Accurate SNR estimates and parameter estimation calculations for ET IMRIs

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Outline

- Motivation to study IMRIs in the context of the ET
- Basic ingredients to perform a Fisher Matrix Analysis
- Beyond inspiral
- Results from selected cases
- Conclusions and future work

Motivation

- To compute estimates of the accuracy with which ET can measure the parameters for IMRIs - the inspirals of small compact objects (CO) (1.4 - 10 $M_\odot$) into black holes of intermediate mass (IMBH).

- In particular: can we obtain reasonable estimates for the binary's extrinsic parameters, i.e., its location in the sky?

- Is a single ET sufficient? Does an IMRI spend enough time in band for the motion of the Earth to provide extrinsic parameter determination?

- A single ET WILL NOT provide satisfactory estimates for the extrinsic parameters, e.g., for a (10 + 100) $M_\odot$ system , $\Delta\theta_s$ ~ 300 and $\Delta\phi_s$ ~ 200, for q = 0.9 and D = 6Gpc.

Ingredients for FM analysis

- Response function - one or two right-angle interferometers at each site, but need to change the angles describing source direction and orientation using the actual location of the various detectors on the Earth.
- Network of ETs, computation of FMs using the appropriate timelags.
- Considering inspiral-only waveforms, an ET network does provide accurate estimates for the extrinsic parameters for IMRIs (Jon's talk).

ET			Network		
$\Delta\theta_s$	$\Delta\phi_s$	SNR	$\Delta\theta_s$	$\Delta\phi_s$	SNR
318	189	248	0.06	0.08	571

Ingredients for FM analysis

- For the binary systems under consideration, not only the inspiral but also the merger and ringdown parts contribute significantly to the SNR (Ajith, CQG, 2009).

- Need to extend this model to implement merger and ringdown consistently.

- To start with we build a model for the inspiral, merger and ringdown of a stellar mass CO onto a non-spinning black hole of intermediate mass.

Beyond inspiral

- Inspiral model taken from Huerta & Gair, 2009. We include the dominant l=2, m=2 and l=2, m=-2 modes. The kludge waveform is computed using the relation

$$\mathrm{h}_{\mathrm{inspiral}} = -(h_+ - i h_\times) = {}_{-2}Y_{22}(\theta,\phi)h_{22}(t) + {}_{-2}Y_{2-2}(\theta,\phi)h_{2-2}(t)$$

- where the angular functions are the spin weight -2 spherical harmonics. This relation reproduces the standard quadrupole approximation

$$h_{\mathrm{inspiral}} = \frac{-4\mu}{D}\,(M\Omega)^{2/3}\left(\frac{1+\cos^2(\theta)}{2}\cos(2\phi) - i\cos(\theta)\sin(2\phi)\right)$$

Beyond inspiral

- To model the merger part we use the Effective One Body approach (EOB) following Buonanno, et al. 2007.

- The various coefficients of the metric tensor

$$ds^2_{\mathrm{eff}} = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

are 3PN Padé approximants. The effective and real Hamiltonians to be considered are

$$H_{\mathrm{eff}} = \mu\sqrt{A(r)\left(1 + \frac{p_\phi^2}{r^2} + \frac{p_r^2}{B(r)} + z_3\frac{p_r^4}{r}\right)} \qquad z_3 = 2(4-3\eta)\eta$$

$$H_{\mathrm{real}} = M\sqrt{1 + 2\eta\left(\frac{H_{\mathrm{eff}} - \mu}{\mu}\right)}$$

Beyond inspiral

- The new radial coordinate and gravitational phase for the merger are computed using the Hamilton equations.

- Assume CO follows a geodesic with fixed angular momentum and energy during plunge, i.e.,

$$\frac{d\mathrm{r}}{dt} = a_1 A(r) \left(\frac{a_2}{B(r)} + \frac{a_3}{r^2} \right), \quad \omega_{\mathrm{plunge}} = a_4 \frac{A(r)}{r^2}$$

- The a_i's depend on the angular and radial momenta, and the real and effective Hamiltonians evaluated at the LSO.

- Use these expressions to build the waveform

Beyond inspiral

- Dominant modes taken into account are

$$h_{22}^{\mathrm{merger}} = -8\sqrt{\frac{\pi}{5}}\frac{\mu}{D}\left(r_\omega\Omega\right)^2 F_{22}(t)e^{-2i\phi}, \quad h_{2-2} = h_{22}^*$$

where $r_\omega = r[\psi(r,p_\phi)]^{1/3}$ is a modified EOB radius and the complex quantity

$$F_{22}(t) = \hat{H}_{\mathrm{eff}}T_{22}\rho_{22}^2 e^{i\delta_{22}}$$

represents a resummed version of all the PN corrections. Using this conventions the waveform takes the form

$$h_{\mathrm{merger}} = {}_{-2}Y_{22}(\theta,\phi)_{22}^{\mathrm{merger}} + {}_{-2}Y_{2-2}(\theta,\phi)h_{2-2}^{\mathrm{merger}}$$

Beyond inspiral

- Merger waveform takes the final form

$$h_{\rm merger} = -4C\,\frac{\mu}{D}\,(r_\omega\Omega)^2\left(\frac{1+\cos^2\theta}{2}\cos(2\chi) - i\cos\theta\sin(2\chi)\right)$$

with $\chi = \phi(t) - \frac{1}{2}\epsilon(t),\ F_{22}(t) = Ce^{i\epsilon}$

- Matching inspiral waveform onto merger part is straightforward.

- Attach ringdown (RD) modes when $\omega_{\rm plunge}$ reaches its maximum.

- Use consistent approach by including “twin modes” with frequency $\omega'_{lmn} = -\omega_{l-mn}$ and damping time $\tau'_{lmn} = \tau_{l-mn}.$

Beyond inspiral

- Following Berti, et al. 2006, the RD waveform is given by

$$h_{\rm ringdown} = \frac{M}{D}\sum_{lmn}\{A_{lmn}e^{-i(\omega_{lmn}t+\phi_{lmn})}e^{-t/\tau_{lmn}}S_{lm}(a\omega_{lmn})+$$
$$A'_{lmn}e^{i(\omega_{lmn}t+\phi'_{lmn})}e^{-t/\tau_{lmn}}S^*_{lm}(a\omega_{lmn})\}$$

- Superpose the dominant tone l=m=2, n=0, and two overtones n=1,2.

- Use one-parameter fit functions (Buonanno et al., 2007) to compute the mass and spin of the final black hole

$$M_f/M = 1+\left(\sqrt{8/9}-1\right)\eta - 0.498\eta^2, \quad a_f/M_f = \sqrt{12}\eta - 2.90\eta^2$$

Beyond inspiral

- Using data from Berti, et al, 2006, build interpolating functions to obtain the various frequencies and decay times

- Compute the spin-weight -2 spheroidal harmonics using the expansion (Poisson, 1995)

$$ {}_{-2}S_{lm}(a\omega_{lmn}) = {}_{-2}Y_{lm} + a\omega_{lmn} S^{(1)}_{lm}(a\omega_{lmn}) $$

$$ S^{(1)}_{lm}(a\omega_{lmn}) = \sum_{l'} c^{l'}_{lm} {}_{-2}Y_{l'm} $$

- Plug these expansions into the waveform and split it into plus and cross polarizations.

- Match plus and cross RD polarizations onto their merger counterparts (24 constants in total!!).

Beyond inspiral

- Compute the various constants evaluating the waveforms and the appropriate high order derivatives at three points

$$t_{\text{light ring}} - dt, \quad t_{\text{light ring}}, \quad t_{\text{light ring}} + dt$$

- Compute overtones' constants at later times and use them as seeds to obtain leading tone's constants at $t_{\text{light ring}} - dt$.

- Add the SNRs from the three waveform phases in quadrature and test convergence of Fisher Matrix estimates.

Results

- Contribution to the SNR by inspiral (I), inspiral plus merger (I + M) and inspiral plus merger plus ringdown (A) for four different configurations.

η	$M/M_\odot$	I	I + M	A
0.014	100	18.98	26.99	31.77
0.083	100	52.58	96.49	183.31
0.003	500	3.20	17.92	18.01
0.019	500	8.62	73.69	74.88

Results

- Fisher matrix estimates

	Mass ratio/IMBH mass			
Parameter	0.014/100	0.083/100	0.0028/500	0.019/500
Δm	0.00036	0.0036	0.0014	0.0048
ΔM	0.00037	0.0035	0.000180	0.00063
$\Delta \phi_0$	0.0068	0.066	0.0018	0.0059
$\Delta \theta_s$	0.044	0.088	0.056	0.056
$\Delta \phi_s$	0.020	0.16	0.26	0.26
$\Delta \theta_k$	0.12	0.12	0.38	0.29
$\Delta \phi_k$	0.20	0.19	0.47	0.34
ΔD	0.088	0.0086	0.42	0.074

Summary and future work

- We have built a waveform model for IMRIs which includes inspiral, merger and ringdown.

- The model allows us to compute parameter accuracy estimates for a network of ETs. We estimate mass precision up to 0.05%, 0.3 radians for the source's location and 10% for the distance to the source.

- Next step - obtain robust results using Monte Carlo simulations and compare SNRs etc. to other models available in the literature, i.e., Ajith, 2009, and EOB-numerical relativity.

- Extend this model for spinning black holes and perform similar analysis.