# *Parametric instability of a Fabry-Perot cavity of Einstein Telescope*


*Kazuhiro Yamamoto*

**Albert Einstein Institute**


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*@Ettore Majorana Foundation and Center for Scientific Culture, Erice, Sicily, Italy*

# *0. Abstract*

**I would like to show not details but outline**

**to evaluate parametric instability in ET interferometer.**

**A cavity without power recycling,**

**signal recycling, resonant sideband extraction**

**Advanced LIGO (U.S.A.) : Serious problem**

**LCGT (Japan) : Not serious problem**

**Einstein Telescope (Europe) : ?**

## *Contents*

# *1. Introduction*

**Advanced LIGO (U.S.A.), LCGT (Japan)**

**Second generation interferometric gravitational wave detector**

**Einstein Telescope (Europe)**

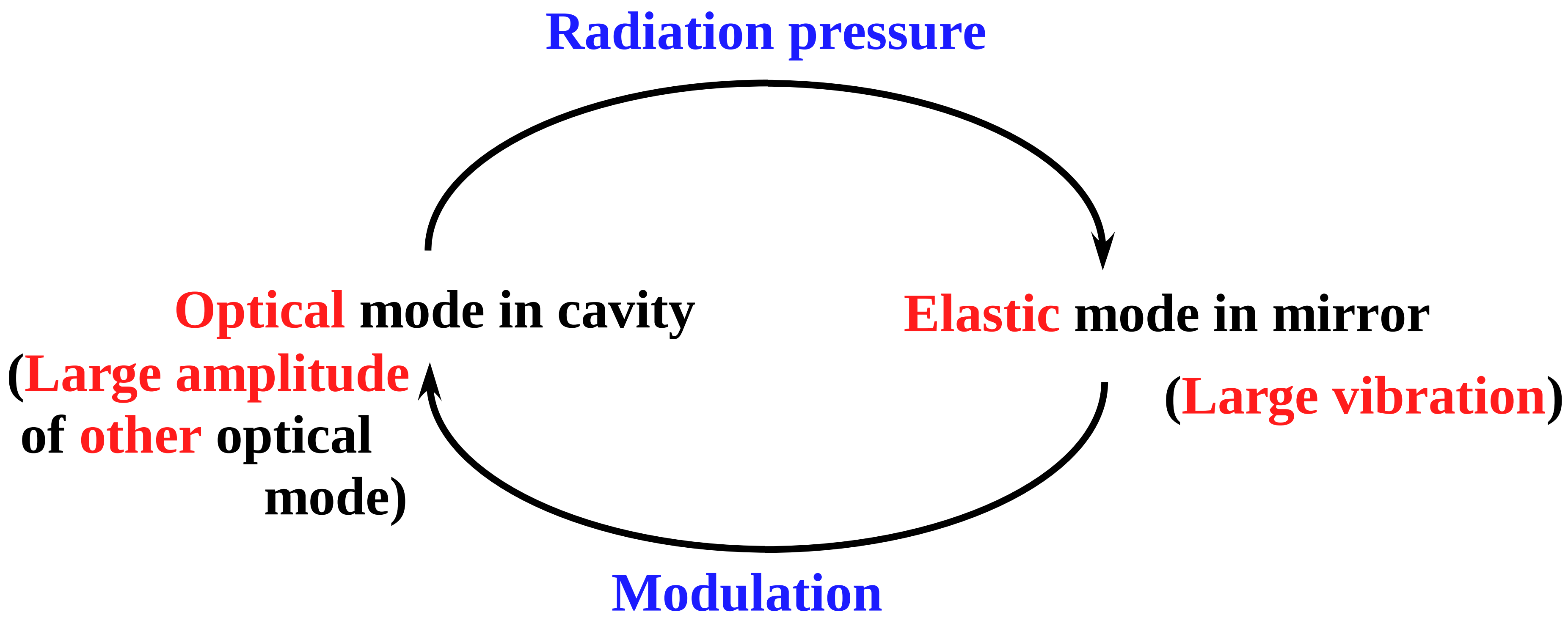
**Third generation interferometric gravitational wave detector**

**Long Fabry-Perot cavity : > 3 km**

**→ Interval of optical mode in cavity : < 10 kHz**

**Interval of elastic mode in mirror : ~ 10 kHz**

Parametric instability Phys. Lett. A 287 (2001) 331.



**Formula of parametric instability** Phys. Lett. A 287 (2001) 331.

$R > 1$ : **instable elastic mode**

**Spatial overlap between optical and elastic modes**

**Q of mirror**

**Power**

$$R \sim \Sigma \frac{4PQ_mQ_o}{McL\omega_m^2} \frac{\Lambda_o}{1+\Delta\omega^2/\delta_o^2} < 4000$$

**(AdLIGO, LCGT)**

**Frequency of elastic mode**

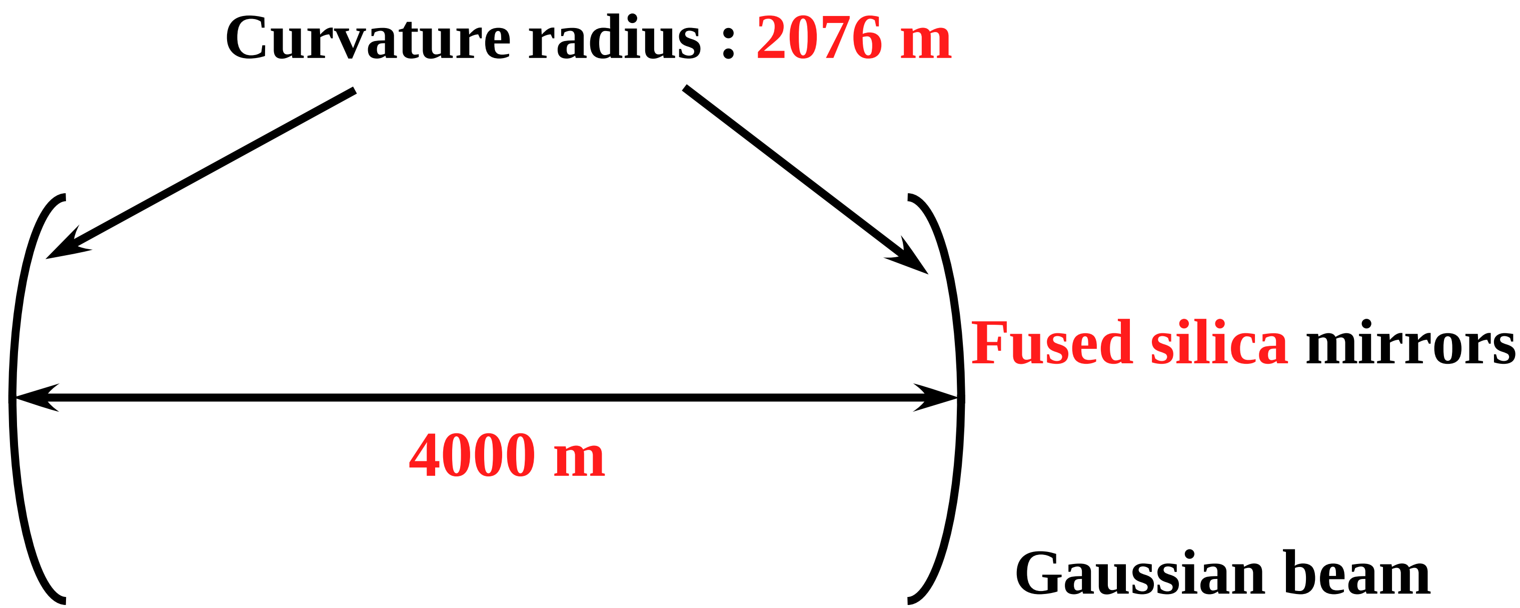
**Width of optical mode**

$$\delta_o^2 = \omega_o/2Q_o$$

**Frequency difference between optical and elastic modes**

# *2. Advanced LIGO*

## *2-1. Specification*




**Power in cavities : 0.83 MW**

**Wavelength : 1064 nm**

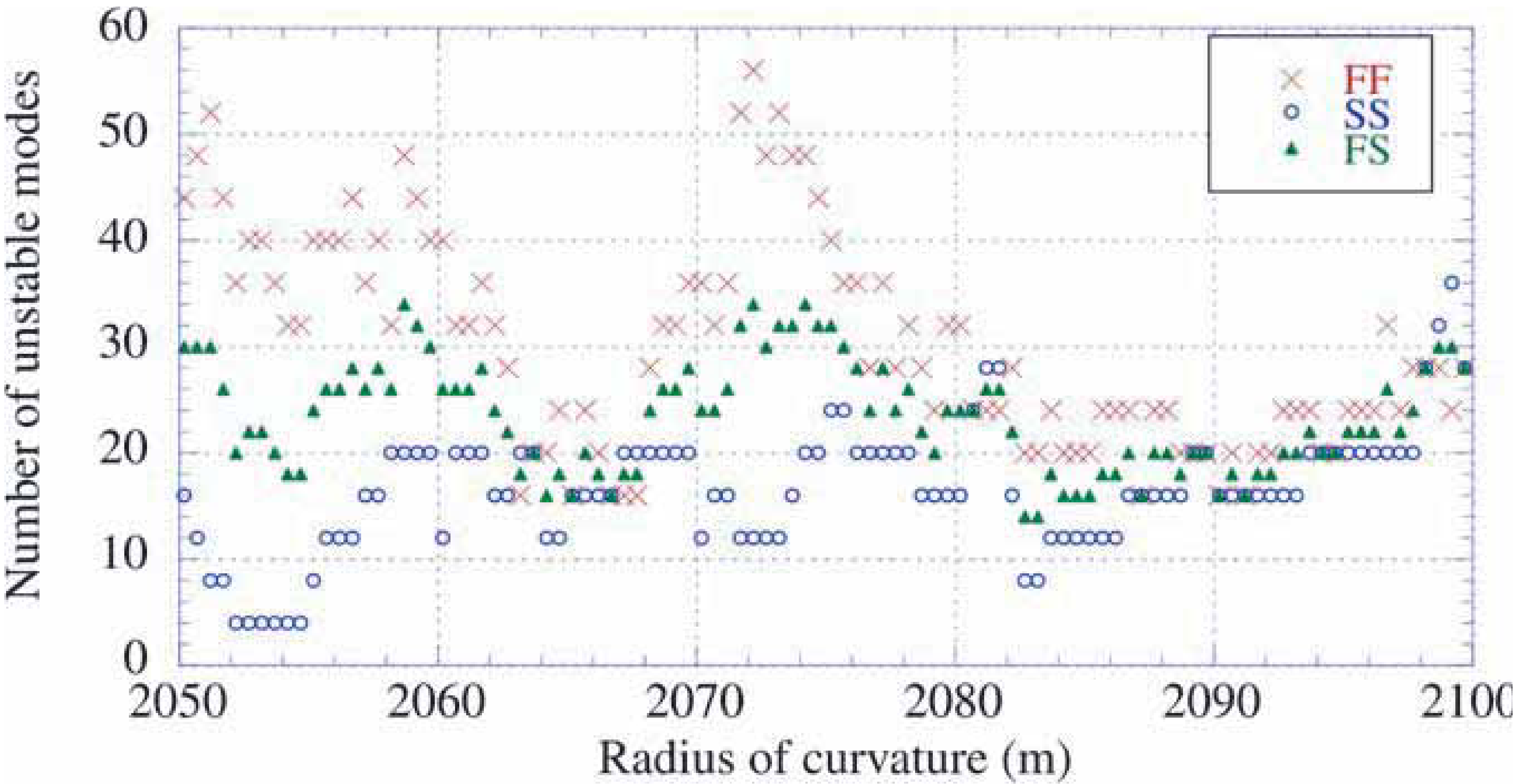
**Study in University of Western Australia**

**Phys. Lett. A 354 (2006) 360.**
**Phys. Lett. A 355 (2006) 419.**

## *2-2. Number of unstable modes*

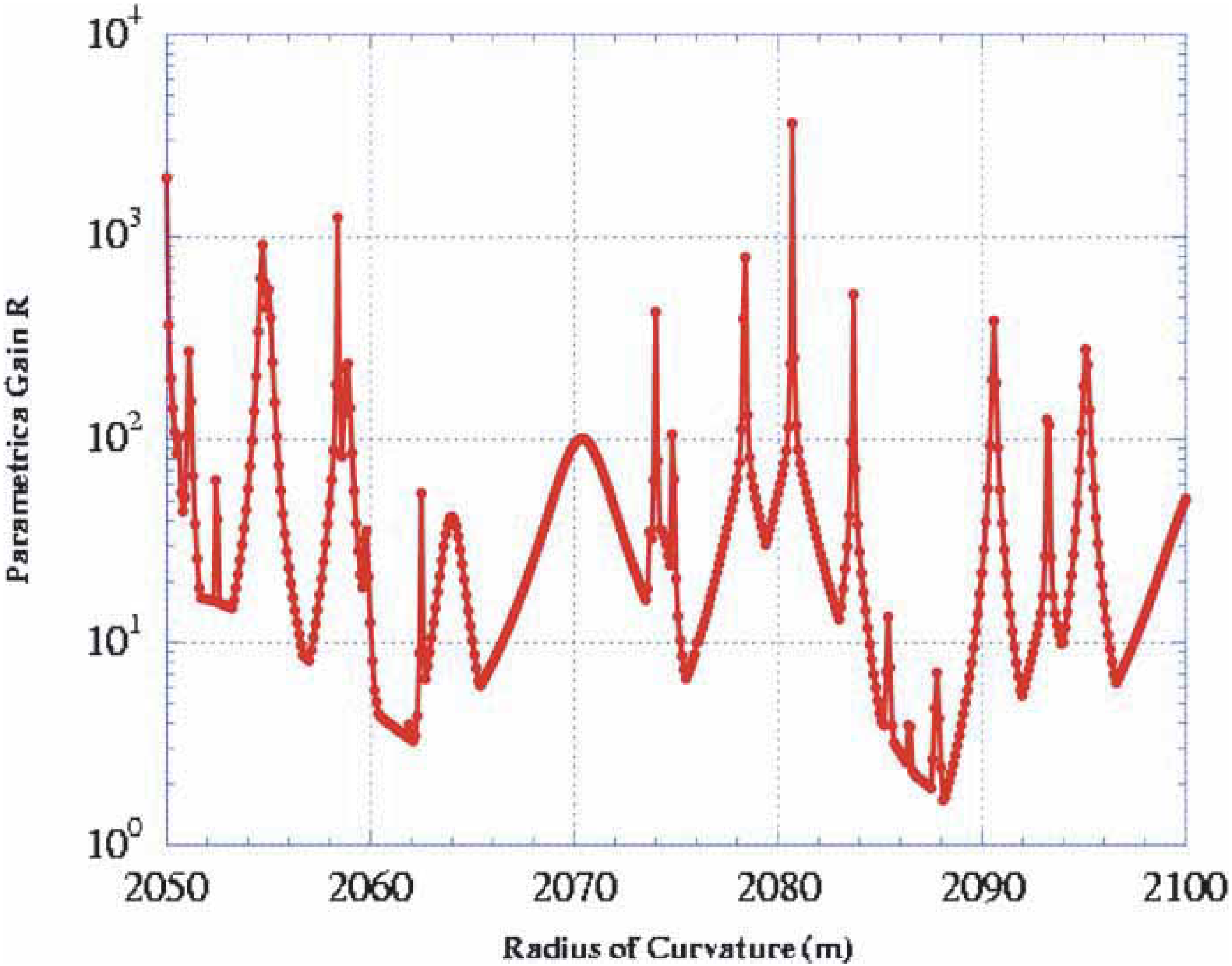
**Phys. Lett. A 355 (2006) 419.**

**FF : Fused silica - Fused silica**



## *2-3. Maximum of R*

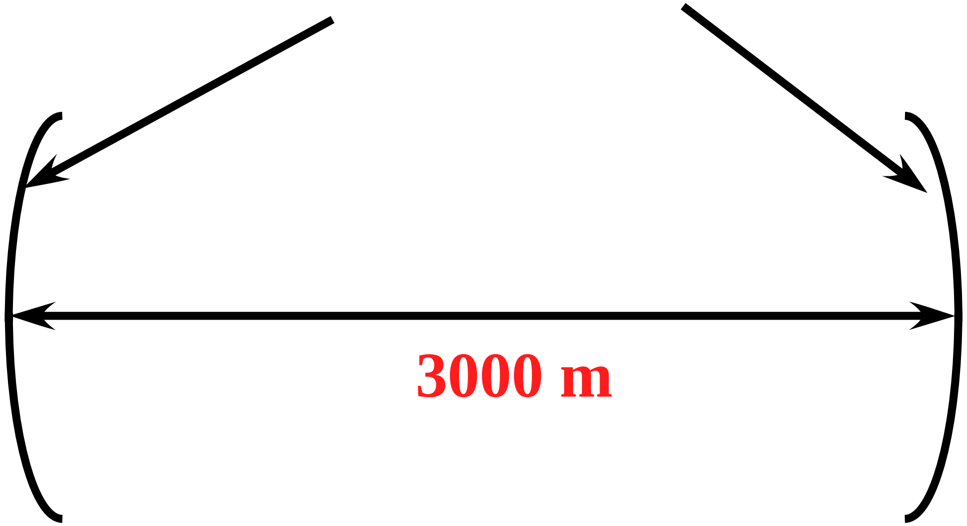
**Phys. Lett. A 354 (2006) 360.**



# 3. LCGT

## *3-1. Specification*

Curvature radius : 7114 m




Sapphire mirrors (20 K)

Power in cavities : 0.41 MW

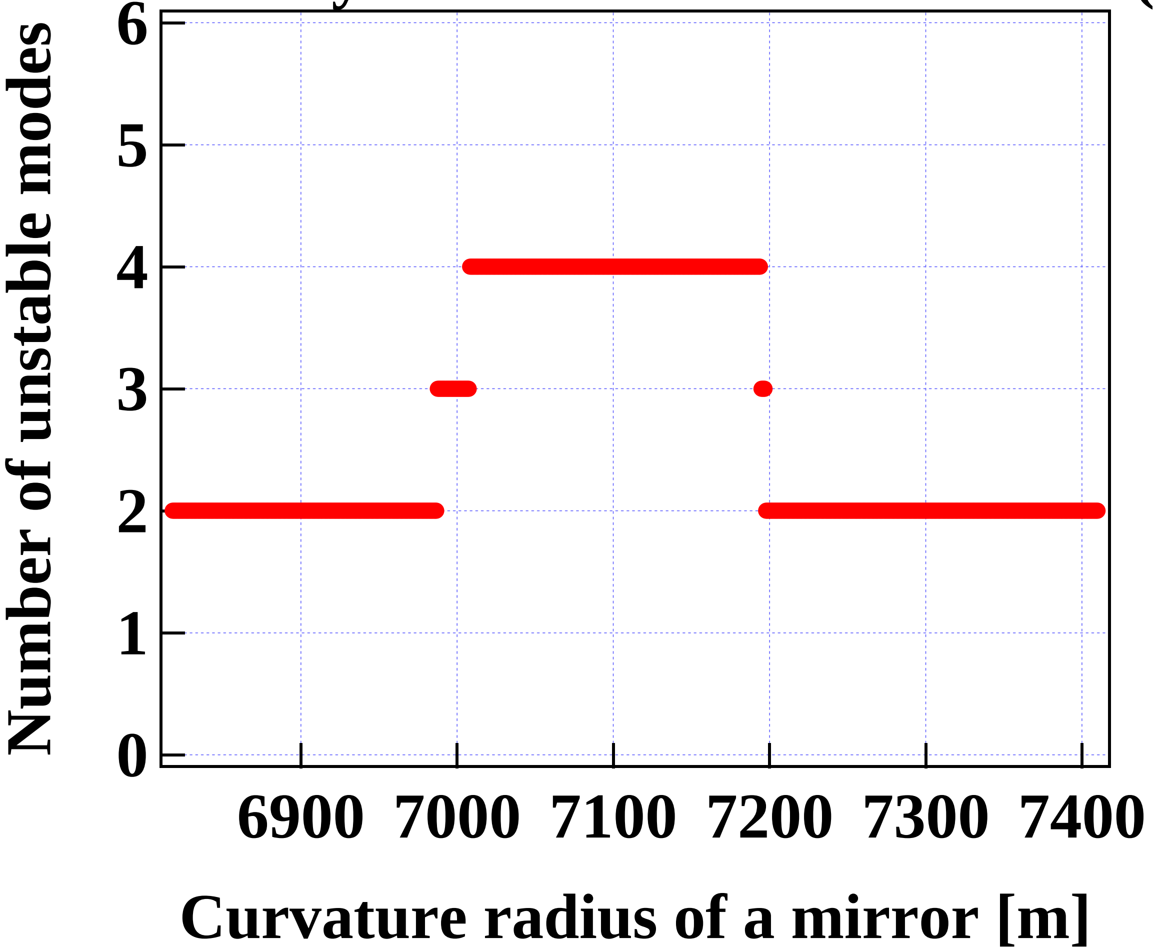
Wavelength : 1064 nm

K. Yamamoto et al., Amaldi7 proceedings
Journal of Physics : Conference Series 122 (2009) 012015

*3-2. Number of unstable modes*

K. Yamamoto et al., Amaldi7 proceedings

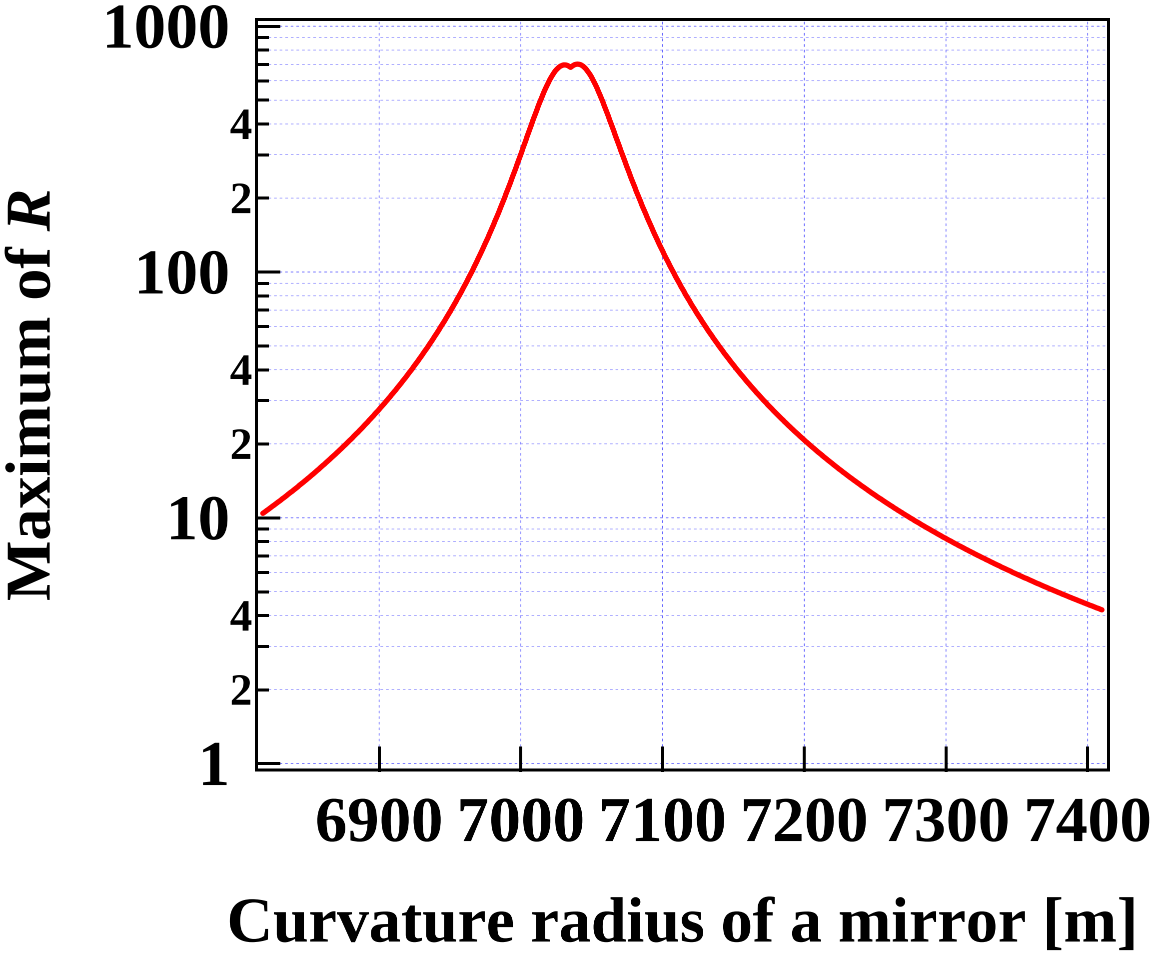
Journal of Physics : Conference Series 122 (2009) 012015



## *3-3. Maximum of R*

**K. Yamamoto et al., Amaldi7 proceedings**

**Journal of Physics : Conference Series 122 (2009) 012015**



# *4. Difference between AdLIGO and LCGT*

**K. Yamamoto et al., Amaldi7 proceedings**
**Journal of Physics : Conference Series 122 (2009) 012015**

## *4-1. Number of unstable modes*

**Advanced LIGO : 20 ~ 60**

**LCGT : 2 ~ 4**

**(i) Elastic mode density : ~ (Sound velocity)$^{-3}$**

**Advanced LIGO (Fused silica) : 6 km/s**

**LCGT (Sapphire) : 10 km/s**

**5 times smaller**

**Cooled mirror for thermal noise suppression (LCGT)**

## (ii) Optical mode density

**Advanced LIGO : 7 modes / FSR**

**LCGT : 3 modes / FSR**

**2 times smaller**

**Larger beam radius for thermal noise reduction (Advanced LIGO)**

**Normal beam radius (Cooled mirror) (LCGT)**

## (iii) Summary

**Product of elastic and optical mode densities : 10 times smaller**

**Calculated number of unstable mode**

**Advanced LIGO : 20 ~ 60**

**LCGT : 2 ~ 4**

## *4-2. Mirror curvature dependence*

**Advanced LIGO : *R* strongly depends on mirror curvature.**

**LCGT : *R* weakly depends on mirror curvature.**

***R* is function of optical mode frequency.**

**Interval of transverse optical mode**

**Advanced LIGO : 15 Hz/m**

**LCGT : 0.58 Hz/m**

**30 times smaller**

**Larger beam radius for thermal noise reduction**

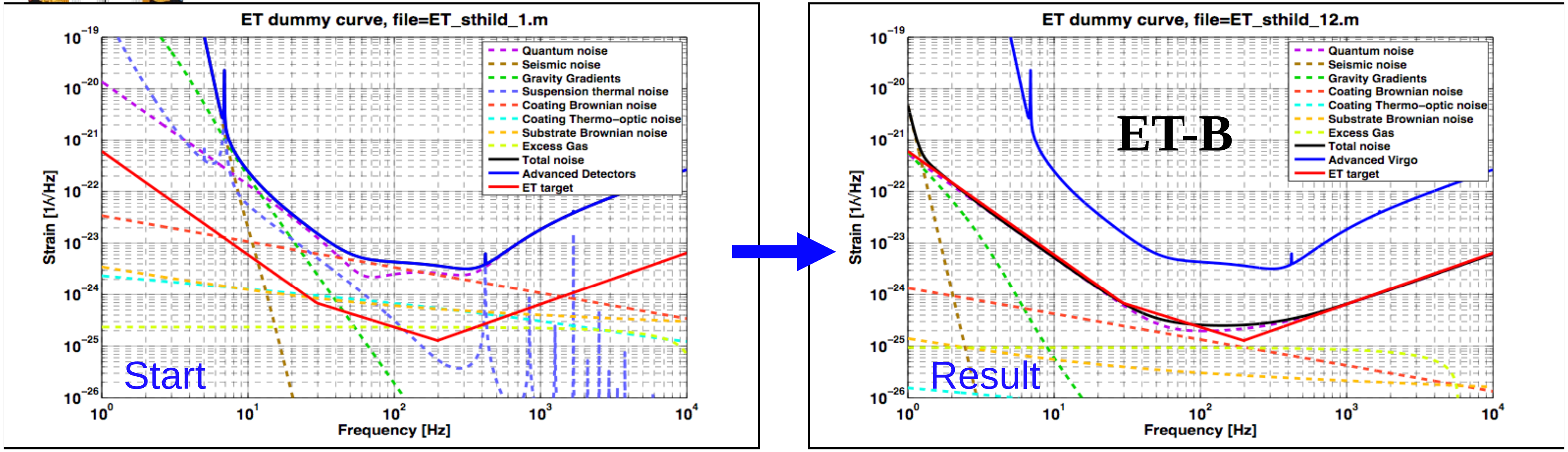
**(Advanced LIGO)**

# *5. Einstein Telescope*

**How much are parameters of Einstein Telescope ?**

# S. Hild *et al.*, arXiv:0810.0604



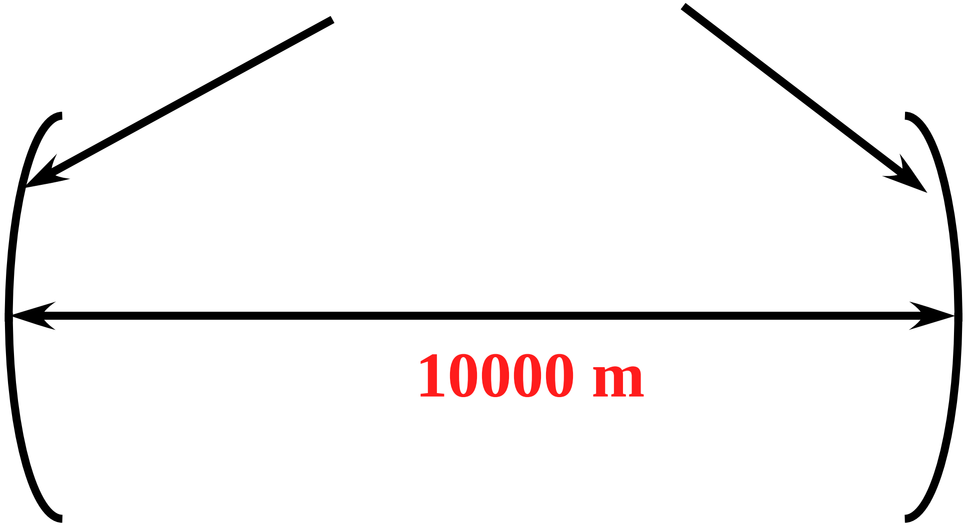



| | advanced detector | potential ET design |
|---|---|---|
| Arm length | 3 km | 10 km |
| SR-phase | detuned (0.15) | tuned (0.0) |
| SR transmittance | 11 % | 10 % |
| Input power (after IMC) | 125 W | 500 W |
| Arm power | 0.75 MW | 3 MW |
| Quantum noise suppression | none | 10 dB |
| Beam radius | 6 cm | 12 cm |
| Temperature | 290 K | 20 K |
| Suspension | Superattenuator | 5 stages of each 10 m length |
| Seismic | $1 \cdot 10^{-7}\,\mathrm{m}/f^2$ for $f > 1\,\mathrm{Hz}$ (Cascina) | $5 \cdot 10^{-9}\,\mathrm{m}/f^2$ for $f > 1\,\mathrm{Hz}$ (Kamioka) |
| Gravity gradient reduction | none | factor 50 required (cave shaping) |
| Mirror masses | 42 kg | 120 kg |
| BNS range | 150 Mpc | 2650 Mpc |
| BBH range | 800 Mpc | 17700 Mpc |

*Specification of ET cavity*

**S. Hild *et al*., arXiv:0810.0604**

**Curvature radius : 5070 m**




**Sapphire or silicon mirrors (20 K)**

**Power in cavities : 3 MW**

**Wavelength : 1064 nm**

**Mass : 120 kg (by S. Hild *et al*) → Thin mirror**

**Same aspect ratio as that of LCGT (0.6)**

**Sapphire : 410 kg, Silicon : 230 kg**

**(Radius : 300 mm, Thickness : 360 mm)**

## *5-1. Upper limit of R*

$$R \sim \sum \frac{4PQ_mQ_o}{McL\omega_m^2} \frac{\Lambda_o}{1+\Delta\omega^2/\delta_o^2}$$

**Comparison with LCGT**

**Power (in a cavity, *P*) : 8 times larger (0.41MW $\longrightarrow$ 3MW)**

**Cavity length (*L*) : 3 times longer (3km $\longrightarrow$ 10km)**

**Beam radius : 4 times larger (3cm $\longrightarrow$ 12cm)**

**Mirror mass (*M*) : 14 times (sapphire) or 8 times (silicon) larger**

**Resonant frequency of elastic modes ($\omega_m$)**
**: 2.4 times (sapphire) or 2.9 times (silicon) smaller**

**Upper limit of *R* is 0.93 times (sapphire) or 2.3 times (silicon) larger.**

**(It is assumed that band width of cavity is same)**

## *5-2. Number of unstable modes*

**(i) Elastic mode density : ~ (Mirror size)$^3$/(Sound velocity)$^3$**

**Sapphire : 10 km/s**

**Silicon : 8.4 km/s**

**Mirror size : 2.4 times larger than that of LCGT**

**14 times larger (Sapphire)**

**23 times larger (Silicon) than that of LCGT**

## (ii) Optical mode density

**Cavity length ($L$) : 3 times (3km $\longrightarrow$ 10km)**

**Free Spectrum Range : 3 times smaller**

**Beam radius : 3 cm $\longrightarrow$ 12 cm !**

**LCGT : 3 modes / FSR**

**Einstein Telescope : 13 modes / FSR**

**4 times larger**

**Total : 14 times larger**

## (iii) Summary

**Product of elastic and optical mode densities (number of unstable modes)**

**190 times larger (Sapphire)**

**310 times larger (Silicon)**

**than that of LCGT.**

**Product of elastic and optical mode densities (number of unstable modes)**

**19 times larger (Sapphire)**

**31 times larger (Silicon)**

**than that of Advanced LIGO.**

**LCGT : Smaller unstable mode number**
**(cooled mirror for thermal noise suppression)**

**ET : Larger unstable mode number**
**(cooled mirror but longer baseline and**
**larger beam for thermal noise suppression)**

## *5-3. Mirror curvature*

**Einstein Telescope**

**Mirror curvature is about a half of cavity length.**

**Cavity length of Einstein Telescope is about 3 times longer.**

**Mirror curvature dependence of interval of transverse optical mode**

**Einstein Telescope : 4 Hz/m**

**Advanced LIGO : 15 Hz/m**

**LCGT : 0.58 Hz/m**

**Mirror curvature dependence of ET is stronger than that of LCGT.**

**Larger beam radius for thermal noise reduction**

**Mirror curvature dependence of ET is weaker than that of Advanced LIGO.**

**Longer baseline for thermal noise reduction**

# *Other methods for instability suppression*

**Investigation in LIGO (University of Western Australia)**

**Phys. Lett. A 355 (2006) 419.**

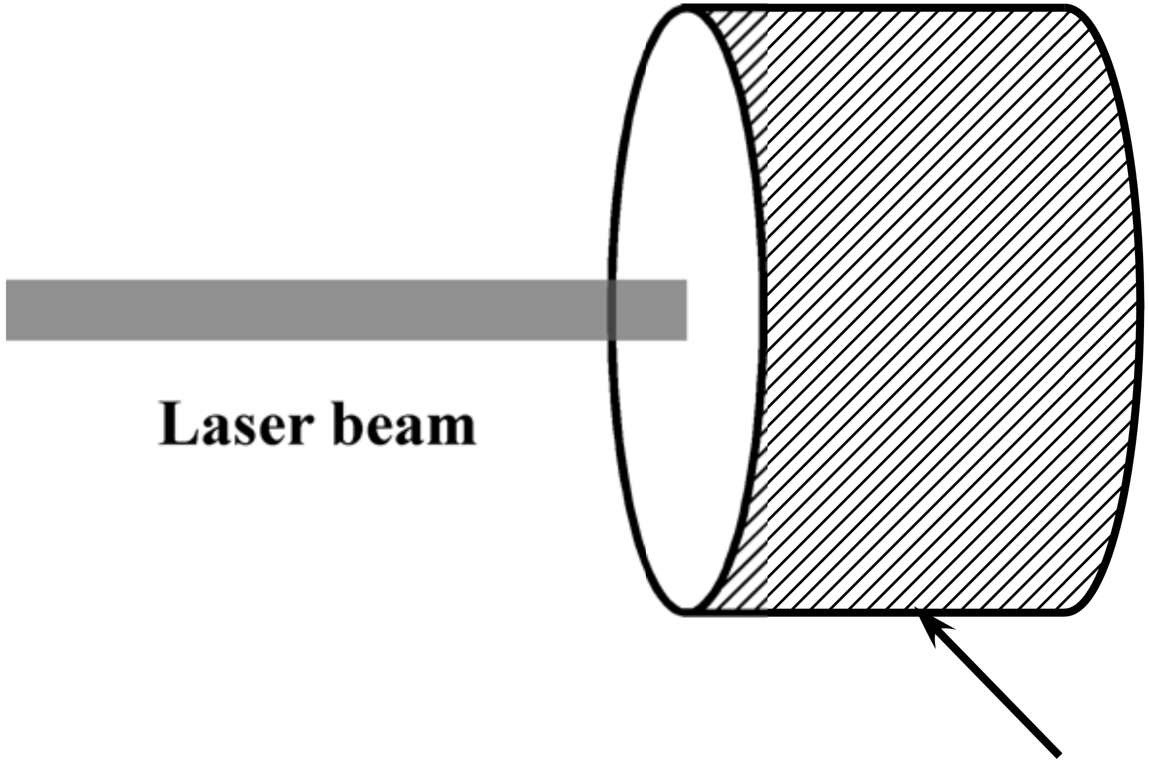
**(1) Thermal tuning**

**(2) Q reduction**

**(3) Feedback control**

## *Q reduction (elastic mode)*

***R*** **is proportional to** ***Q*****.**




**Coating (dissipation)**
**It reduces Q values effectively,**
**but does not increase thermal noise effectively.**

**K. Yamamoto et al., Phys. Lett. A 305 (2002) 18.**
**S. Gras et al., Phys. Lett. A 333 (2004) 1.**
**S. Gras et al., J. Phys. : Conf. Ser. 32 (2006) 251.**

For LCGT

$Q = 10^8 \longrightarrow Q = 10^6$

(Almost all modes become stable)

0.2 mm thickness $Ta_2O_5$ coating on cylindrical surface

K. Yamamoto et al.,Phys. Rev. D 74 (2006) 022002.

Thermal noise of cylindrical surface coating

is comparable with that of reflective coating

and a few times smaller than that of LCGT goal sensitivity.

K. Yamamoto et al., Amaldi7 proceedings
Journal of Physics : Conference Series 122 (2009) 012015

K. Yamamoto et al., Phys. Lett. A 305 (2002) 18.

**For Einstein Telescope**

**Upper limit of *R* is comparable with that of LCGT.**
***R* is proportional to *Q*.**

$$Q = 10^{8} \longrightarrow Q = 10^{6}$$

**(Almost all modes become stable)**

**0.48 mm (Sapphire) or 0.2 mm (Silicon) thickness $Ta_2O_5$ coating on cylindrical surface**

**Thermal noise of cylindrical surface coating is comparable with (Sapphire) or 2 times larger (Silicon) than that of ET goal sensitivity.**

# 7. Summary and future work

**(1) Upper limit of *R* (strength of instability)**

**Upper limit of *R* of Einstein Telescope is almost same as that of Advanced LIGO and LCGT.**

**(2) Number of unstable modes and mirror curvature dependence**

**LCGT : Less unstable modes and weak curvature dependence**

**Cooled mirror for thermal noise reduction (sapphire mirror and normal beam radius)**

**ET : Many unstable modes and strong curvature dependence**

**Cooled mirror but larger beam radius and longer arm for thermal noise reduction**

**(3) Instability suppression for ET**

**Q reduction (elastic mode) by cylindrical surface loss is effective, but thermal noise is comparable with (or larger than) goal sensitivity.**

**(4) Future work : Full interferometer and instability suppression**

**Do not be too pessimistic,**

**but pay attention.**