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Testing General Relativity Using Bayesian Model Selection

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Introduction

Tests of General Relativity (GR) are among the main science drivers for the ET.

Instrument potential investigations have been performed:

- Fisher information matrix, e.g.:
  - Arun & Will 09;
  - Arun et al. 06;
  - Stavridis & Will 09;
  - Berti, Buonanno & Will 05;

Bayesian Inference

- Bayes Theorem provides a simple and rigorous way of selecting among models
- Difficult computational implementation: large N dimensional integral
- Exploratory study:

  → calculate Bayes factors between competing models;

  → estimate parameters and evaluate biases;

  → combine naturally multiple observations.

- We started considering a Massive Graviton (MG) theory:

  → simplicity

- We developed our method on the Advanced LIGO instrument:

  → computationally inexpensive

Method

Given some data {d} and two models H_i and H_j, depending on a set of parameters $\{\Theta_k\}$, we define the odds ratio O_{ij} as:

$$O_{ij} = \frac{P(H_i \mid d)}{P(H_j \mid d)} = \underbrace{\frac{P(H_i)}{P(H_j)}}_{\text{prior odds}} \frac{P(d \mid H_i)}{P(d \mid H_j)} = \frac{P(H_i)}{P(H_j)} B_{ij}$$

Bayes factor

$$P(d \mid H_{\{i,j\}}) = \int d\vec{\theta}\, p(\vec{\theta})\, p(d \mid H_{\{i,j\}}, \vec{\theta})$$ evidence

- In the frequency domain: $$d(f) = h(f) + n(f)$$

gravitational wave   noise

- n(f) is Gaussian and stationary:

$$p(n) \propto e^{-(n|n)/2} = e^{-(d-h_m|d-h_m)/2} \propto p(d \mid h_m, \vec{\theta})$$

(.|.) is the usual inner product, m=GR,MG.

- We calculate the evidence using the *nested sampling* algorithm described in Veitch & Vecchio 2009, LIGO-P0900117.

Model

- GWs from inspiral binaries:

$$h(f) = 30^{-1/2}\pi^{-2/3}\frac{M^{5/6}}{D_L} f^{-7/6} e^{i\Psi(f)}$$

and:

$$\Psi(f) = 2\pi f t_0 - \phi_0 + \Psi_N(\eta)[\sum_{k=0}^{4}\Psi_k(\eta)(\pi M f)^{(k-3)/5} - \beta_g(\pi M f)^{2/3}]$$

$$\beta_g = \frac{128}{3}\frac{\pi^2 D M}{\lambda_g^2(1+z)}$$

- λ_g: graviton Compton wavelength.
- GR corresponds to the case $\beta_g = 0$.

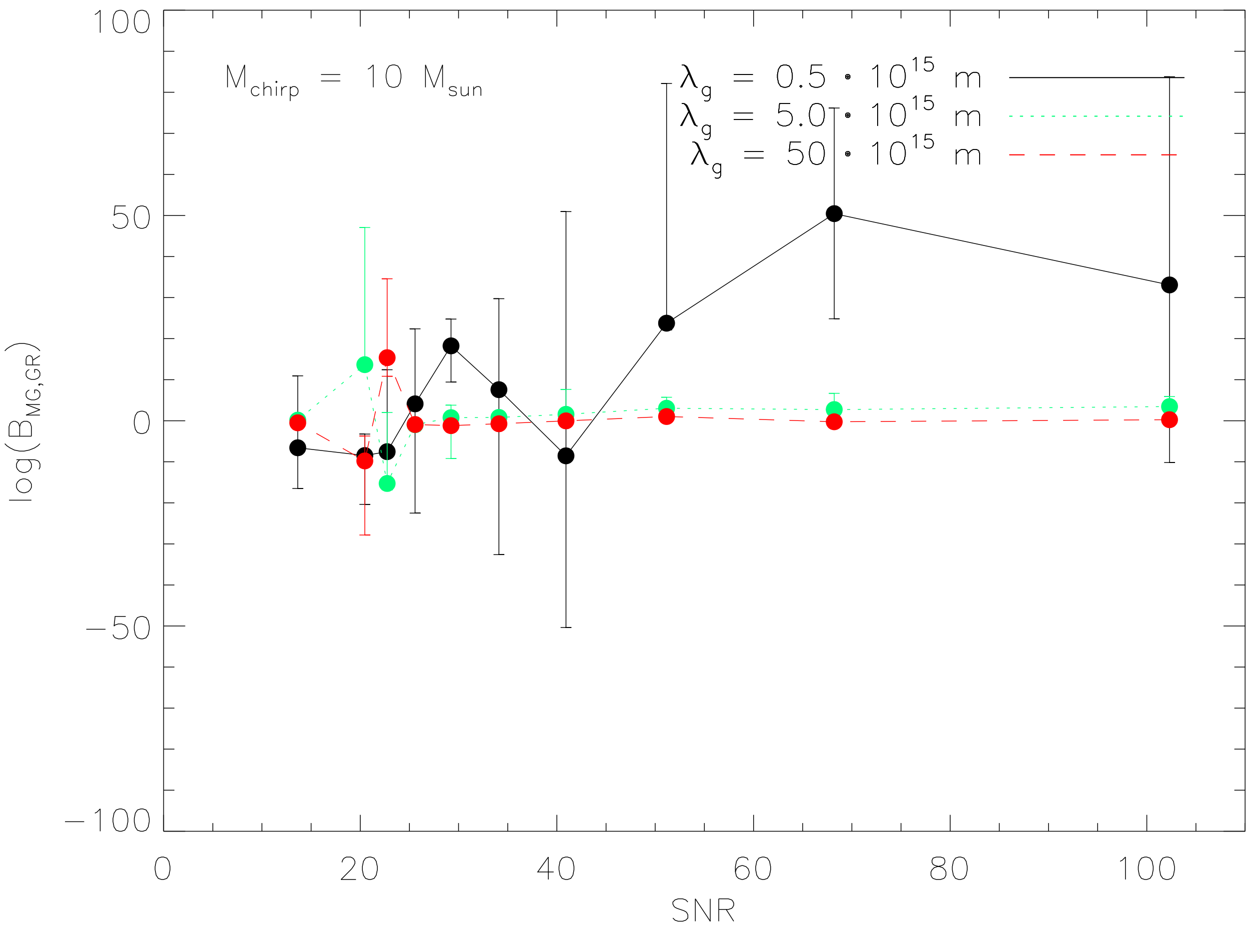
Bayes Factors

- Injected a MG waveform and analyzed the data using both the MG and the GR models.

- Calculated $B_{MG,GR}$ for a range of SNRs, M_{chirp} and 3 values of λ_g.

- All other parameters were kept fixed.

Bayes Factors



Example:
M_{chirp} =10 M_{sun}
η=0.2495

10 realizations of the noise per SNR

NB: From Solar System observations:

$$\lambda_g > 2.8\times10^{15}\,m$$

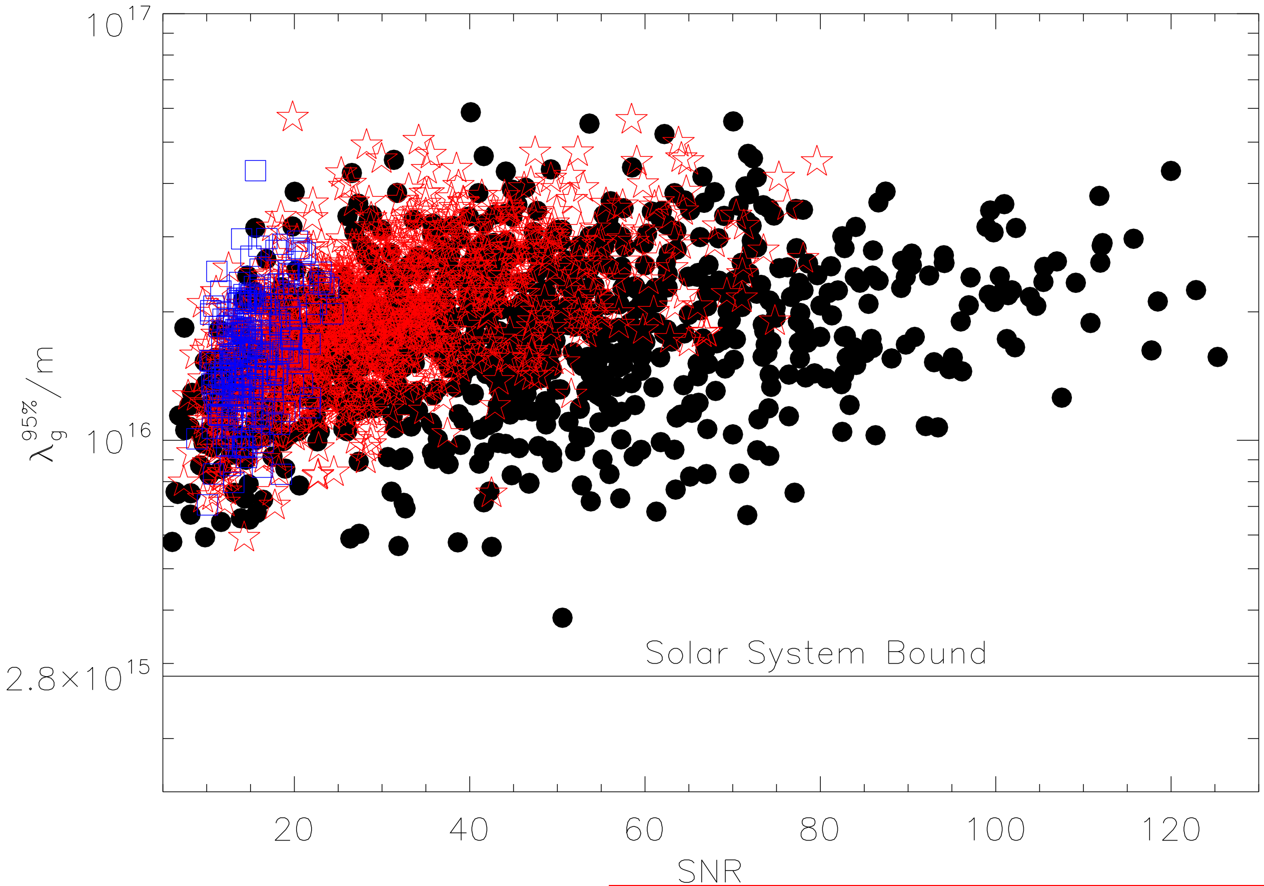
Lower Limit on λ_g

- Simulated 3000 sources using the GR model:
  - 3 values of M_{chirp}
  - Random sky position and polarization/inclination
  - Fixed η=0.2495 and D_L=20Mpc

- Analyzed the data using the MG template

- Calculated the 95% probability interval on λ_g

Lower Limit on λ_g

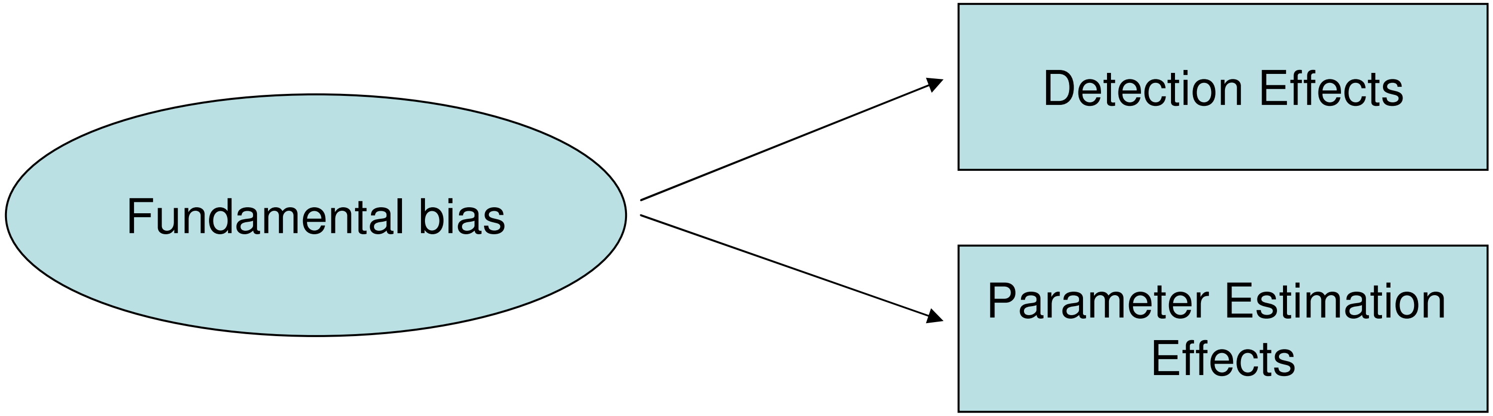


M_{chirp}= 1.2 M_{sun} [blue]
M_{chirp}= 5 M_{sun} [red]
M_{chirp}= 10M_{sun} [black]

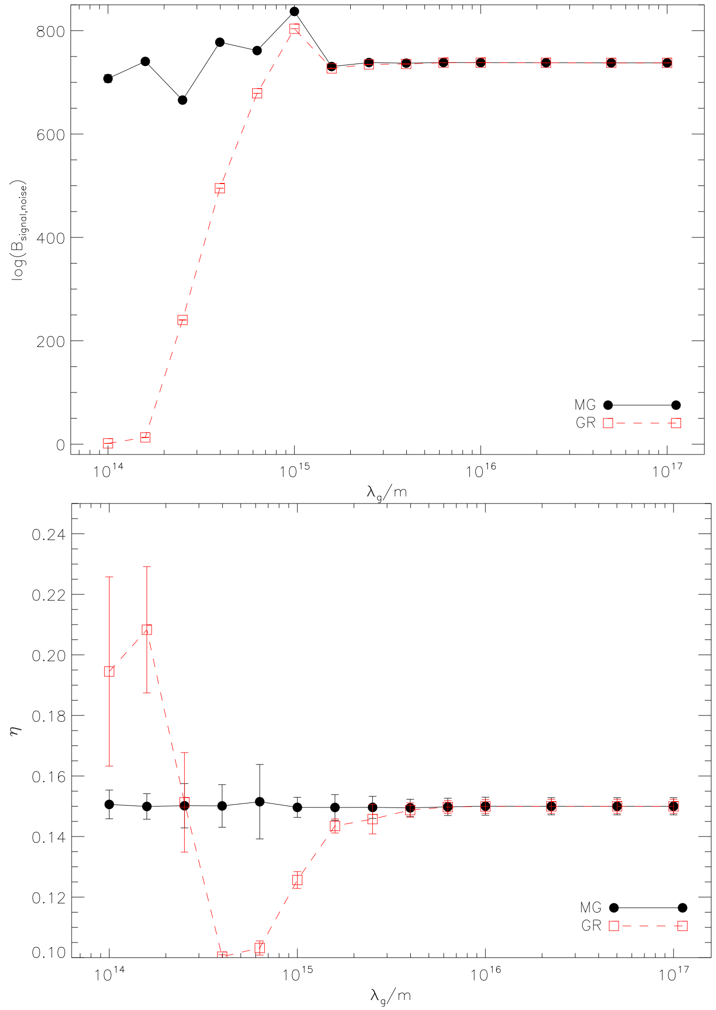
$$\boxed{\bar{\lambda}_g^{95\%} \approx 2\times10^{16}\, m}$$

Bias in GW astronomy

The assumption that GR is the correct theory of Nature might lead to fundamental bias (Yunos & Pretorius, arxiv:09093328).



Bias in the parameter estimation



GW source with the MG model.

SNR=41

if $\lambda_g < 10^{15}$ m GR is affected by bias.

Combination of multiple observations

- ET will observe a large number of sources.
- It is crucial to take advantage of this.
- Bayes theorem offers a natural way to extract information from the combination of independent observations.
- Given N independent observations $\{d_i\}$ in fact:

$$P(\lambda_g \mid d_1,\ldots,d_N) \propto P(\lambda_g)\prod_{i=1}^{N} P(\lambda_g \mid d_i)$$

Combination of multiple observations

- We generated 50 datasets with the GR model.
- All the injection parameters were randomly chosen.
- We analyzed the data using the MG template and we calculated the combined PDF.
- Numerically it is non trivial.

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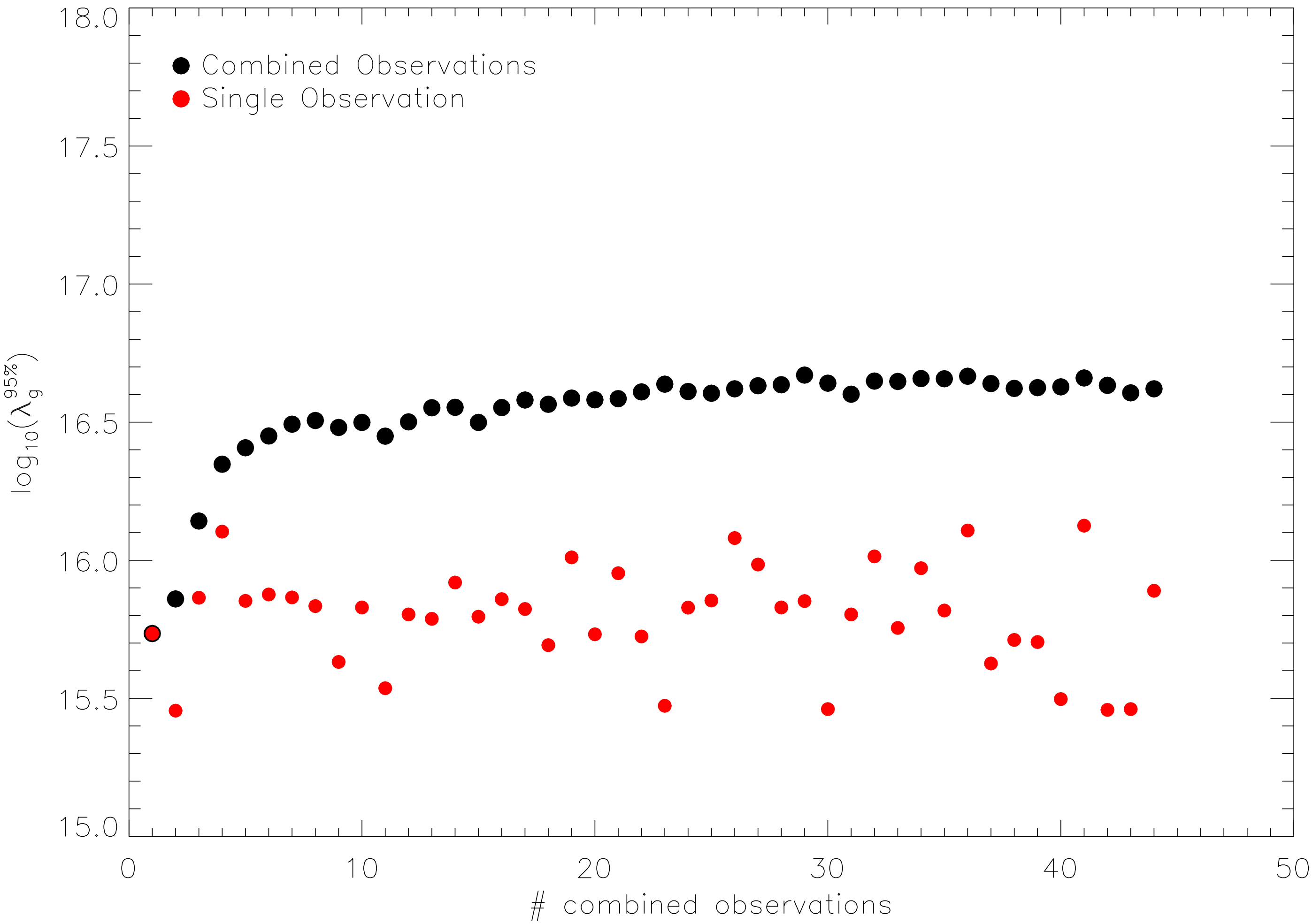
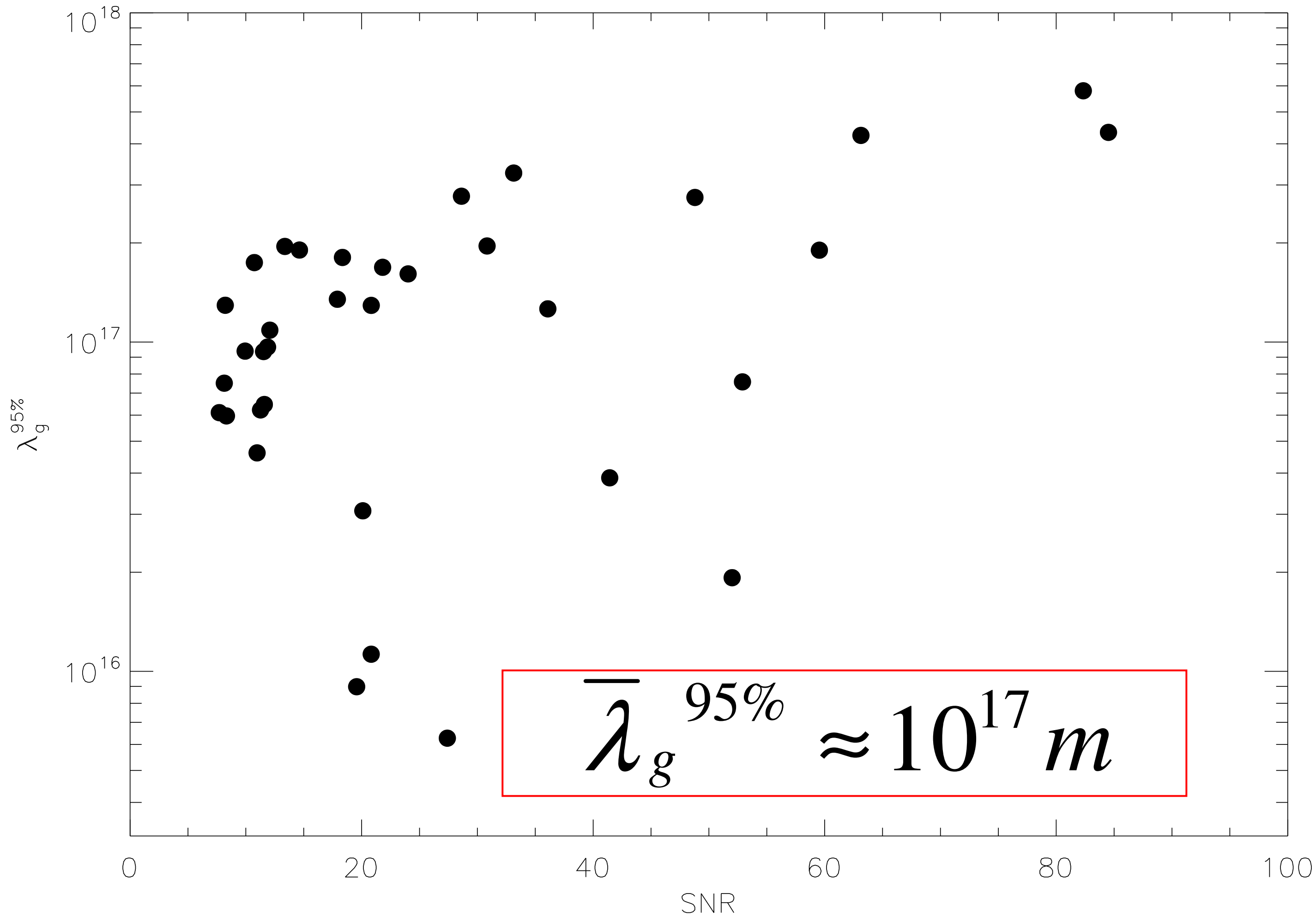
Combination of multiple observations



Einstein Telescope

- Broadband design sensitivity (Hild et al 08).
- $f_{low} \approx 1\text{Hz} \Rightarrow$ run time is huge (~ 20 days).
- Simulated 100 random GR sources with D_L=1Gpc.
- Analysed with the MG template using low accuracy integration (100 Live points).
- Computed the 95% lower limit on λ_g.

ET preliminary results



Summary

- Rigorous framework to perform tests of GR using GWs observations.
- General method applicable to any waveform and any number of observations.
- So far built the infrastructure on Advanced LIGO and a simple alternative theory: massive graviton.
- Future work: systematic study of ET performance.