TIGER's tail: Testing the no-hair theorem with black hole ringdowns.

J. Meidam, M. Agathos, C. Van Den Broeck,
J. Veitch and B. S. Sathyaprakash

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Meidam et al., Phys. Rev D 90 (2014) 064009

- *No-hair theorem*: Final stationary black hole vacuum state is determined only by its mass M and spin J.
- Quasinormal modes are characterized by mode frequencies and damping times $\omega_{lm}(M,J)$ and $\tau_{lm}(M,J)$.
- Deviations from predicted mass and spin dependence can be written as:

$$\omega_{lm} = \omega_{lm}^{\mathrm{GR}}(M,J)(1+\delta\hat{\omega}_{lm}), \qquad (1)$$

$$\tau_{lm} = \tau_{lm}^{\mathrm{GR}}(M,J)(1+\delta\hat{\tau}_{lm}) \qquad (2)$$

- Use multiple detections from ET to test the no-hair theorem

Previous work

- Two methods:
  1. $M = 500 - 1000\,\mathrm{M}_{\odot}$, $D_{\mathrm{L}} = 6$ Gpc ($z \sim 1$):
     Determined mausurability of $\delta\hat{\omega}_{22}$, $\delta\hat{\omega}_{33}$ and $\delta\hat{\tau}_{22}$.
     $\Rightarrow$ Accuracies of a few percent for the first two, and about 10% for the third.
  2. $M = 500\,\mathrm{M}_{\odot}$, compare evidences for two models:
     - model1:$\{\delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}\}$ free parameters,
     - model2: parameters are zero, i.e. the GR prediction.

     $\Rightarrow$ Deviation of a few percent in $\delta\hat{\omega}_{22}$ could be discriminated from GR.

Gossan et al. Phys. Rev. D 85, 124056 (2014)

Using multiple detections

- Coalescence rates in the range $500 - 1000\,\mathrm{M}_\odot$ estimated between one and a few thousand per year.
- Maximally exploit available set of detections by combining information from many sources.
  1. Combining posteriors for $\delta\hat{\omega}_{22}$, $\delta\hat{\omega}_{33}$ and/or $\delta\hat{\tau}_{22}$ not possible unless assuming GR is correct.
  2. Bayesian model selection well suited.
     However, if $\{\delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}\}$ allowed to vary at the same time, one may be penalized if correct model involves a smaller number of additional parameters.

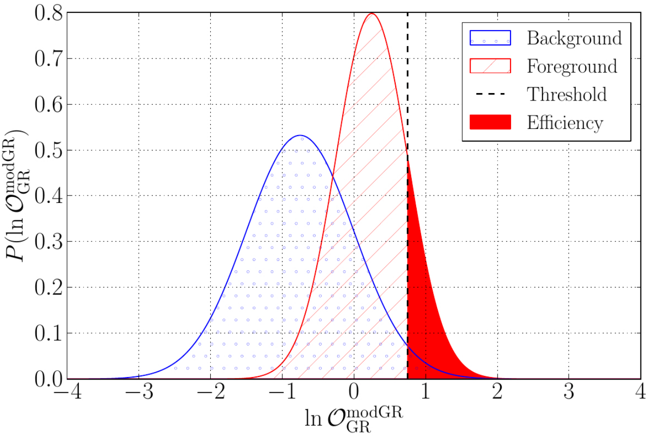
Test Infrastructure for GEneral Relativity

- TIGER is designed and well tested for binary neutron star sources and advanced detectors.
- TIGER is well suited for low SNR and information from a population of sources can be combined.
- Simply put: TIGER compares evidences between two hypotheses:
  - $\mathcal{H}_{\mathrm{GR}}$: GR is correct
  - $\mathcal{H}_{\mathrm{modGR}}$: One or more parameters deviate from zero.
- Use odds ratio to compare models

$$\mathcal{O}_{GR}^{\mathrm{modGR}} \equiv \frac{P(\mathcal{H}_{\mathrm{modGR}}|d,I)}{P(\mathcal{H}_{\mathrm{GR}}|d,I)} \tag{3}$$

Li et al. Phys. Rev. D 85, 082003 (2012)   Agathos et al. Phys. Rev. D 89, 082001 (2014)

What are the odds?



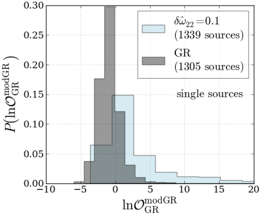
Hypotheses

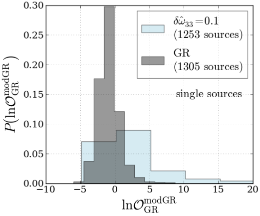
- The dimensionless parameterized deformations are $\{\delta\hat{\omega}_{22},\ \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}\}$. Recall that

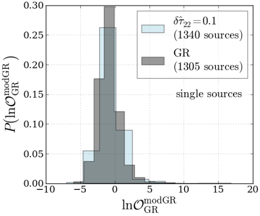
$$\omega_{lm} = \omega_{lm}^{\mathrm{GR}}(M,J)(1+\delta\hat{\omega}_{lm}),$$
$$\tau_{lm} = \tau_{lm}^{\mathrm{GR}}(M,J)(1+\delta\hat{\tau}_{lm})$$

- The four cases analyzed in this work are:
  1. $(\delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}) = (0.1, 0, 0)$.
  2. $(\delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}) = (0, 0.1, 0)$.
  3. $(\delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}) = (0, 0, 0.1)$.
  4. $(\delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}) = (0, 0, 0.25)$.

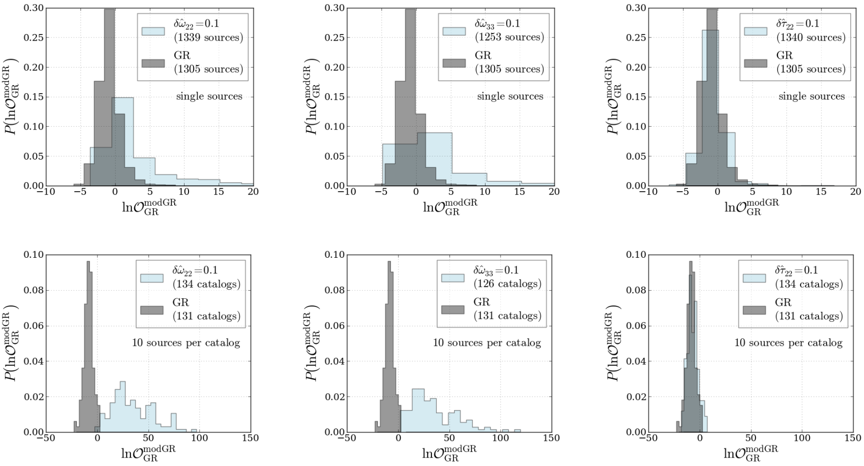
Single sources vs. combined sources



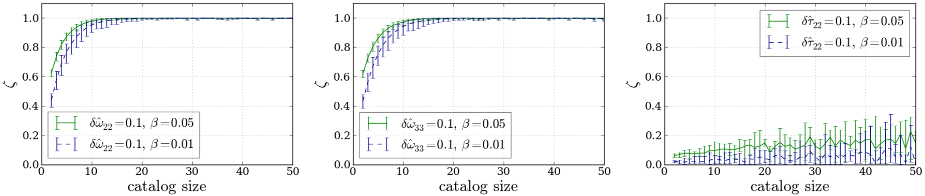




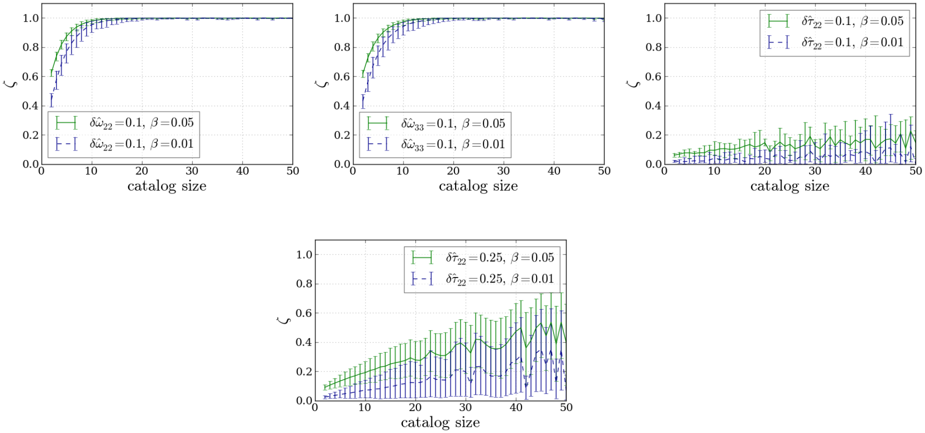
Single sources vs. combined sources



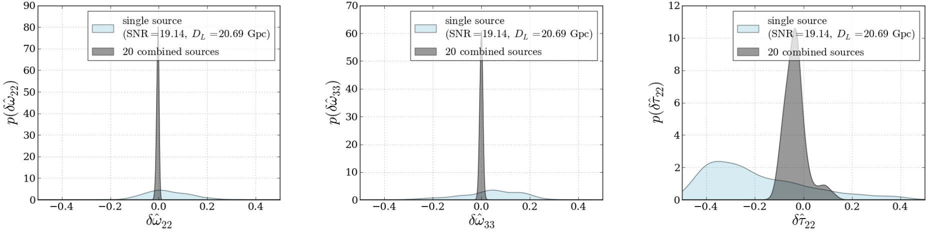
Efficiency


ζ
catalog size
$\delta\hat{\omega}_{22}=0.1,\ \beta=0.05$
$\delta\hat{\omega}_{22}=0.1,\ \beta=0.01$
$\delta\hat{\omega}_{33}=0.1,\ \beta=0.05$
$\delta\hat{\omega}_{33}=0.1,\ \beta=0.01$
$\delta\hat{\tau}_{22}=0.1,\ \beta=0.05$
$\delta\hat{\tau}_{22}=0.1,\ \beta=0.01$

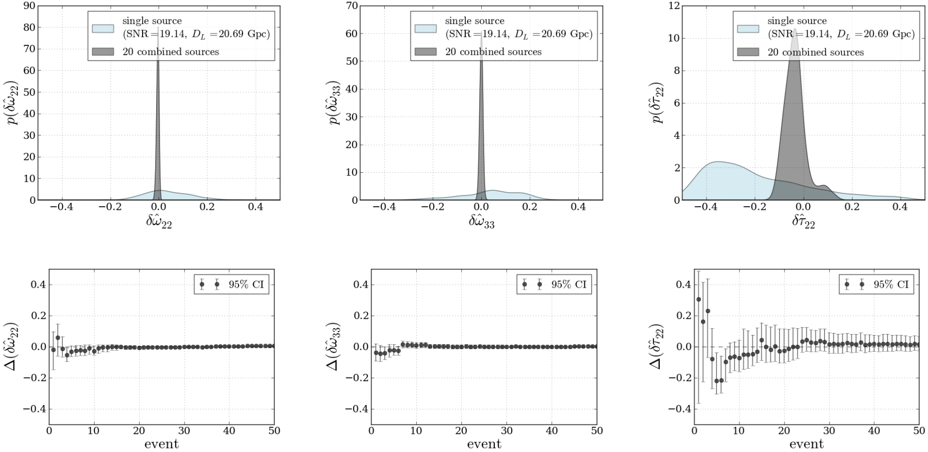
Efficiency


ζ
catalog size
δω̂22 = 0.1, β = 0.05
δω̂22 = 0.1, β = 0.01
δω̂33 = 0.1, β = 0.05
δω̂33 = 0.1, β = 0.01
δτ̂22 = 0.1, β = 0.05
δτ̂22 = 0.1, β = 0.01
δτ̂22 = 0.25, β = 0.05
δτ̂22 = 0.25, β = 0.01

Parameter estimation



Parameter estimation



Summary

- Previous work has shown how deviation of a few percent in ω_{22}, ω_{33} and τ_{22} could be observed up to $z \sim 1$ ($D_\mathrm{L} \sim 6$ Gpc).
- We used the TIGER framework to maximally exploit the available set of detections from ET in a low SNR scenario.
- We show that a deviation of a few percent can be observed up to $z \sim 5$ ($D_\mathrm{L} \sim 50$ Gpc), when combining $\mathcal{O}(10)$ sources.
- When GR is not in doubt, upper limits can be placed on deviations.

Thank you

- For violations of the no-hair theorem we found the following efficiencies:

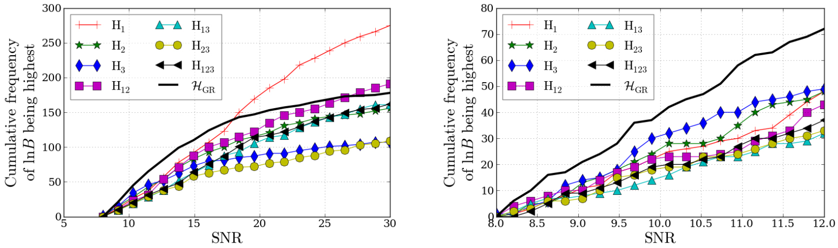
	single source	10 sources	50 sources
$\delta\hat{\omega}_{22} = 0.10$	$\sim 60 - 65\%$	$> 98\%$	$> 99\%$
$\delta\hat{\omega}_{33} = 0.10$	$\sim 60 - 65\%$	$> 95\%$	$> 99\%$
$\delta\hat{\tau}_{22} = 0.10$	$\sim 7 - 10\%$	$\sim 7 - 18\%$	$\sim 7 - 18\%$
$\delta\hat{\tau}_{22} = 0.25$	$\sim 10 - 14\%$	$\sim 12 - 30\%$	$\sim 20 - 70\%$

- When GR is not in doubt, the following constraints could be placed on the free parameters:

	single source	20 sources	50 sources
$\delta\hat{\omega}_{22}$	0.10	0.0051	~ 0.001
$\delta\hat{\omega}_{33}$	0.13	0.0051	~ 0.001
$\delta\hat{\tau}_{22}$	0.21	0.0480	~ 0.1

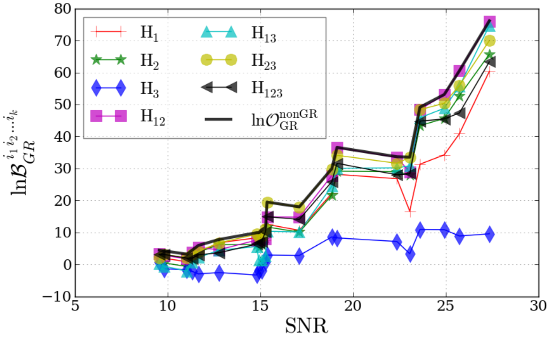
More advantages to using TIGER

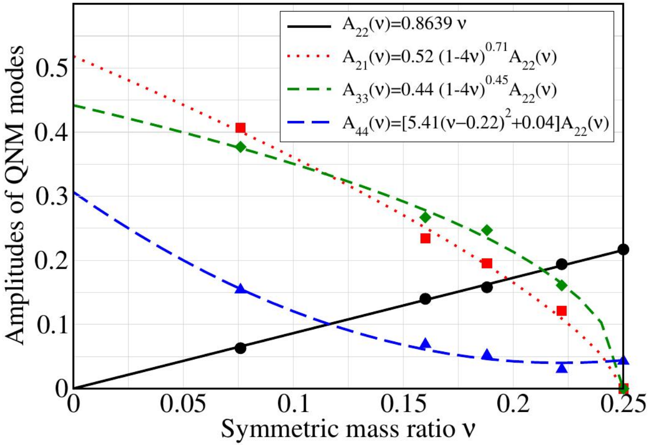
- Combining sources greatly improves ability to test the no-hair theorem.
- Using multiple subhypotheses has a significant impact in finding deviations.



More advantages to using TIGER

- As sources are combined, two observations can be made:
  1. H_{123} ($\{\delta\hat{\omega}_{22}, \delta\hat{\omega}_{33}, \delta\hat{\tau}_{22}\}$ are left free) is depricated compared to some others!
  2. The correct hypothesis is not always the most dominant one!





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