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# Strong lensing of gravitational waves from double compact binaries - predictions for the Einstein Telescope

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BASED ON
results obtained with:

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UNIVERSITY OF SILESIA
IN KATOWICE

# Einstein Telescope


University of Silesia has joined the Polish Einstein Telescope Consortium

- Increased sensitivity
great expectations

- Big catalogs of inspiral events
up to cosmological distances

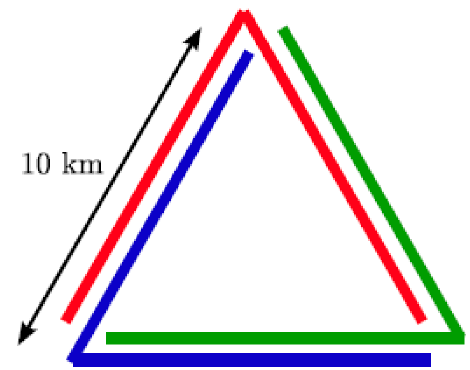


**Figure 6:** Three nested detectors in a triangular arrangement will form the final Einstein Telescope geometry.

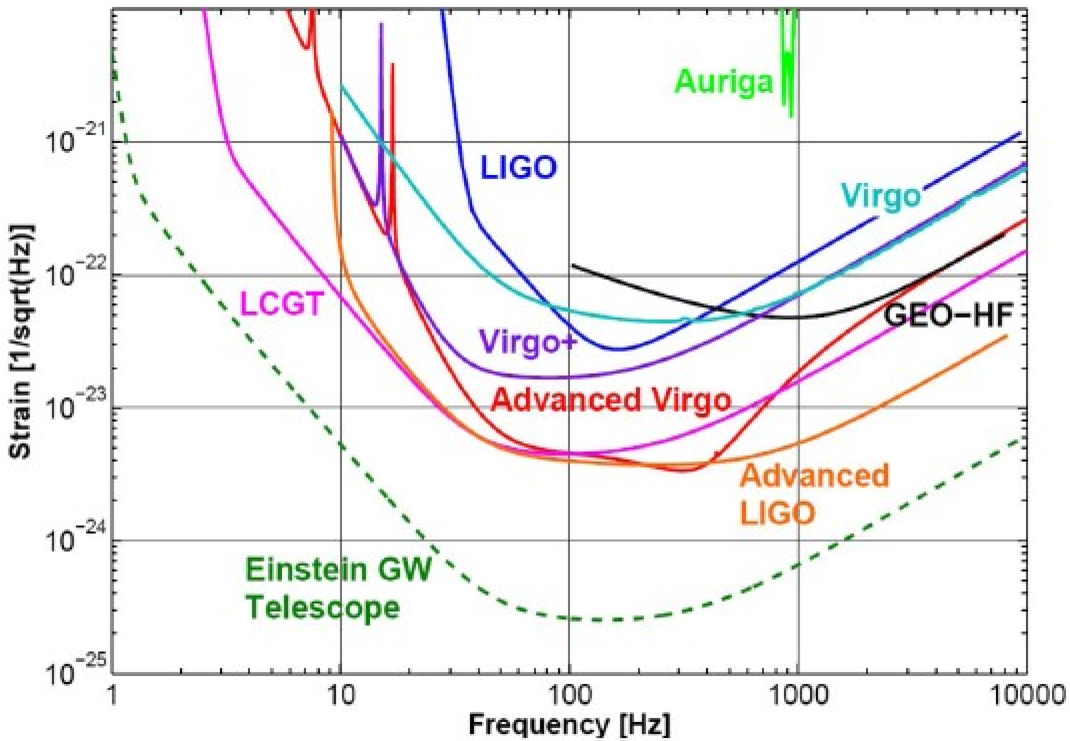


**Figure 5:** Sensitivities of gravitational wave detectors from the first to the third generation.

From ET

Design Study

**Table 2:** Expected coalescence rates per $\mathrm{Mpc}^3$ per Myr in the local universe ($z \simeq 0$). Also shown are predicted event rates in Advanced LIGO (aLIGO) and ET.

| Source | BNS | NS-BH | BBH |
|---|---|---|---|
| Rate ($\mathrm{Mpc}^{-1}$ $\mathrm{Myr}^{-1}$) | 0.1–6 | 0.01–0.3 | $2 \times 10^{-3}$–0.04 |
| Event Rate ($\mathrm{yr}^{-1}$) in aLIGO | 0.4–400 | 0.2–300 | 2–4000 |
| Event Rate ($\mathrm{yr}^{-1}$) in ET | $\mathcal{O}(10^3$–$10^7)$ | $\mathcal{O}(10^3$–$10^7)$ | $\mathcal{O}(10^4$–$10^8)$ |

ET will see sources to much higher distances than 2nd generation experiments

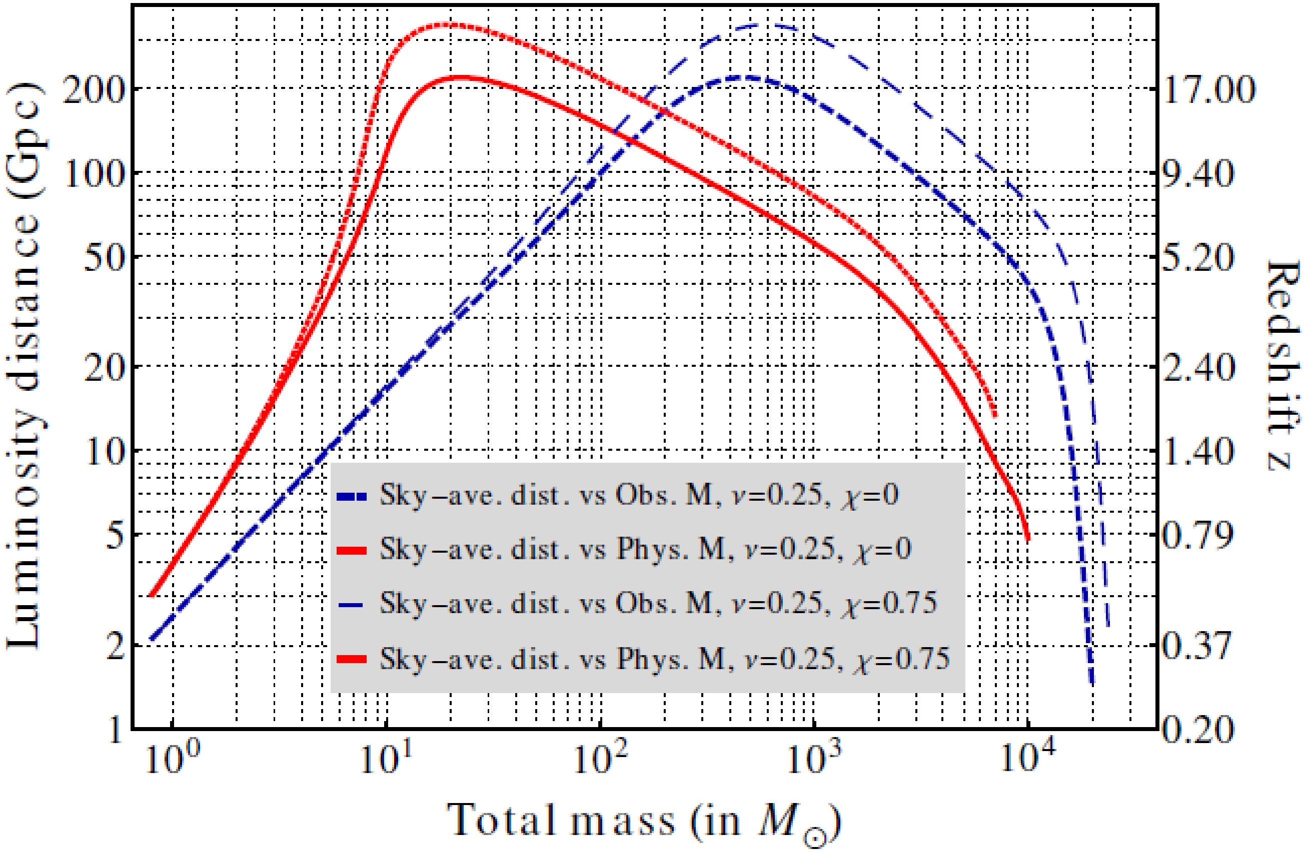


**Figure 18:** ET's distance reach for signals from coalescing compact binaries as a function of the *intrinsic* (red curves) and *observed* (blue curves) total mass, averaged over sky position and binary's orientation relative to the line-of-sight. We assume that a source is visible if it produces an SNR of at least 8 in ET. Solid red and short-dashed blue curves correspond to binaries composed of non-spinning objects. Dotted red and long-dashed blue curves correspond to binaries composed of objects whose spins are aligned with the orbital angular momentum of the binary, with spin parameter $\chi = 0.75$.


# Einstein Telescope

- Increased sensitivity
  great expectations

- Big catalogs of inspiral events up to cosmological distances

- Multi-messenger astrophysics

- Some of them would be gravitationally lensed

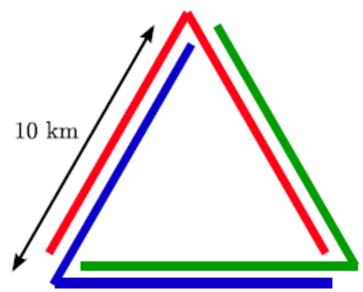


**Figure 6:** Three nested detectors in a triangular arrangement will form the final Einstein Telescope geometry.

# Based on papers

Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

## Strong gravitational lensing of gravitational waves from double compact binaries — perspectives for the Einstein Telescope

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**Abstract.** Gravitational wave (GW) experiments are entering their advanced stage which should soon open a new observational window on the Universe. Looking into this future, the Einstein Telescope (ET) was designed to have a fantastic sensitivity improving significantly

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## Strong gravitational lensing of gravitational waves in Einstein Telescope

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**Abstract.** Gravitational wave experiments have entered a new stage which gets us closer to the opening a new observational window on the Universe. In particular, the Einstein

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# 1. DCO rate prediction

$$\dot N(>\rho_0|z_s) = \int_0^{z_s} \frac{d\dot N(>\rho_0)}{dz} dz$$

yearly detection rate

Rate of inspiral events originating from sources in the redshift interval [z, z+ dz]

$$\frac{d\dot N(>\rho_0)}{dz_s} = 4\pi\left(\frac{c}{H_0}\right)^3 \frac{\dot n_0(z_s)}{1+z_s}\,\frac{\bar r^2(z_s)}{E(z_s)}\,C_\Theta(x(z_s))$$

Detector's reach

Original configuration $r_0 = 1527\ Mpc$

„xylophone" configuration $r_0 = 1918\ Mpc$

cosmology - ΛCDM

$$E(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$$

intrinsic inspiral rate

StarTrack population synthesis evolutionary code

DOUBLE COMPACT OBJECTS II: COSMOLOGICAL MERGER RATES


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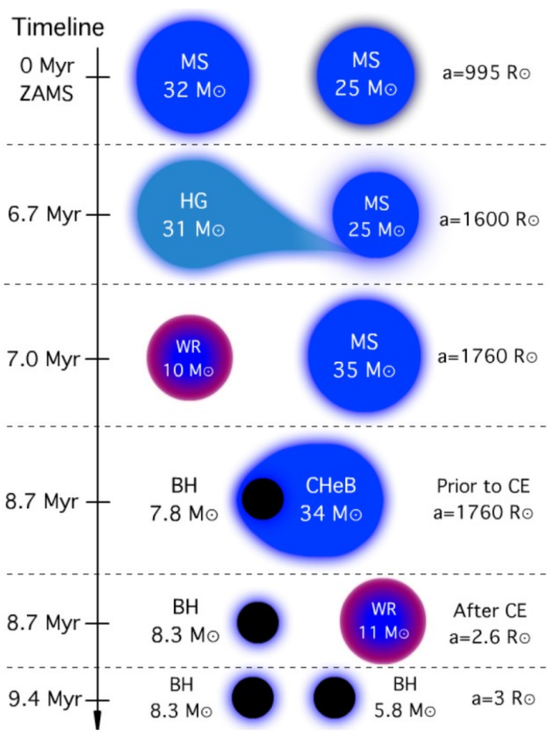
Data available from https://www.syntheticuniverse.org

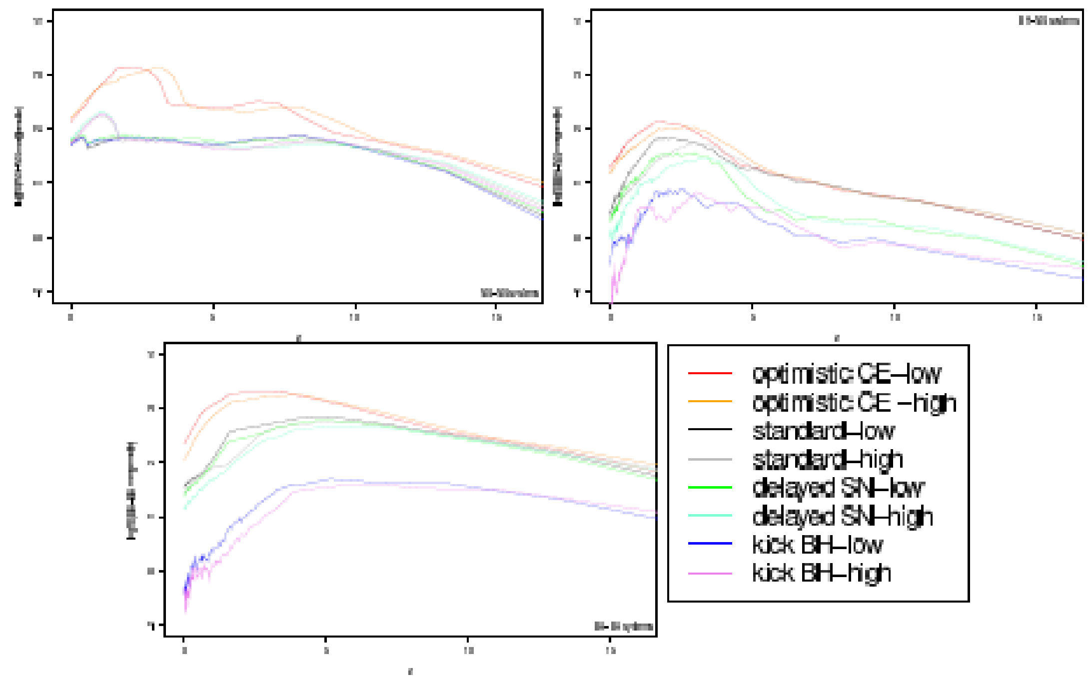
# Merger rates according to M. Dominik et al. 2013

Thoroughly motivated assumptions:

•star formation rates
•galaxy mass distribution
•stellar populations and their metallicities
•galaxy metallicity evloution „high-end” „low-end”




**StarTrack**




Four binary evolution scenarios:

•Standard
•Optimistic Common Envelope (OCE)
•Delayed SN explosion
•High BH kick

We assumed median values of DCO chirp masses

| Yearly detection rate | standard | optimistic CE | delayed SN | high BH kicks |
| --- | --- | --- | --- | --- |
| $\dot{N}(>\rho_0)$ $[yr^{-1}]$ for NS-NS | | | | |
| initial design | 3834.8 | 11631.8 | 4303.4 | 3683.1 |
| "xylophone" design | 6948.8 | 23399.3 | 7832.8 | 6879.7 |
| $\dot{N}(>\rho_0)$ $[yr^{-1}]$ for BH-NS | | | | |
| initial design | 3468.1 | 11539.3 | 1939.9 | 434.1 |
| "xylophone" design | 6004.5 | 17704.5 | 3302.5 | 715.8 |
| $\dot{N}(>\rho_0)$ $[yr^{-1}]$ for BH-BH | | | | |
| initial design | 105703.3 | 494917.5 | 81848.4 | 4955.6 |
| "xylophone" design | 144801.9 | 619555.5 | 113879.8 | 7411.1 |
| $\dot{N}(>\rho_0)$ $[yr^{-1}]$ for total | | | | |
| initial design | 113006.2 | 518088.5 | 88091.7 | 9072.8 |
| "xylophone" design | 157755.2 | 660659.3 | 125015.1 | 15006.6 |

**Table 1.** Yearly detection rate of inspiralling DCOs of different classes under different evolutionary scenarios. "High-end" metallicity evolution assumed. Predictions for the Einstein Telescope in the initial and "xylophone" configuration.

„low-end" metallicity scenario predictions are ca. 47% higher (for total)

# 2. Lensing statistics

SIS lens model – the simplest realistic case

Typical scale for galaxy lens = 1”

will not be resolved in ET

1D velocity dispersion

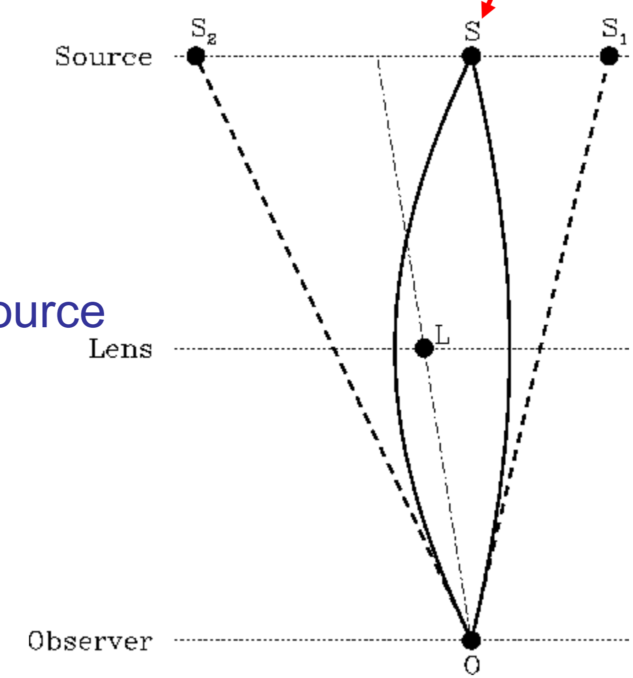
Einstein radius
$$\vartheta_E = 4\pi \frac{D_{ls}}{D_s} \frac{\sigma^2}{c^2}$$

Two images form on the opposite side of the lens

$$R_A = \beta + \vartheta_E \qquad R_B = \vartheta_E - \beta$$

angle between directions to the lens and to the source

source




Time delay between images in SIS lens

$$\Delta t_{SIS} = \frac{1+z_l}{2c} \frac{D_l D_s}{D_{ls}} (R_A^2 - R_B^2)$$

$$\Delta t_{SIS} = \frac{2(1+z_l)}{c} \frac{D_l D_s}{D_{ls}} \vartheta_E \beta = \frac{8\pi}{H_0} \tilde{r}_l \beta \frac{\sigma^2}{c^2}$$

The only way to see lensed GW source - **register 2 time delayed waveforms**
(of similar temporal structure i.e. frequency drift)
with different amplitudes

$$h_{\pm} = \sqrt{\mu_{\pm}}\, h(t) = \sqrt{\frac{1}{y} \pm 1}\, h(t)$$

Time delay between signals

$$\Delta t = \frac{32\pi^2}{c} \left(\frac{\sigma}{c}\right)^4 \frac{d_A(z_l) d_A(z_l, z_s)}{d_A(z_s)} (1 + z_l) y = \frac{32\pi^2}{H_0} \left(\frac{\sigma}{c}\right)^4 \frac{\tilde{r}_l \tilde{r}_{ls}}{\tilde{r}_s} y = \Delta t_0 y$$

Second image would be observed if it is magnified above the detector's threshold

$$\sqrt{\frac{1}{y} - 1}\, \rho_{intr.} \geq 8.$$

↑

intrinsic SNR

Hence the impact parameter should be

$$y \leq y_{max} := \left[1 + \left(\frac{8.}{\rho_{intr.}}\right)^2\right]^{-1}$$

$$S_{cr} = \pi \theta_E^2 y_{max}^2$$

## Elementary cross section

$$S_{cr} = 16\pi^3 \left(\frac{\sigma}{c}\right)^4 \left(\frac{\tilde{r}_{ls}}{\tilde{r}_s}\right)^2 y_{max}^2$$

## Differential lensing cross section

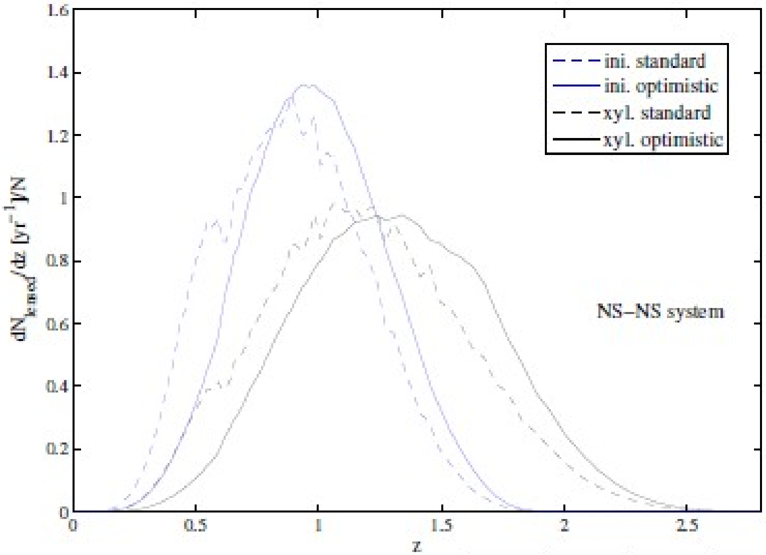
$$\frac{d^2\tau}{d\sigma dz_l} = \frac{dn}{d\sigma} S_{cr}(\sigma, z_l, z_s) \frac{dV}{dz_l} = 4\pi \left(\frac{c}{H_0}\right)^3 \frac{\tilde{r}_l^2}{E(z_l)} S_{cr}(\sigma, z_l, z_s) \frac{dn}{d\sigma}$$

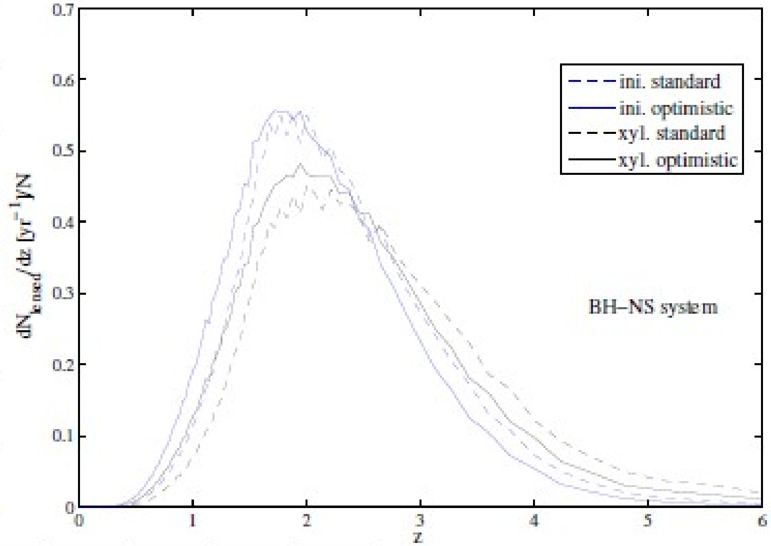
$$\frac{dn}{d\sigma} = n_* \left(\frac{\sigma}{\sigma_*}\right)^\alpha \exp\left(-\left(\frac{\sigma}{\sigma_*}\right)^\beta\right) \frac{\beta}{\Gamma(\frac{\alpha}{\beta})} \frac{1}{\sigma}$$

Schechter function – parameters after Choi et al. 2007

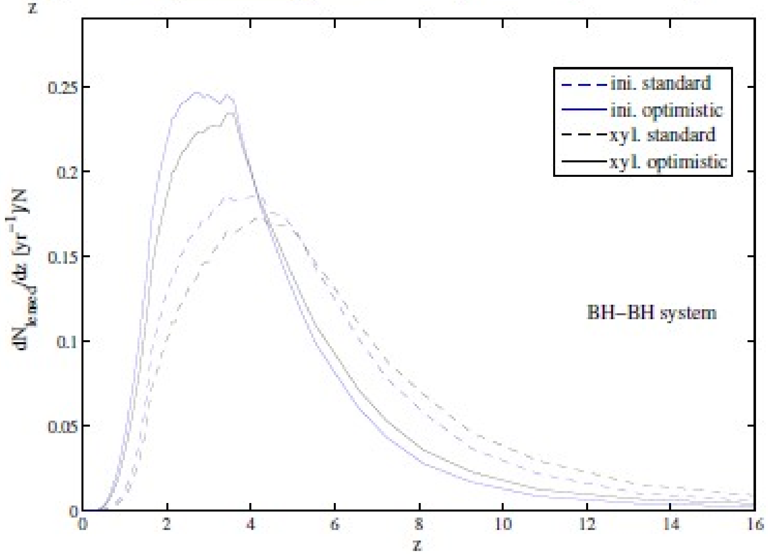
| | $n_*$ [$Mpc^{-3}$ ] | $\sigma_*$ [km/s] | α | β |
|---|---|---|---|---|
| Choi et al. 2007 | 8. $10^{-3}$ $(H_0/100)^3$ | 161 | 2.32 | 2.67 |
| Mitchell et al. 2005 | 4.1 $10^{-3}$ $(H_0/100)^3$ | 88.8 | 6.5 | 1.93 |
| Bernardi et al. 2013 | 2.6 $10^{-3}$ $(H_0/70)^3$ | 159.6 | 0.41 | 2.59 |

# Differential lensing rate of DCO inspiralling binaries




$$\frac{1}{\dot{N}_{\rm lensed}} \frac{d\dot{N}_{\rm lensed}}{dz}$$



Our considerations so far were strictly valid for infinitely long survey. If the survey lasts a finite time $T_{surv}$ some of the lensed transient signals (i.e. those occuring near the start or the end of the survey) would be missed because of the time delay (3.2). Consequently the cross section for lensing should be modified appropriately (see e.g. [20, 24]):

$$S_{cr} = 2\pi\theta_E^2 \int_0^{y_{max}} f(\Delta t(y, z_l, z_s, \sigma))\; y\; dy \tag{3.11}$$

Under assumption of uniform distribution of arrival times, one can write: $f(\Delta t) = 1 - \frac{\Delta t}{T_{surv}}$, and:

$$S_{cr} = \pi\theta_E^2 \left( y_{max}^2 - \frac{2}{3}\frac{\Delta t_0}{T_{surv}} y_{max}^3 \right)$$

Substituting this to (3.7), after calculating the integrals:

$$\mathcal{F}_8 = \int_0^{\infty} \left(\frac{\sigma}{c}\right)^8 \frac{dn}{d\sigma} d\sigma = n_* \left(\frac{\sigma_*}{c}\right)^8 \frac{\Gamma\left(\frac{8+\alpha}{\beta}\right)}{\Gamma\left(\frac{\alpha}{\beta}\right)}$$

$$\int_0^{z_s} \frac{\tilde{r}_{ls}^3 \tilde{r}_l^3}{\tilde{r}_s^3 E(z_l)} dz_l = \int_0^{\tilde{r}_s} \frac{\tilde{r}_{ls}^3 \tilde{r}_l^3}{\tilde{r}_s^3} d\tilde{r}_l = \frac{1}{140}\tilde{r}_s^4$$

and denoting: $\mathcal{F}_* = 16\pi^3 \mathcal{F}_4$ and $\Delta t_* = \frac{32\pi^2}{H_0}\tilde{r}_s \left(\frac{\sigma_*}{c}\right)^4 y_{max}$ one has:

$$\tau_{\Delta t} = \frac{\mathcal{F}_*}{30}\left(\frac{c}{H_0}\right)^3 \tilde{r}_s^3 y_{max}^2 \left[1 - \frac{1}{7}\frac{\Gamma\left(\frac{\alpha+8}{\beta}\right)}{\Gamma\left(\frac{\alpha+4}{\beta}\right)}\frac{\Delta t_*}{T_{surv}}\right] = \tau \left[1 - \frac{1}{7}\frac{\Gamma\left(\frac{\alpha+8}{\beta}\right)}{\Gamma\left(\frac{\alpha+4}{\beta}\right)}\frac{\Delta t_*}{T_{surv}}\right] \tag{3.12}$$

# Predictions of lensed inspiral signals

| ET configuration $T_{surv}$ | standard (1yr; 5yrs;continuous) | optimistic (1yr; 5yrs;continuous) | delayed SN (1yr; 5yrs;continuous) | high BH kicks (1yr; 5yrs;continuous) |
|---|---|---|---|---|
| NS-NS | | | | |
| initial design | (0.06; 0.07; 0.07) | (0.2; 0.2; 0.2) | (0.07; 0.07; 0.08) | (0.06; 0.06; 0.07) |
| xylophone | (0.2; 0.2; 0.2) | (0.7; 0.8; 0.8) | (0.2; 0.2; 0.2) | (0.2; 0.2; 0.2) |
| BH-NS | | | | |
| initial design | (0.4; 0.5; 0.5) | (1.1; 1.3; 1.3) | (0.2; 0.3; 0.3) | (0.05; 0.05; 0.05) |
| xylophone | (0.9; 1.1; 1.1) | (2.1; 2.4; 2.5) | (0.5; 0.6; 0.6) | (0.1; 0.1; 0.1) |
| BH-BH | | | | |
| initial design | (30.3; 36.1; 37.6) | (99.1; 116.0; 120.2) | (24.7; 29.5; 30.7) | (1.8; 2.2; 2.3) |
| xylophone | (45.8; 54.9; 57.2) | (136.7; 160.8; 166.8) | (37.8; 45.4; 47.3) | (3.0; 3.6; 3.8) |
| TOTAL | | | | |
| initial design | (30.8; 36.7; 38.2) | (100.4; 117.4; 121.7) | (25.0; 29.8; 31.0) | (1.9; 2.3; 2.4) |
| xylophone | (46.9; 56.2; 58.5) | (139.5; 164.2; 170.1) | (38.5; 46.2; 48.1) | (3.3; 4.0; 4.1) |

**Table 3.** Expected numbers of lensed GW events from inspiralling DCOs of different classes under different evolutionary scenarios. "High-end" metallicity evolution assumed. Predictions for the Einstein Telescope in the initial and "xylophone" configuration.

BH-BH systems contribute 91 – 95%; NS-NS systems 1 – 4%

| Velocity Distribution Function | Mitchell | Choi | Bernardi |
|---|---|---|---|
| NS-NS systems | | | |
| initial design | | | |
| standard scenario | 0.05 | 0.07 | 0.09 |
| optimistic OCE | 0.2 | 0.2 | 0.3 |
| "xylophone" design | | | |
| standard scenario | 0.1 | 0.2 | 0.2 |
| optimistic OCE | 0.6 | 0.8 | 1. |
| BH-NS systems | | | |
| initial design | | | |
| standard scenario | 0.4 | 0.5 | 0.7 |
| optimistic CE scenario | 0.9 | 1.3 | 1.7 |
| "xylophone" design | | | |
| standard scenario | 0.8 | 1.1 | 1.4 |
| optimistic CE scenario | 1.8 | 2.5 | 3.2 |
| BH-BH systems | | | |
| initial design | | | |
| standard scenario | 26.5 | 37.6 | 48.2 |
| optimistic CE scenario | 84.8 | 120.2 | 153.8 |
| "xylophone" design | | | |
| standard scenario | 40.3 | 57.2 | 73.2 |
| optimistic CE scenario | 117.6 | 166.8 | 213.6 |

# Dependence of lensed GW events on the assumed VDF

differences are noticeable

only for BH-BH systems

# If we discover such lensed GW source

1. Identification of double images: time delayed, the same waveform, different amplitudes.

$$h_{\pm} \propto \frac{A_{\pm}}{d_L}$$

2. Amplitude ratio → source position y

$$y = \frac{1 - |A_-/A_+|}{1 + |A_-/A_+|}.$$

3. Known y means known amplification → infer luminosity distance $d_L(z_s)$

Breaks degeneracy w.r.t. y in time delay → distance ratio $\frac{\tilde{r}_l \tilde{r}_{ls}}{\tilde{r}_s}$

4. Seek for optical galaxy (now we approximately know at which $z_l$ )

5. If found – spectroscopical follow up …

# Other perspectives:

Study the amplification bias in the GW catalogs expected from E.T.