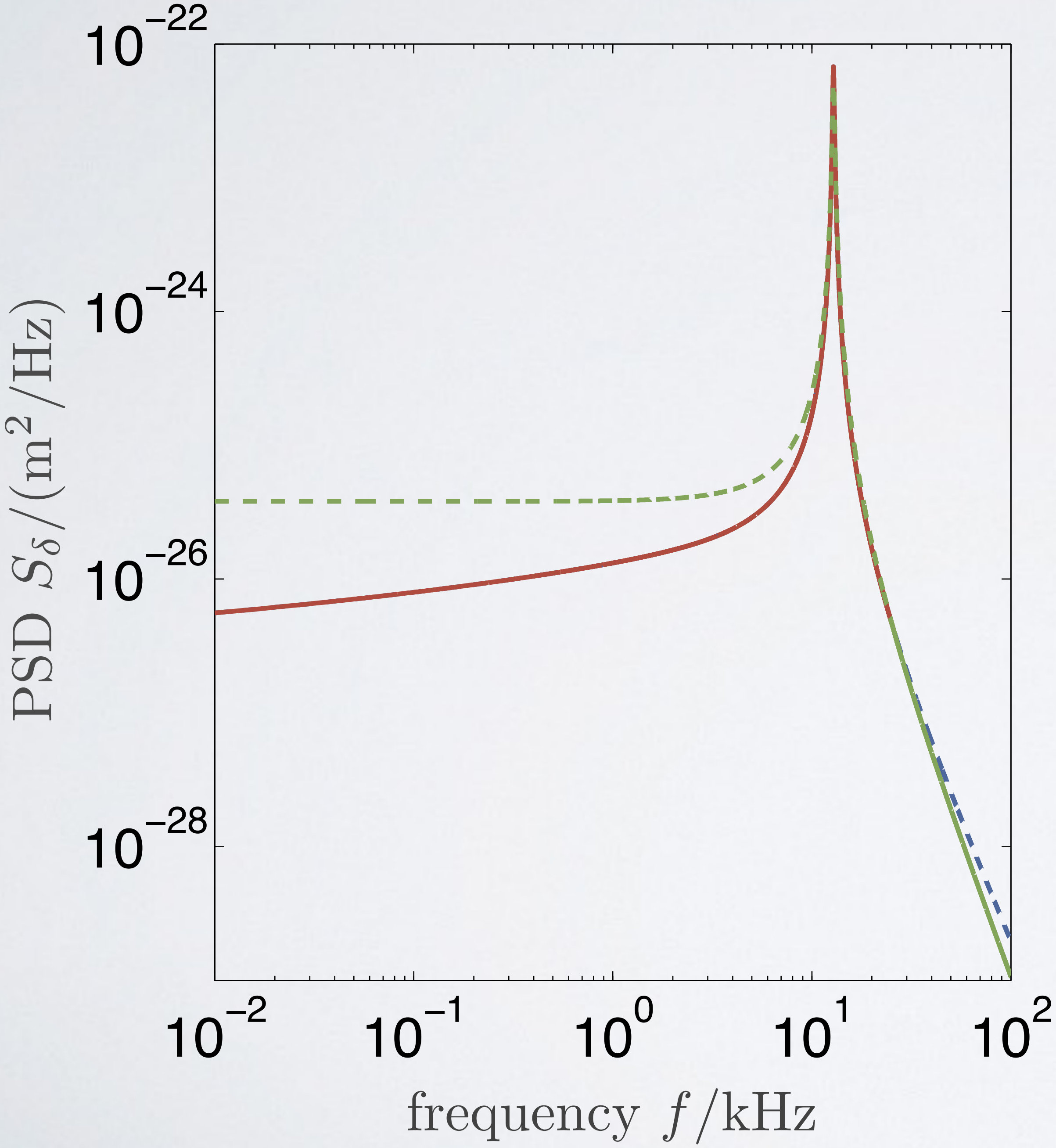
$$S_\delta(\omega) \propto \mathrm{Im}\left[\frac{1}{G(\omega)}\right] \xrightarrow{\textbf{KK}} \mathrm{Re}\left[\frac{1}{G(\omega)}\right] = \frac{2}{\pi}\mathcal{PP}\int_0^\infty \frac{\Omega}{\Omega^2-\omega^2}\mathrm{Im}\left[\frac{1}{G(\omega)}\right] d\Omega$$

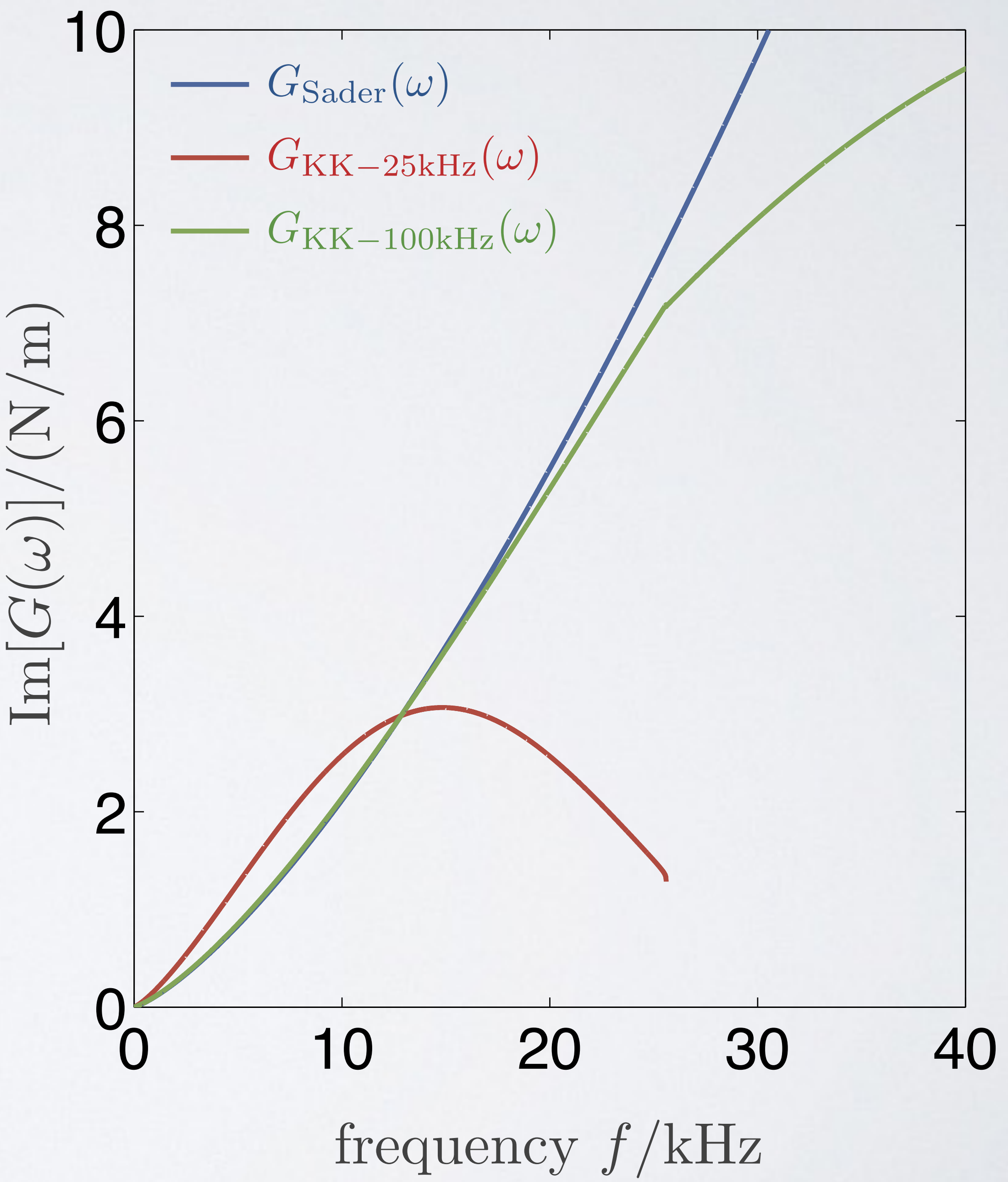




Synthetic signal : Sader model

$$S_\delta(\omega) \propto \mathrm{Im}\left[\frac{1}{G(\omega)}\right] \xrightarrow{\textbf{KK}} \frac{1}{G(\omega)} = \mathcal{KK}\left[S_\delta(\omega)\right]$$

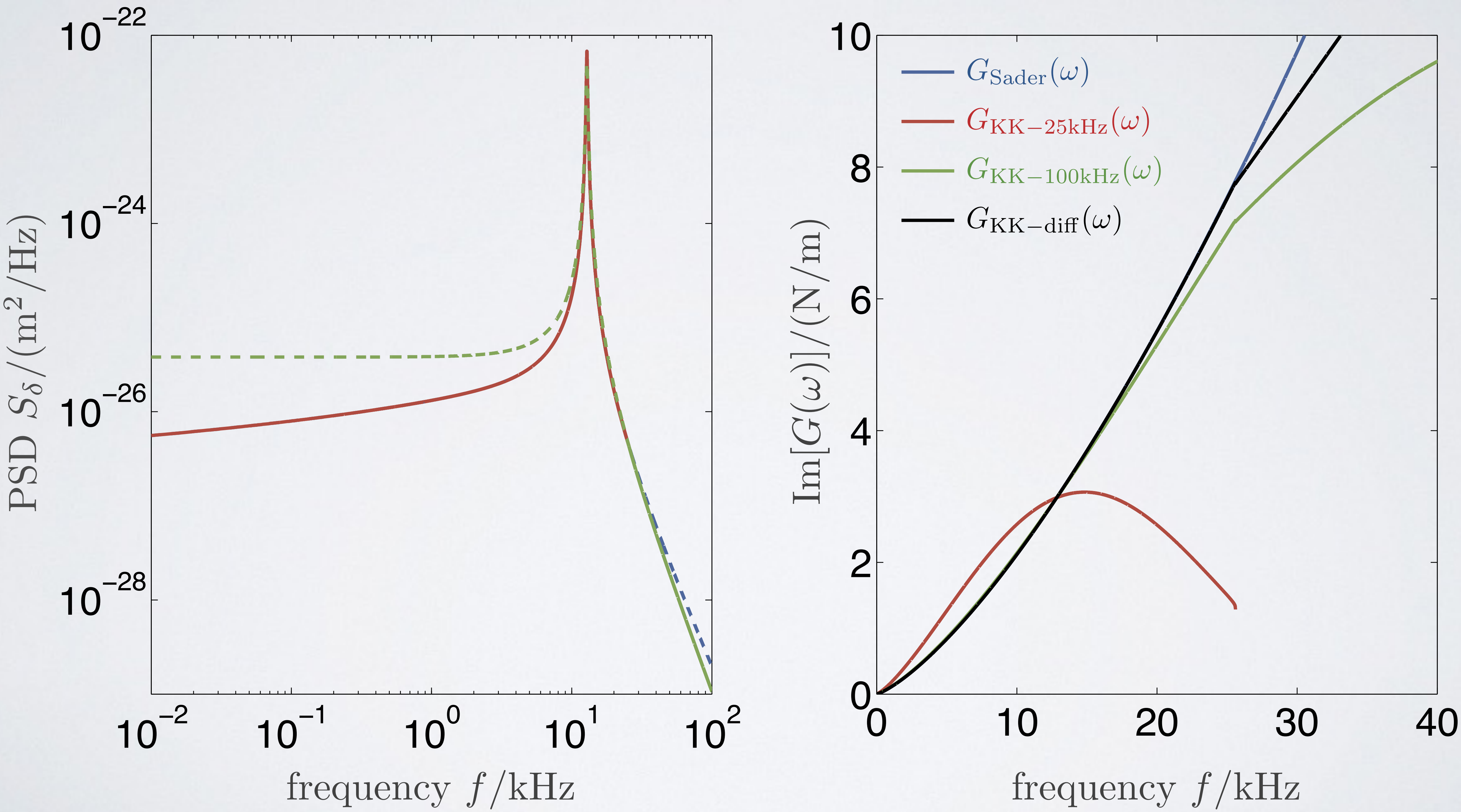




Reconstruction of the full response

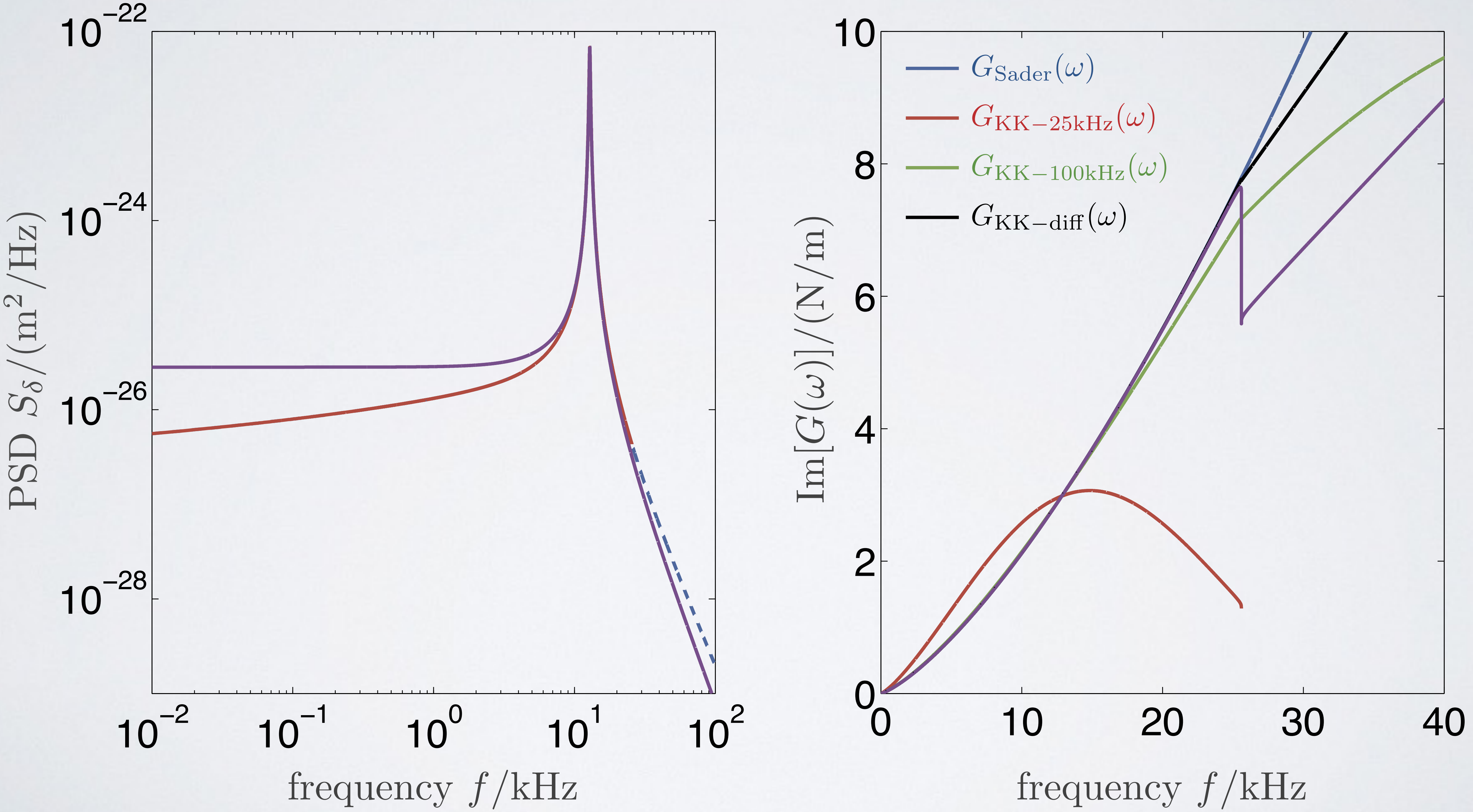
Synthetic signal : Sader model

$$S_\delta(\omega) \propto \mathrm{Im}\left[\frac{1}{G(\omega)}\right] \xrightarrow{\textbf{KK}} \frac{1}{G(\omega)} - \frac{1}{G_0(\omega)} = \mathcal{KK}\left[S_\delta(\omega) - S_{\delta 0}(\omega)\right]$$



Synthetic signal : Sader model

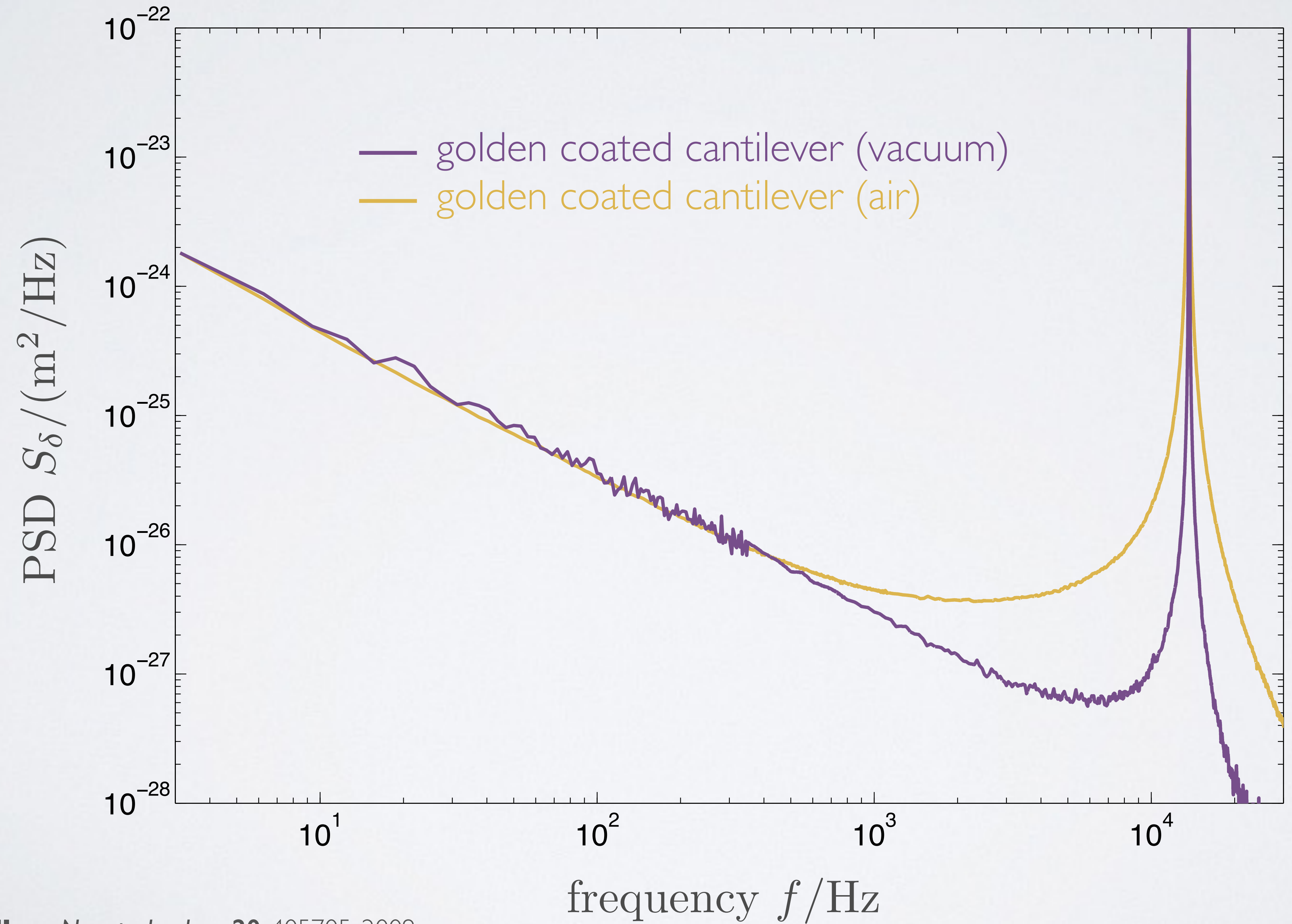
$$S_\delta(\omega) \propto \mathrm{Im}\left[\frac{1}{G(\omega)}\right] \xrightarrow{\textbf{KK}} \frac{1}{G(\omega)} - \frac{1}{G_0(\omega)} = \mathcal{KK}\left[S_\delta(\omega) - S_{\delta 0}(\omega)\right]$$



Cantilever internal damping: 1/f noise ?

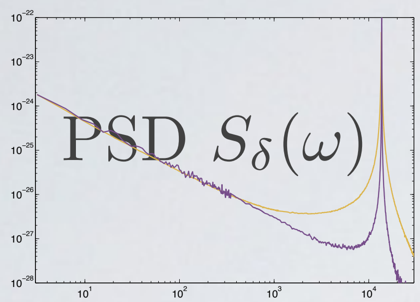
FDT : $$\mathrm{Im}\left[\frac{1}{G(\omega)}\right] = -\frac{\omega}{4k_BT}S_\delta(\omega)$$

Kramers-Kronig : $$\mathrm{Re}\left[\frac{1}{G(\omega)}\right] = \frac{2}{\pi}\mathcal{PP}\int_0^\infty \frac{\Omega}{\Omega^2-\omega^2}\mathrm{Im}\left[\frac{1}{G(\Omega)}\right]d\Omega$$

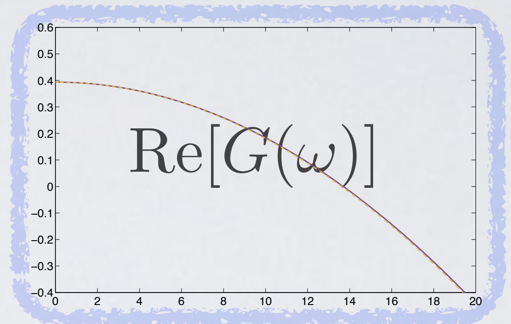


P. Paolino, L. Bellon, *Nanotechnology*, **20**, 405705, 2009.

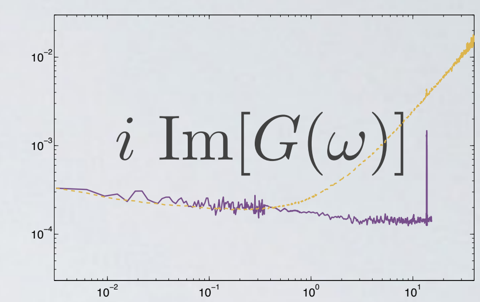
Internal damping: mechanical response

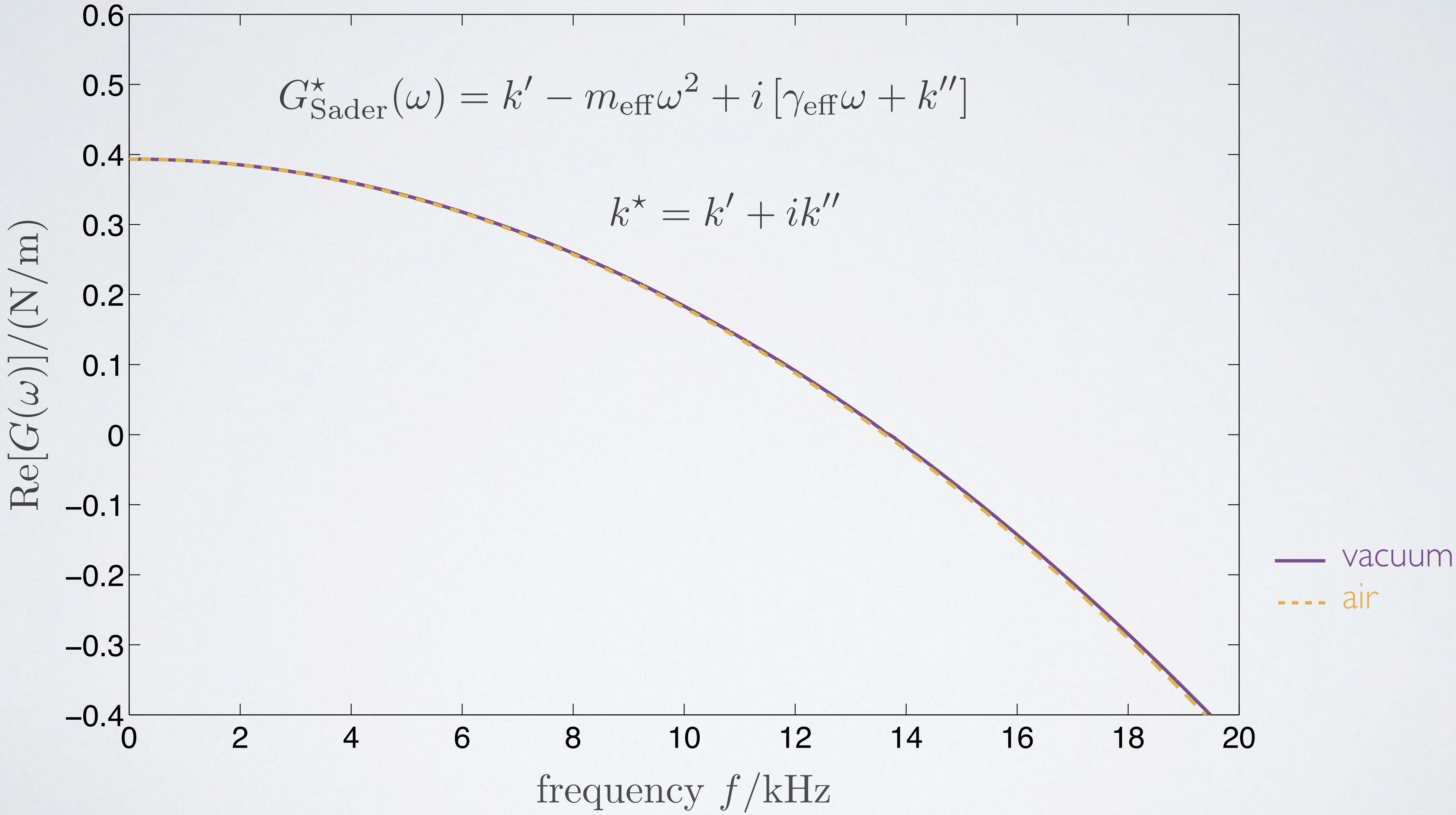


FDT^{-1} + KK →



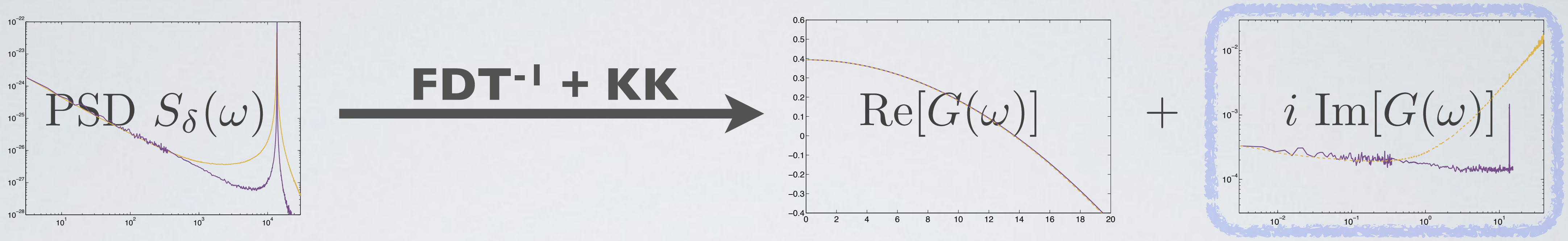
+

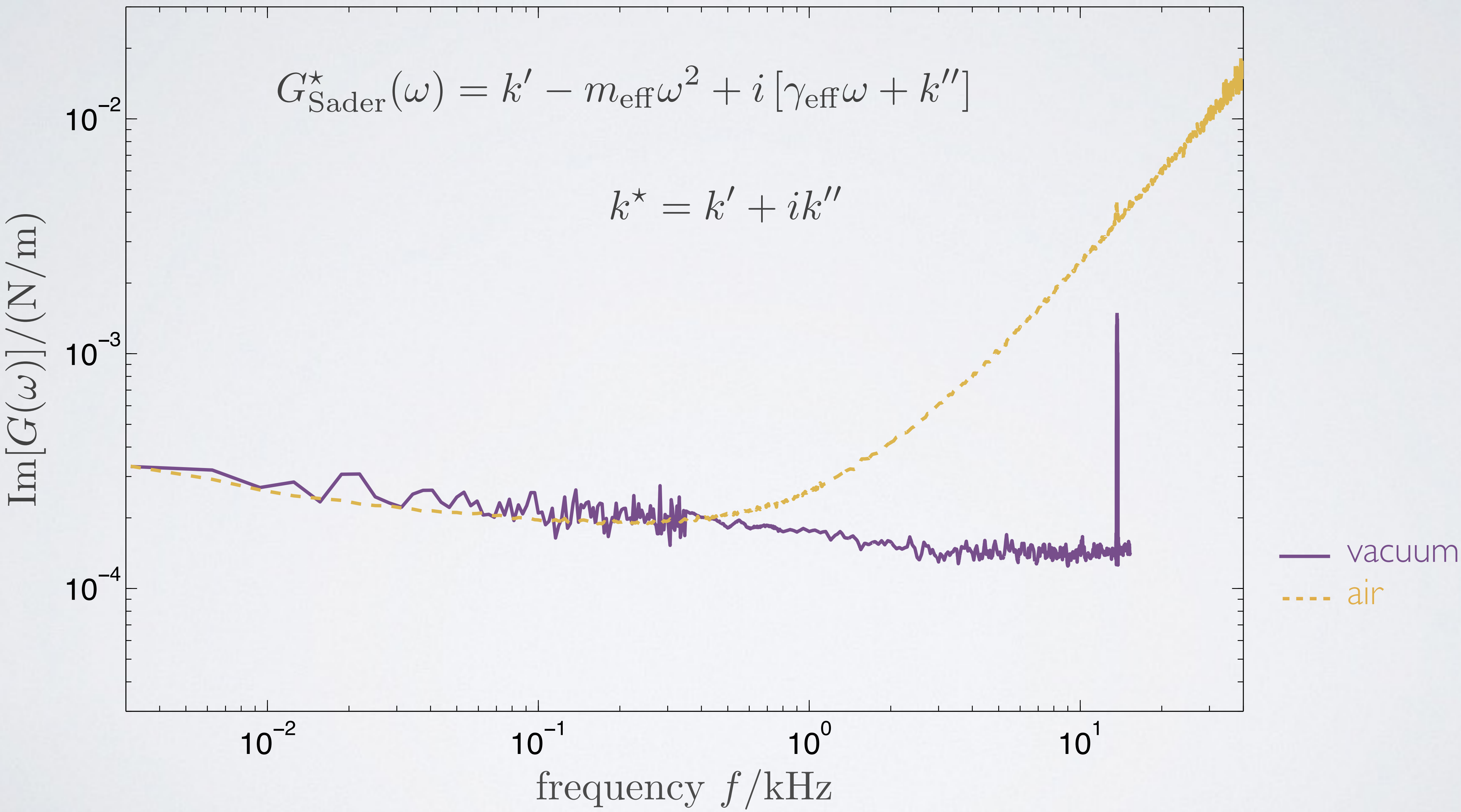




P. Paolino, L. Bellon, *Nanotechnology*, **20**, 405705, 2009.

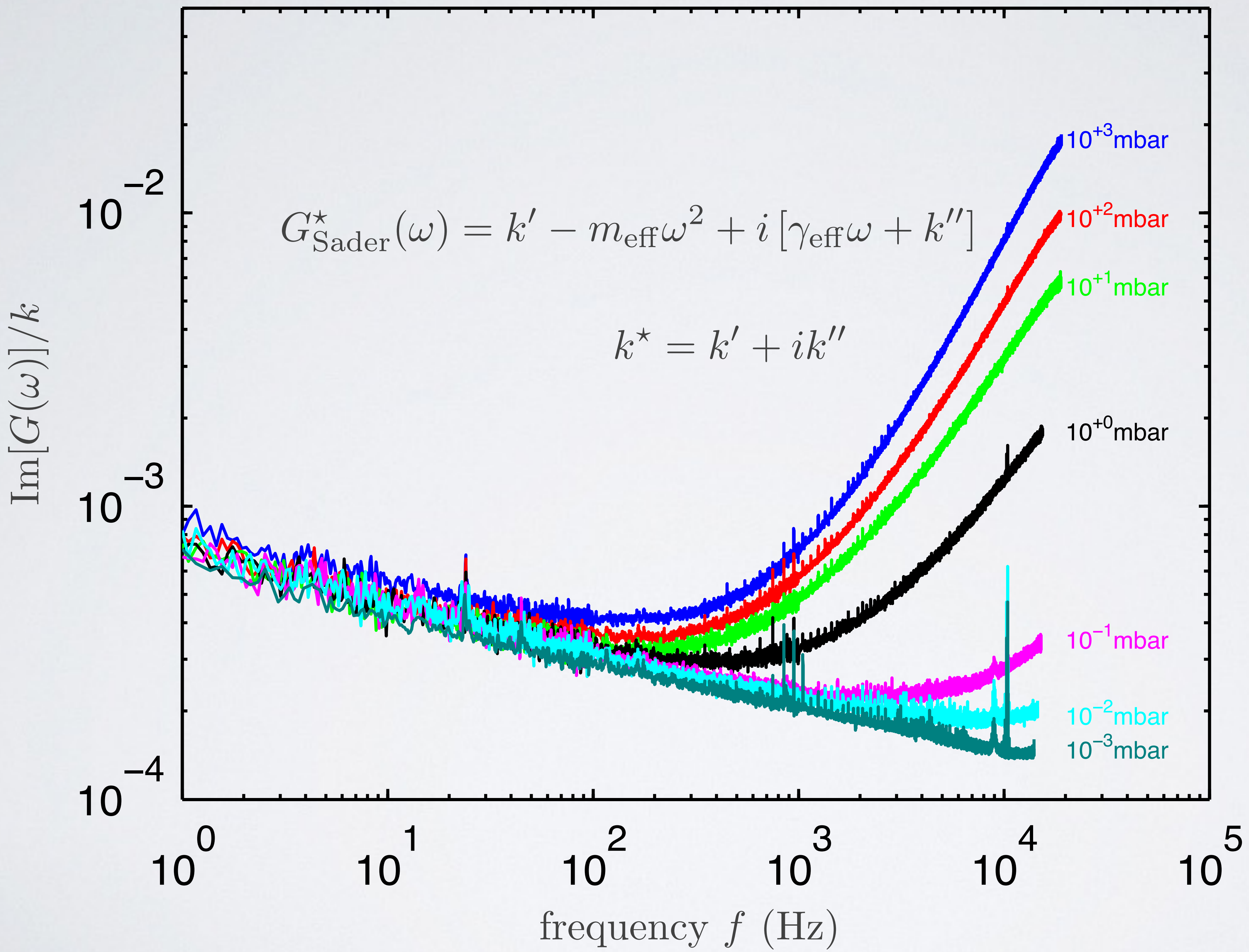
Internal damping: mechanical response





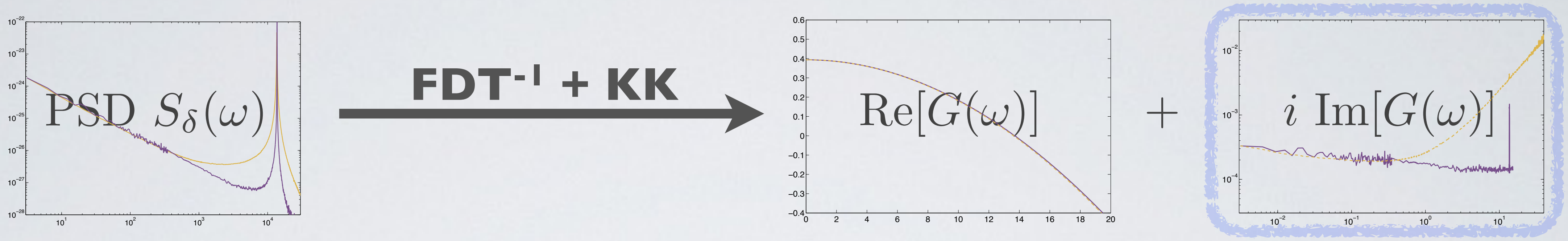
P. Paolino, L. Bellon, *Nanotechnology*, **20**, 405705, 2009.

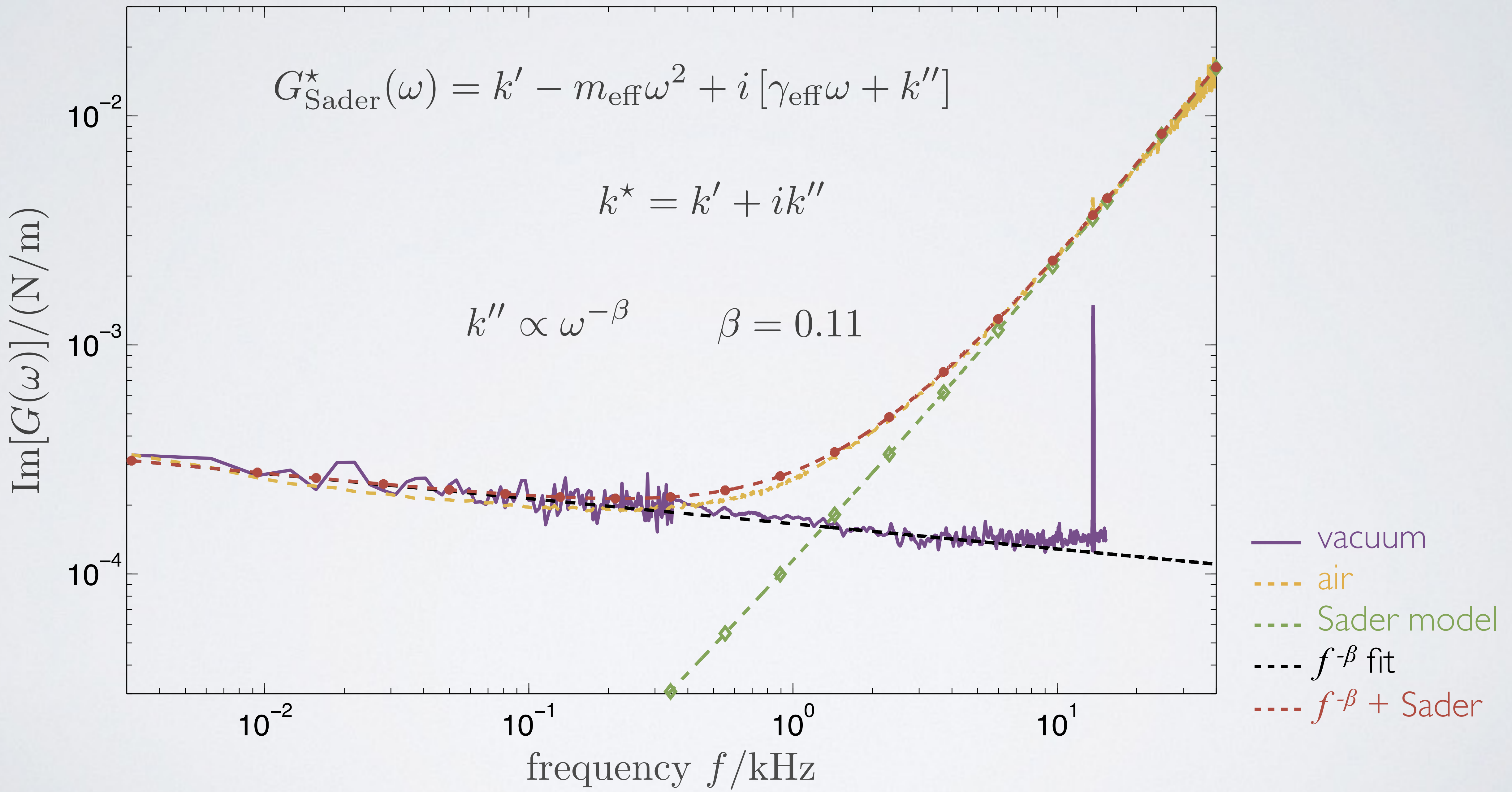
Internal damping: viscoelasticity



T. Li, L. Bellon, *EPL* **98**, 14004, 2012.

Internal damping: viscoelasticity





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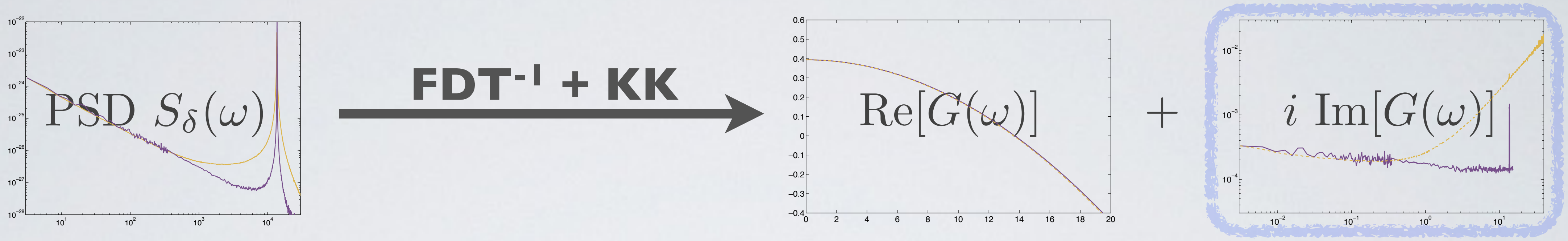
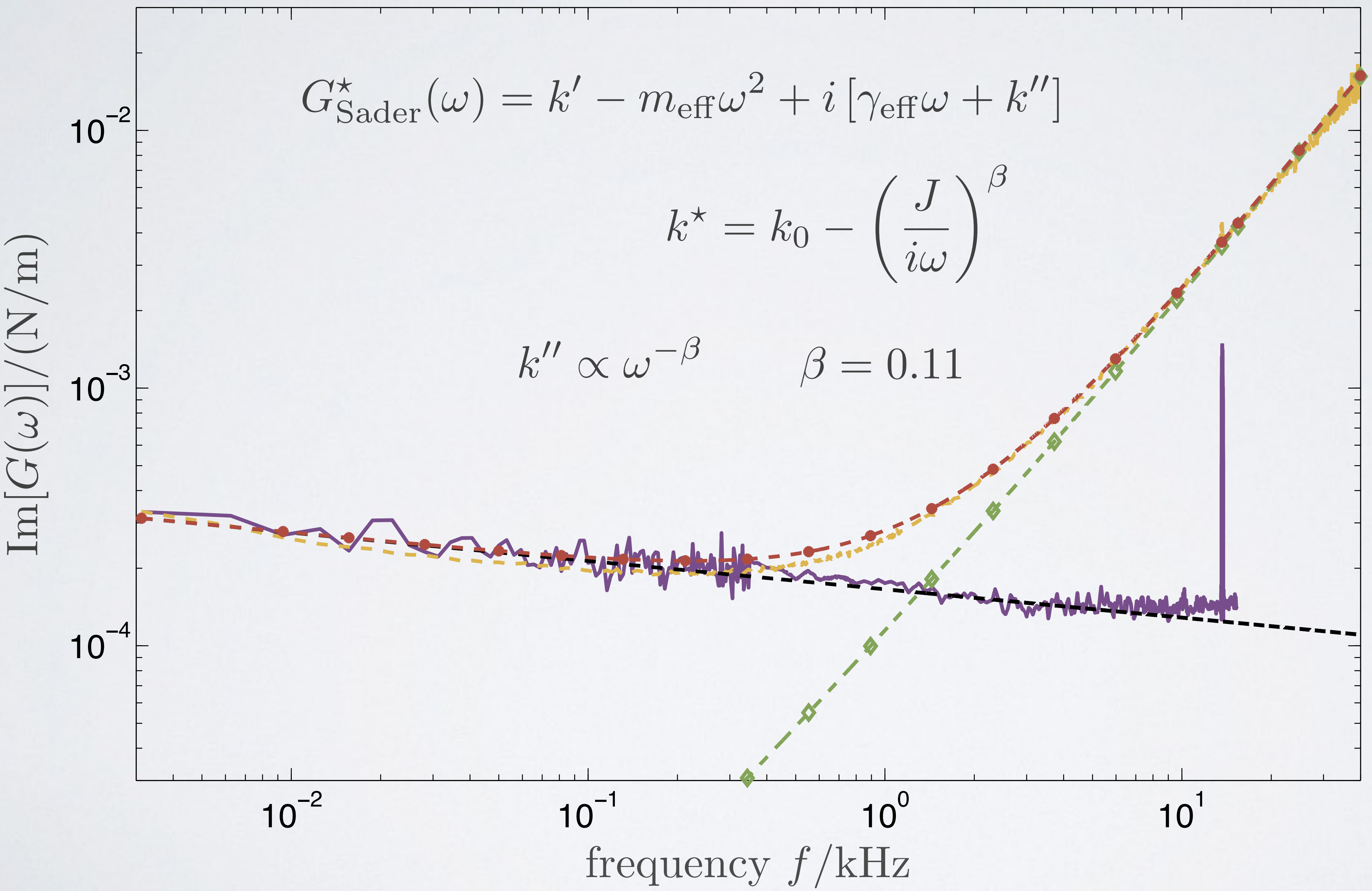
Internal damping: viscoelasticity





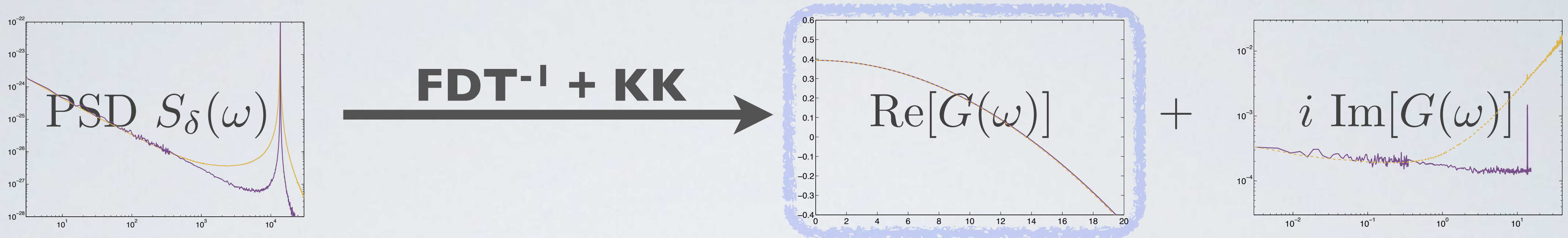
$$k' = k_0 - \cos(\tfrac{\pi}{2}\beta)\left(\frac{J}{\omega}\right)^\beta$$

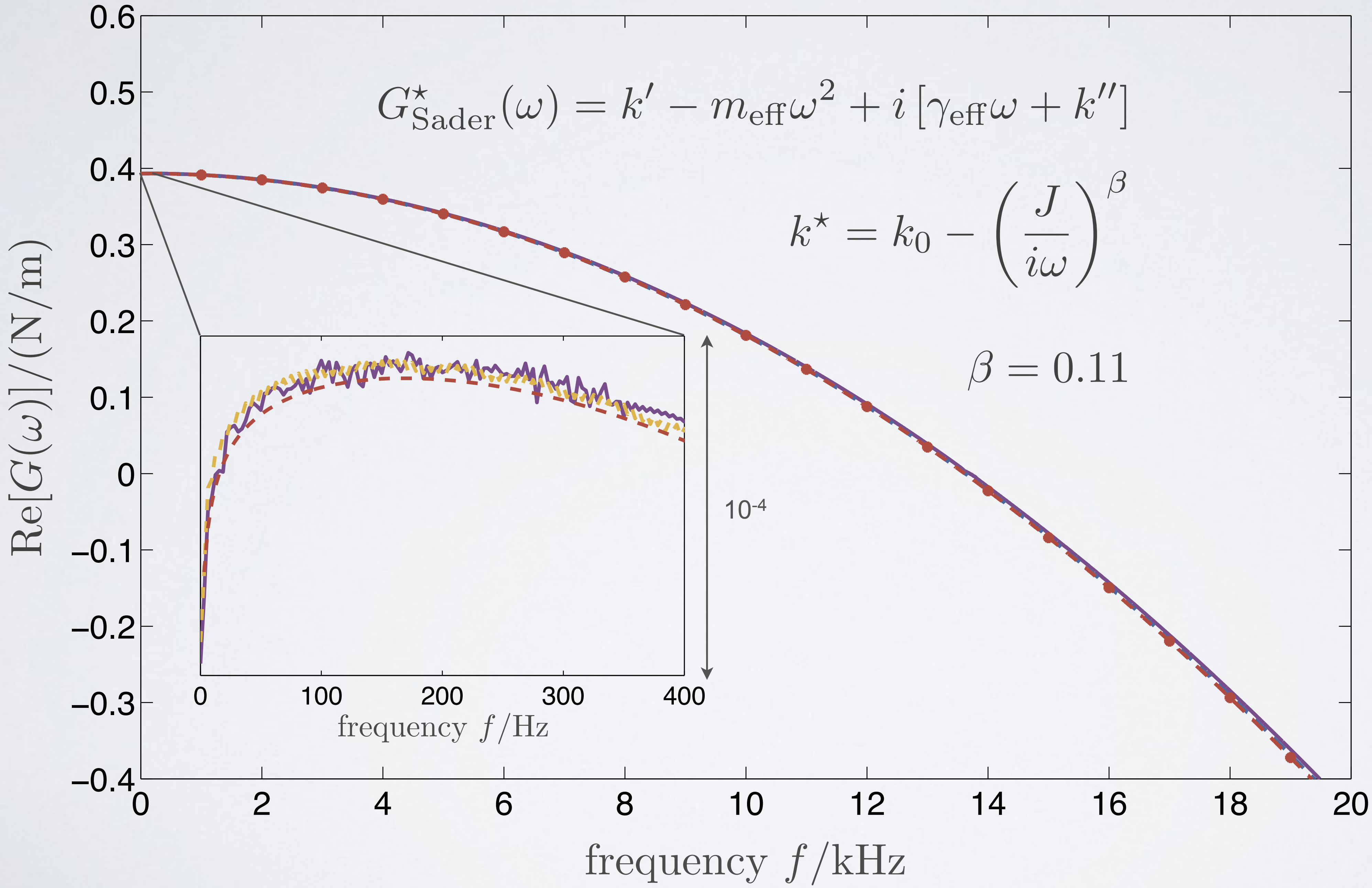
$$k'' = \sin(\tfrac{\pi}{2}\beta)\left(\frac{J}{\omega}\right)^\beta$$

vacuum
air
Sader model
$f^{-\beta}$ fit
$f^{-\beta}$ + Sader

P. Paolino, L. Bellon, *Nanotechnology*, **20**, 405705, 2009.

Internal damping: viscoelasticity



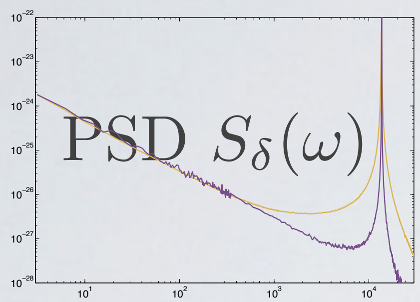


$$k' = k_0 - \cos(\frac{\pi}{2}\beta)\left(\frac{J}{\omega}\right)^\beta$$

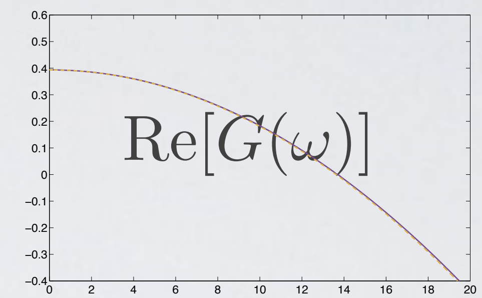
$$k'' = \sin(\frac{\pi}{2}\beta)\left(\frac{J}{\omega}\right)^\beta$$

P. Paolino, L. Bellon, *Nanotechnology*, **20**, 405705, 2009.

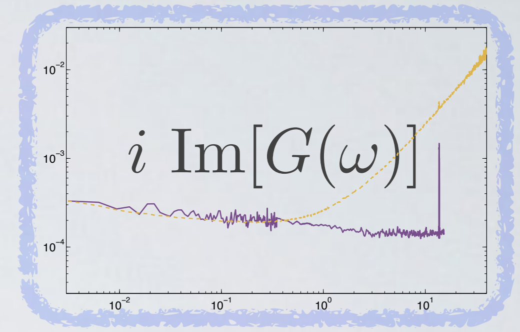
Internal damping: viscoelasticity

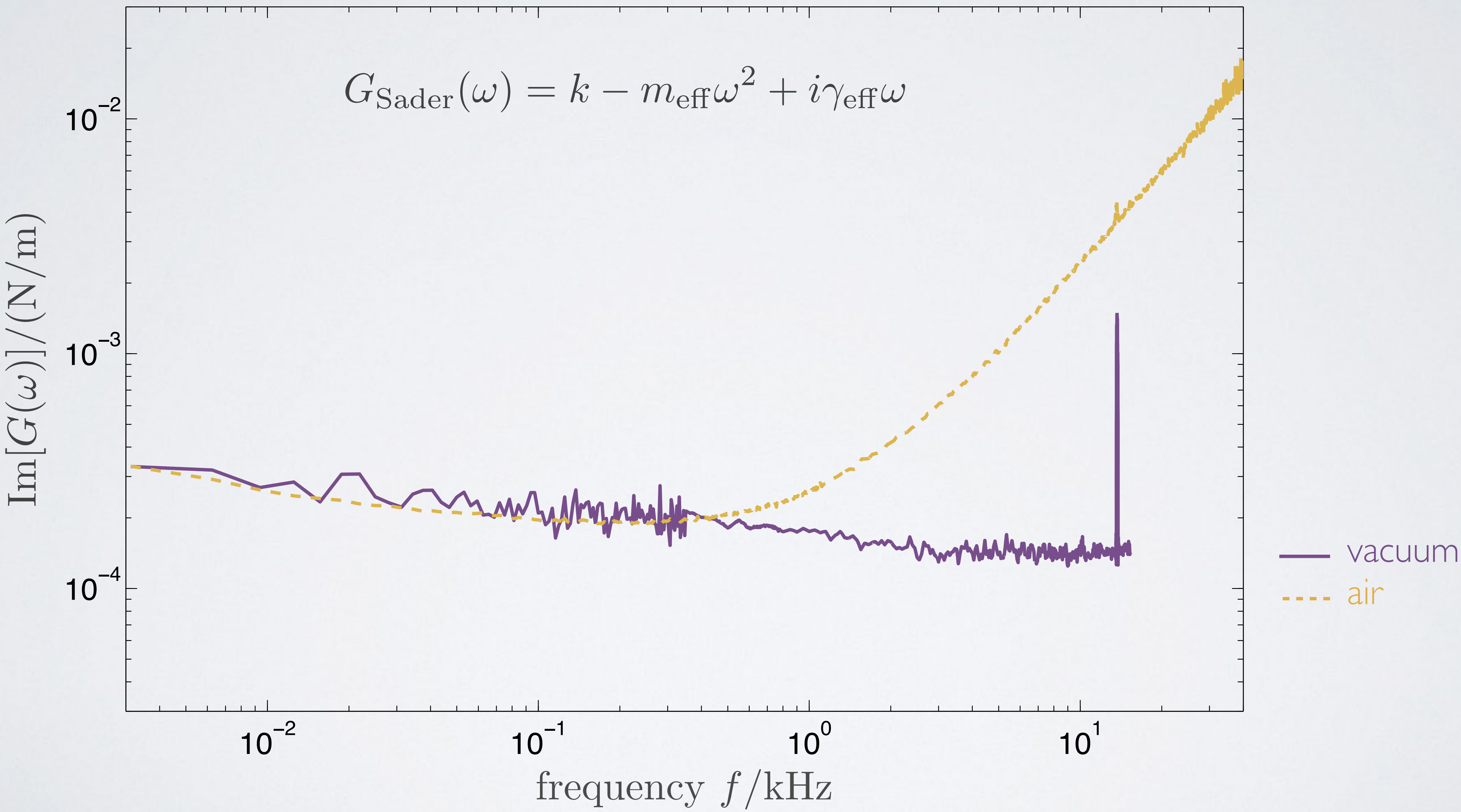


FDT^{-1} + KK →

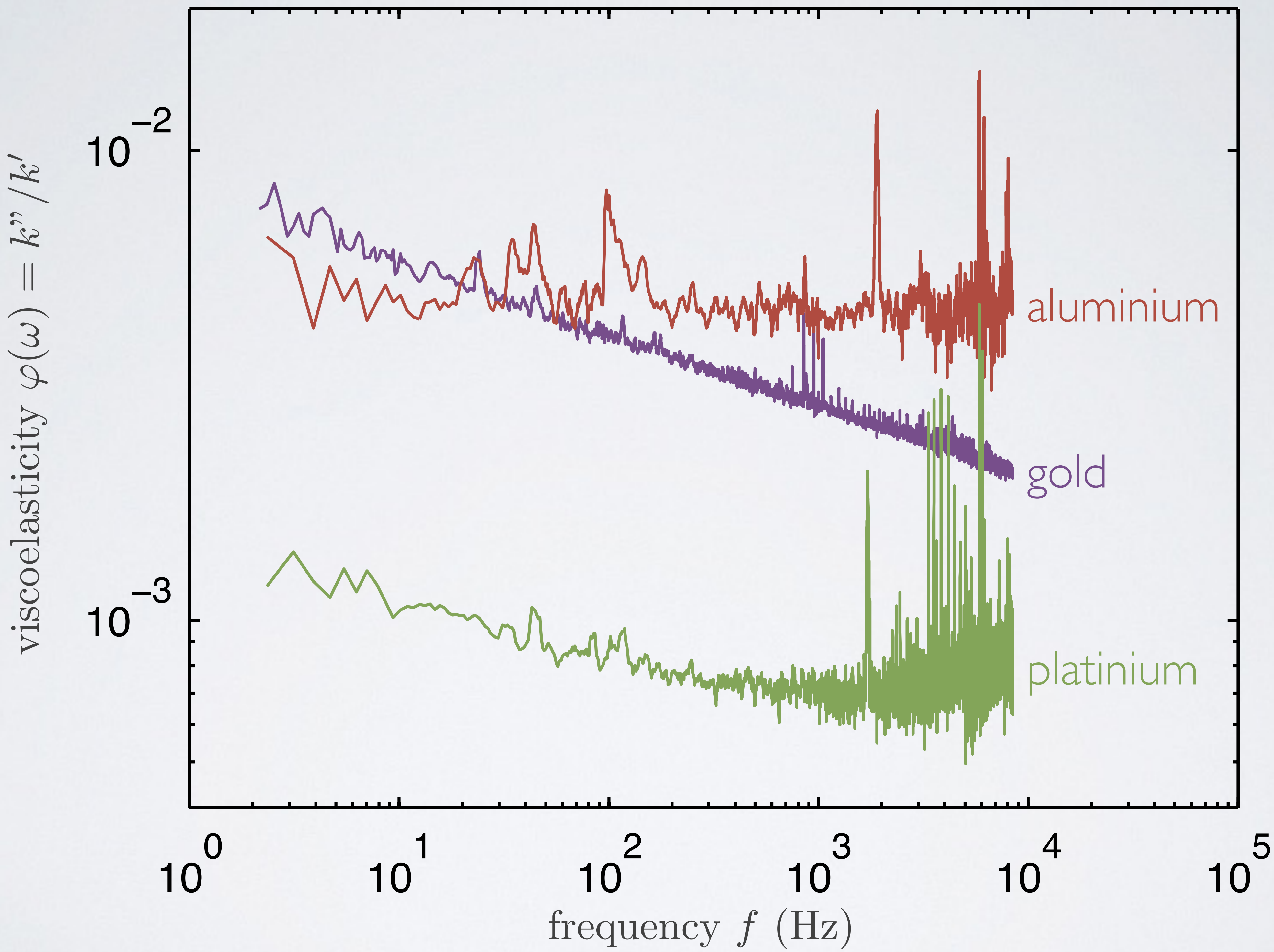


+



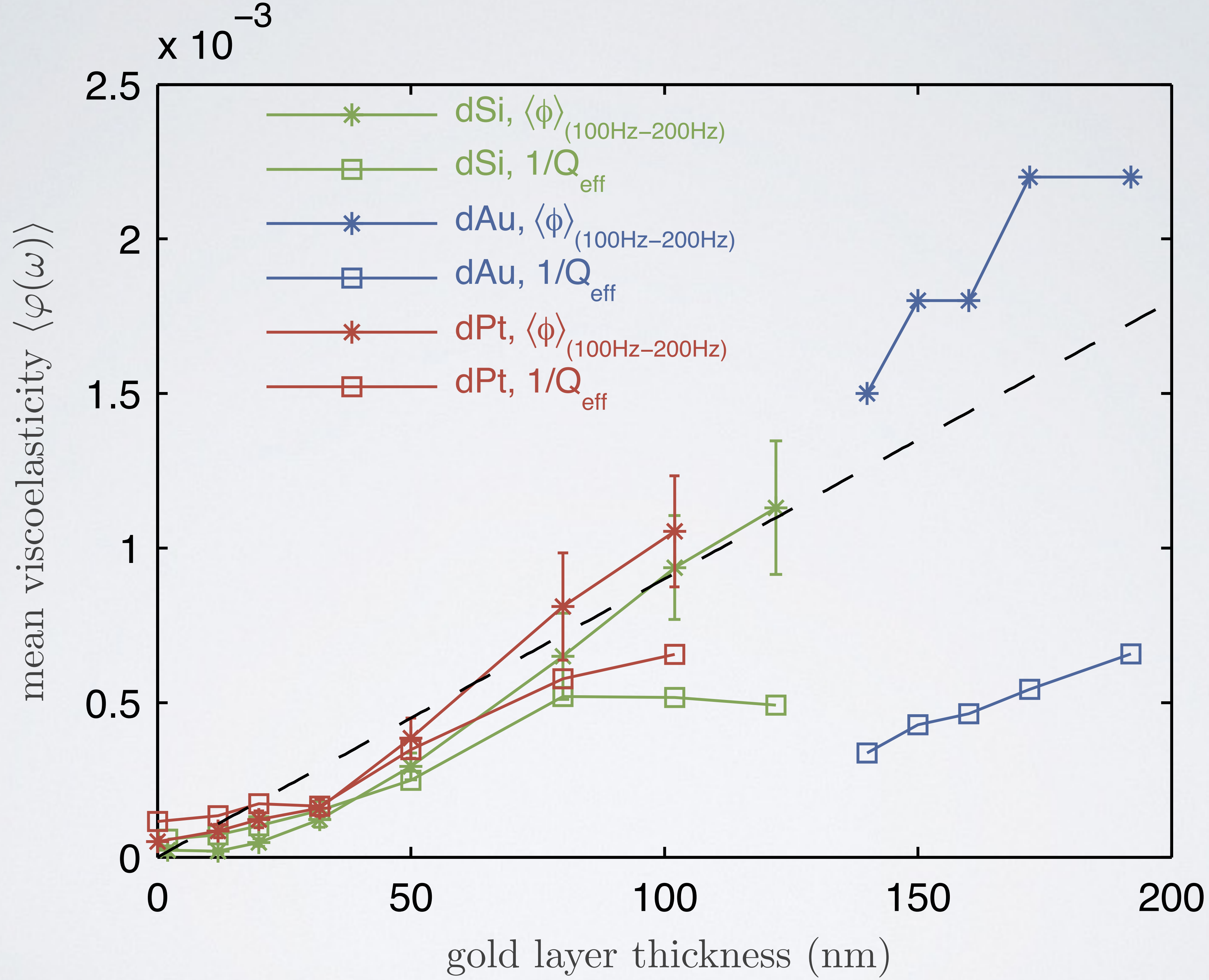


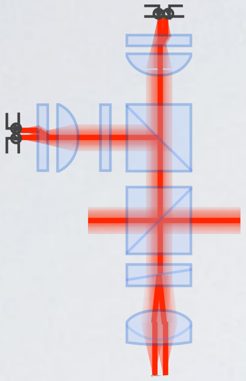
Internal damping: viscoelasticity



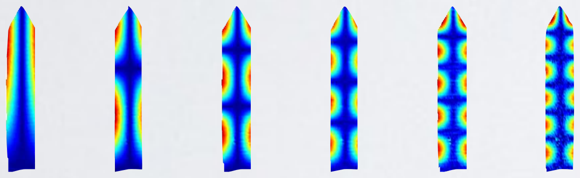
T. Li, L. Bellon, *EPL* **98**, 14004, 2012.

Internal damping: viscoelasticity

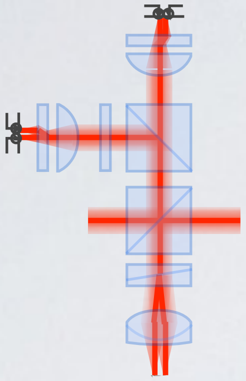


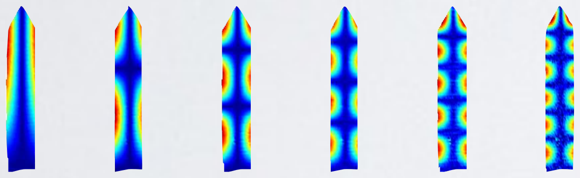


- Quadrature phase interferometry
  - Experimental setup
  - Measurement of thermal noise



- Micro-cantilever response from thermal noise
  - Full measurement of response with Kramers-Kronig relations
  - Viscoelasticity of coating layer

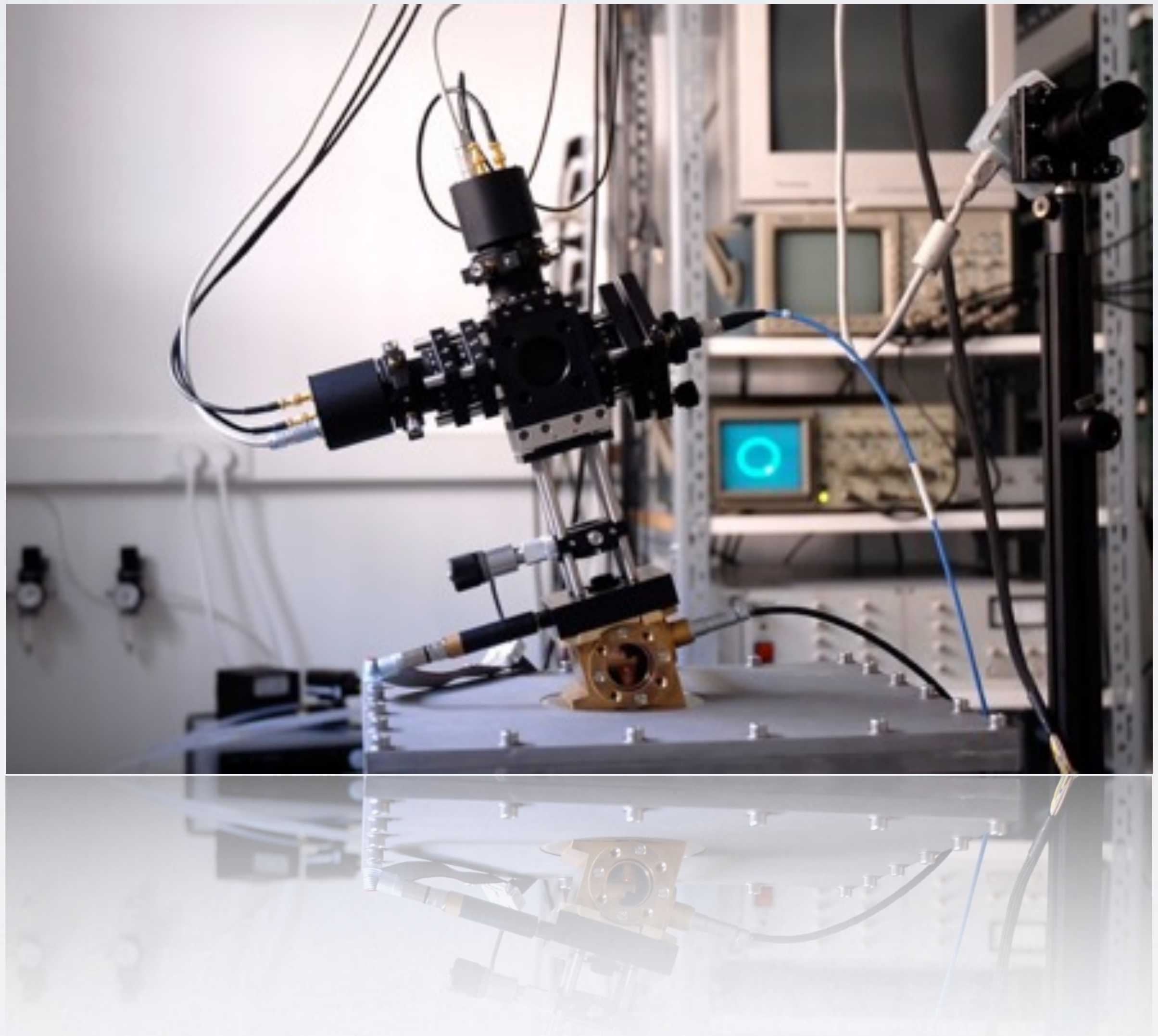




- Lowering the baseline noise
- Cryogenic operation
- Viscoelasticity of dielectric coatings
- Beyond FDT : fluctuations in out of equilibrium systems

Order 0 message:

Original instruments create innovative physics



Order n+1 steps: *http://perso.ens-lyon.fr/ludovic.bellon/*