Finite element calculations of gravity gradient noise for subterranean gravitational wave detectors

16 October 2009

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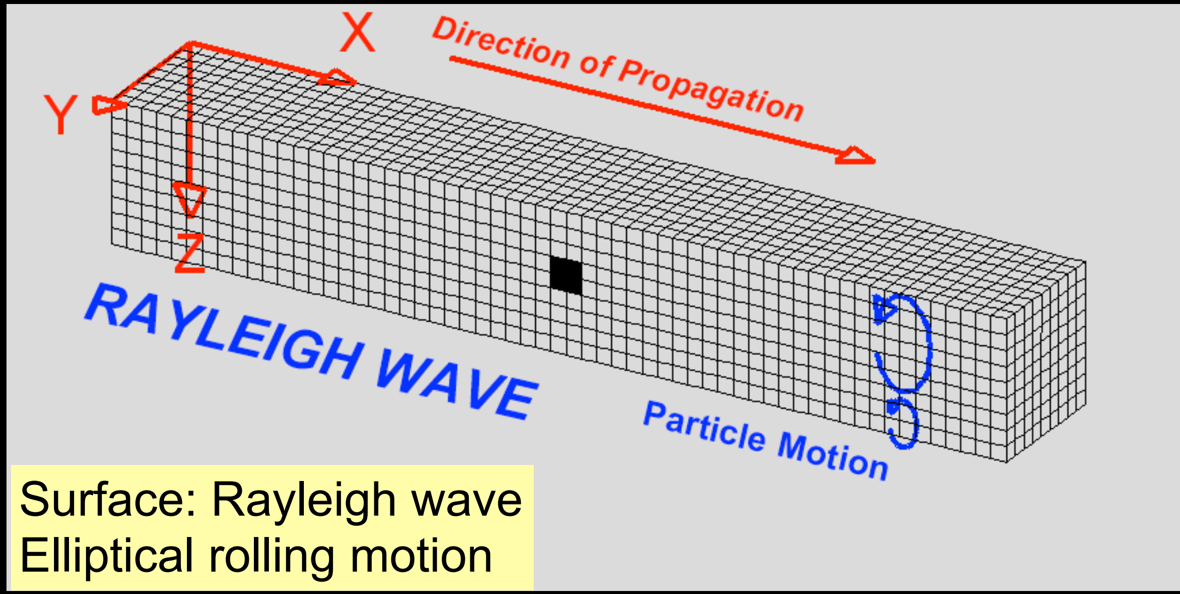
2nd ET Workshop Erice

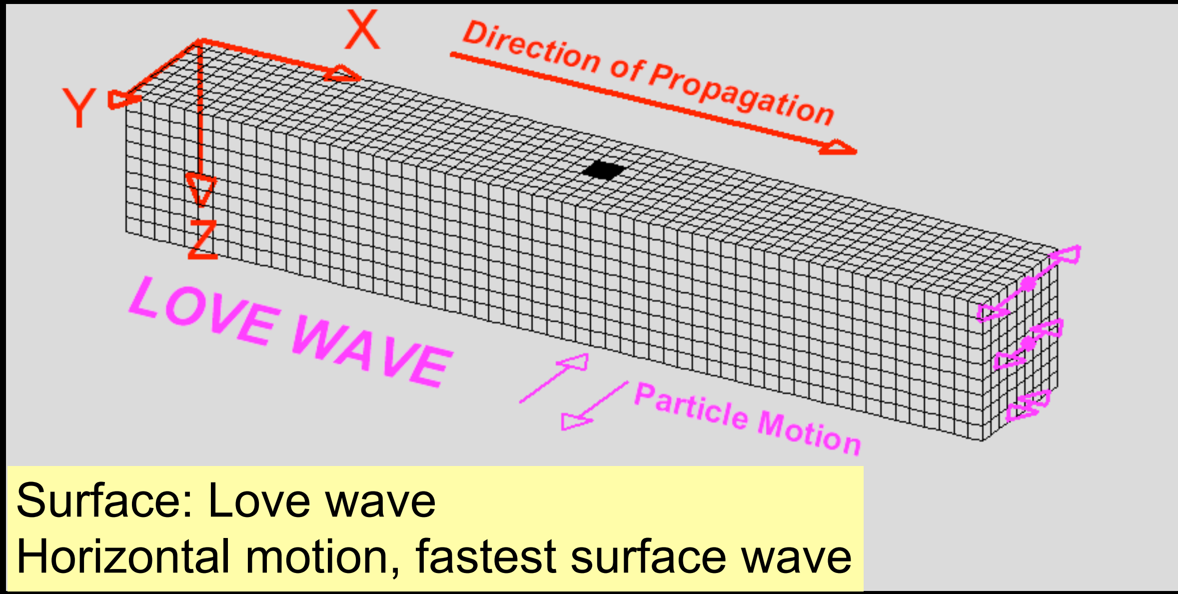
Why we need finite element simulations for GGN

- FEA provide solutions to the wave equation
- Solutions to the wave equation constitute various wave types
  - *Surface: Rayleigh, Love waves*
  - *Body: Pressure, Shear, Head waves*
- Allows to simulate realistic geologies
  - *Multi-layer systems*
  - *Faults / reflections / scattering centers*
- Can be used to test and verify subtraction algorithms

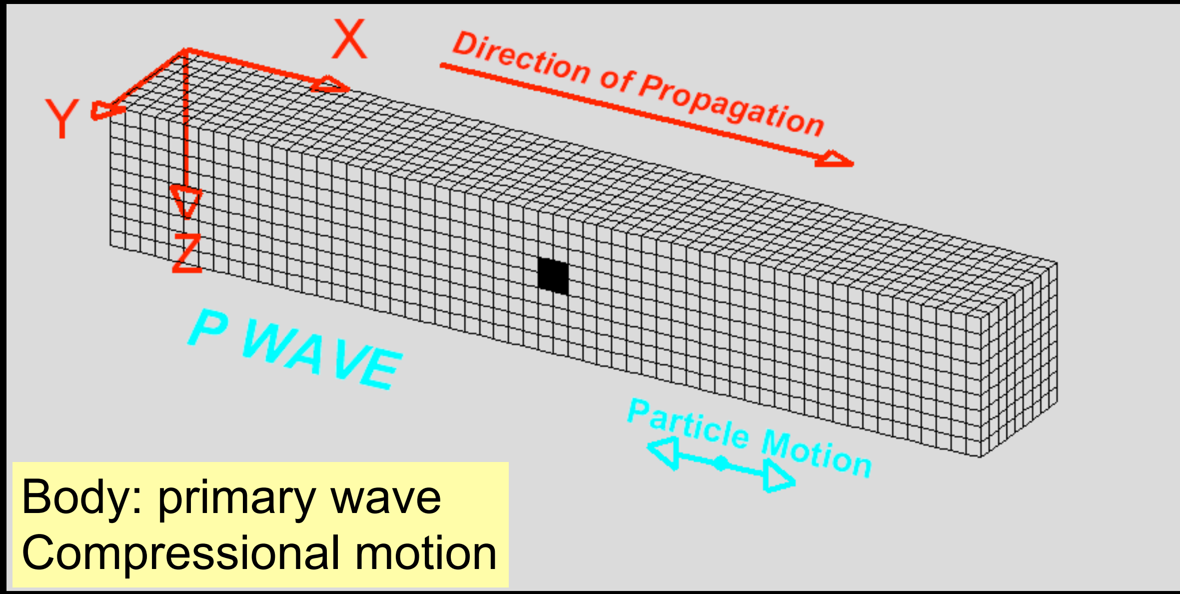
Solutions to the wave equation include surface and body waves



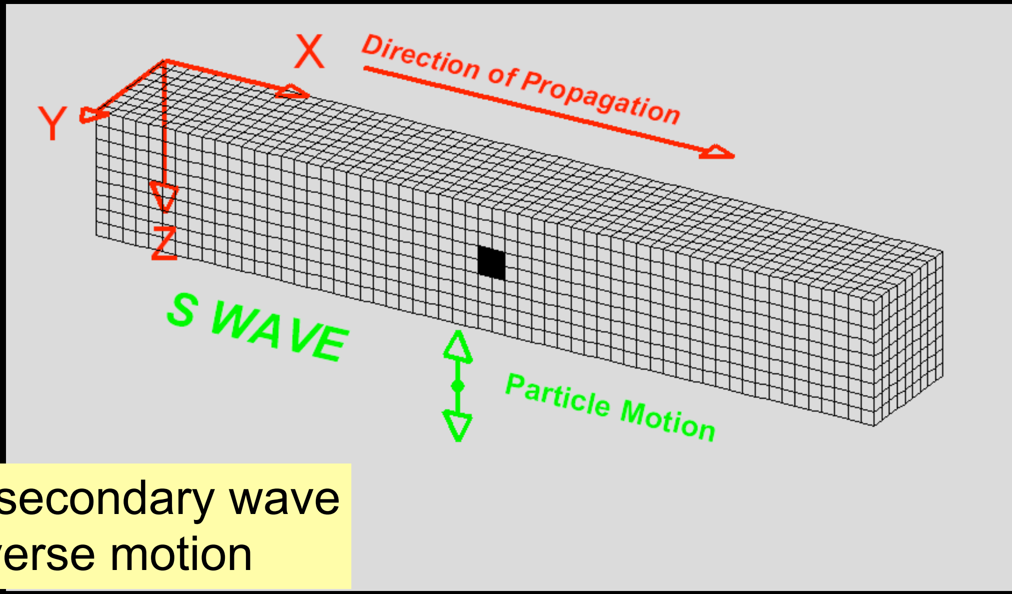
Surface: Rayleigh wave
Elliptical rolling motion



Surface: Love wave
Horizontal motion, fastest surface wave

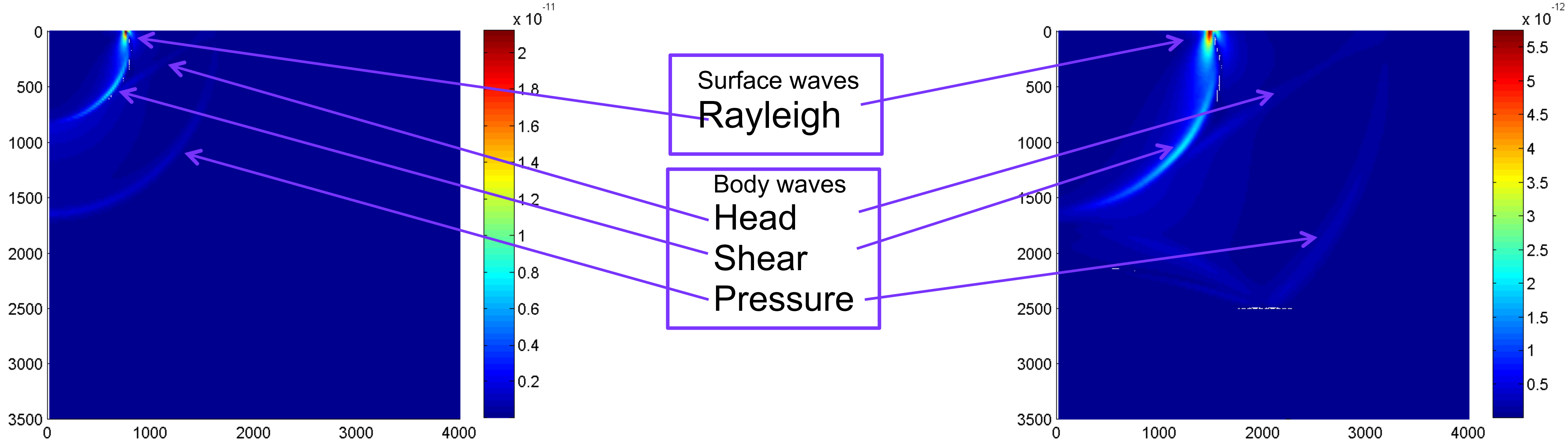


Body: primary wave
Compressional motion



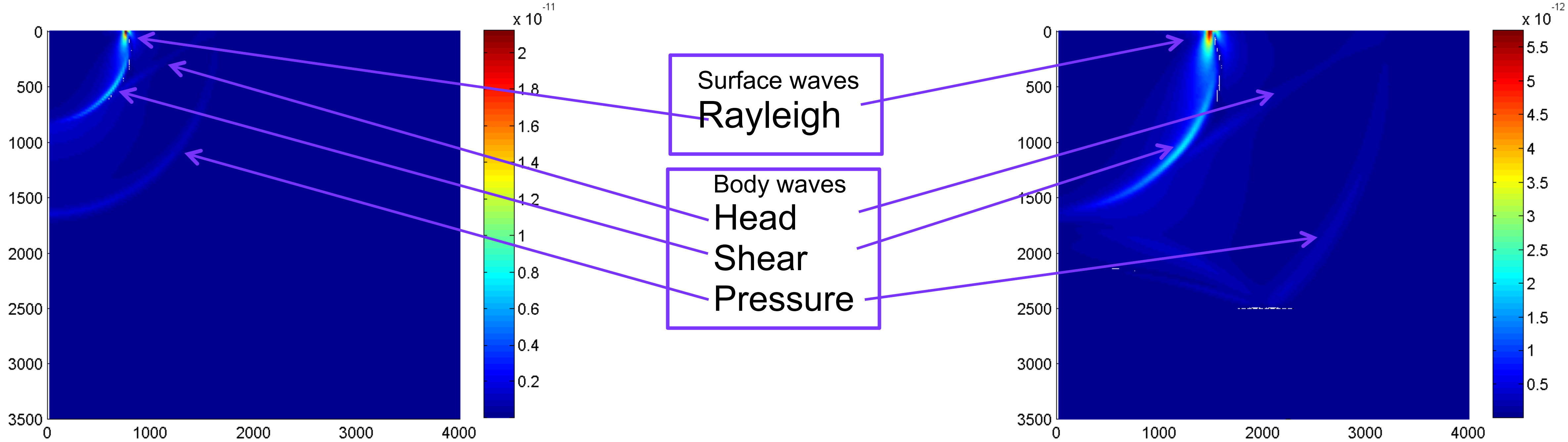
Body: secondary wave
Transverse motion

Finite element solutions for all wave fields



- Reaction to vertical point oscillation
  - Two layer geology
- Wave attenuation has two components
  - Geometrical (expansion of wave fronts) ~ r^n
    - Rayleigh, n=-1/2
    - Body waves at depth, n=-1
  - Material (damping)

Finite element solutions for all wave fields



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$$A_r = \frac{A_o}{\sqrt{r/r_o}} \exp\{-\pi f r / cQ\} \qquad \text{then} \quad 1/Q = \alpha c/\pi f = 2\delta f / f$$

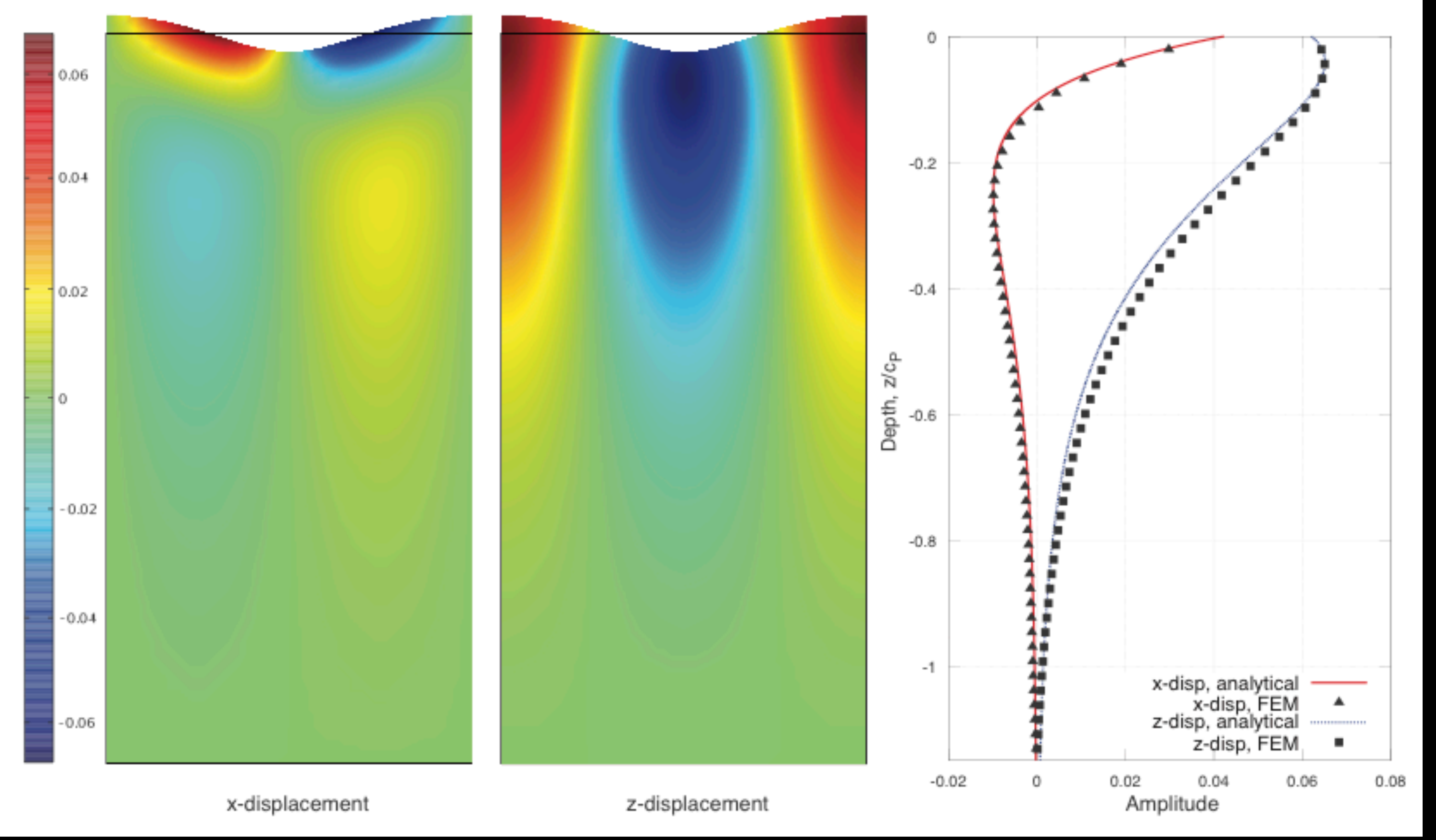
in which
$c =$ velocity ,
$f =$ frequency
$\alpha =$ attenuation$/l$
$\delta f =$ half width at .707 response

Example: sandstone, $\alpha = 3.5 \times 10^{-8}$ f sec/cm, a plane wave disturbance at 1 Hz would be attenuated over 10 km by less than 4%

FE models are good approximations of wave equation solutions - can be compared to analytical results

$$\xi_x = A c_R \left[e^{-\sqrt{c_R^2 - c_P^2}\, z} - \frac{2\sqrt{(c_R^2 - c_P^2)(c_R^2 - c_S^2)}}{c_S^2} e^{-\sqrt{c_R^2 - c_S^2}\, z} \right] e^{i(cx - \omega t - \frac{\pi}{2})}$$

$$\xi_z = A\sqrt{c_R^2 - c_P^2} \left[e^{-\sqrt{c_R^2 - c_P^2}\, z} - \frac{2c_R^2)}{2c_R^2 - c_S^2} e^{-\sqrt{c_R^2 - c_S^2}\, z} \right] e^{i(cx - \omega t)}$$



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FE models are good approximations of wave equation solutions - can be compared to analytical results

- Rayleigh waves
  - *cR - Rayleigh wave speed*
  - *cP/cS - P/S-wave speed*

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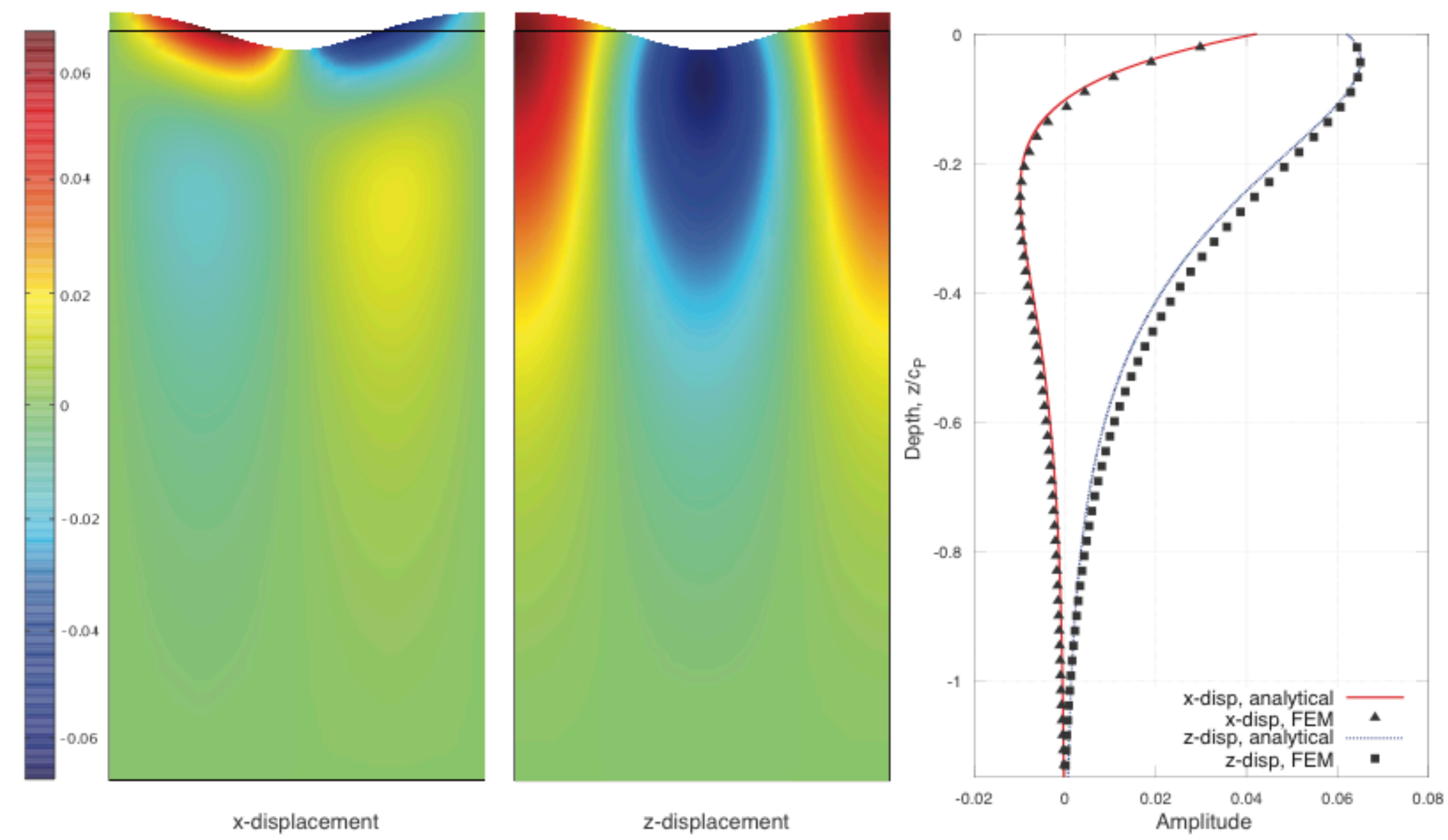

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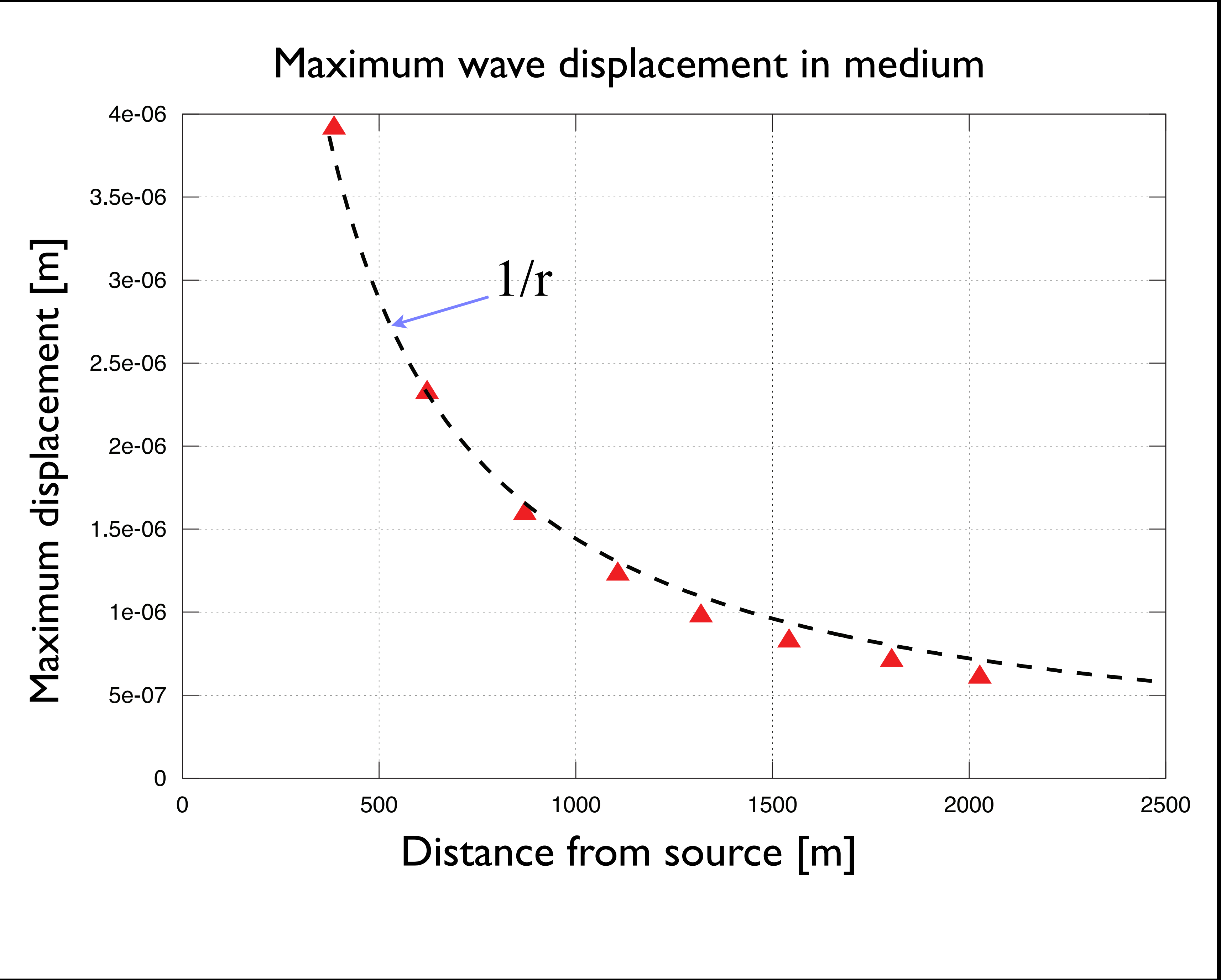
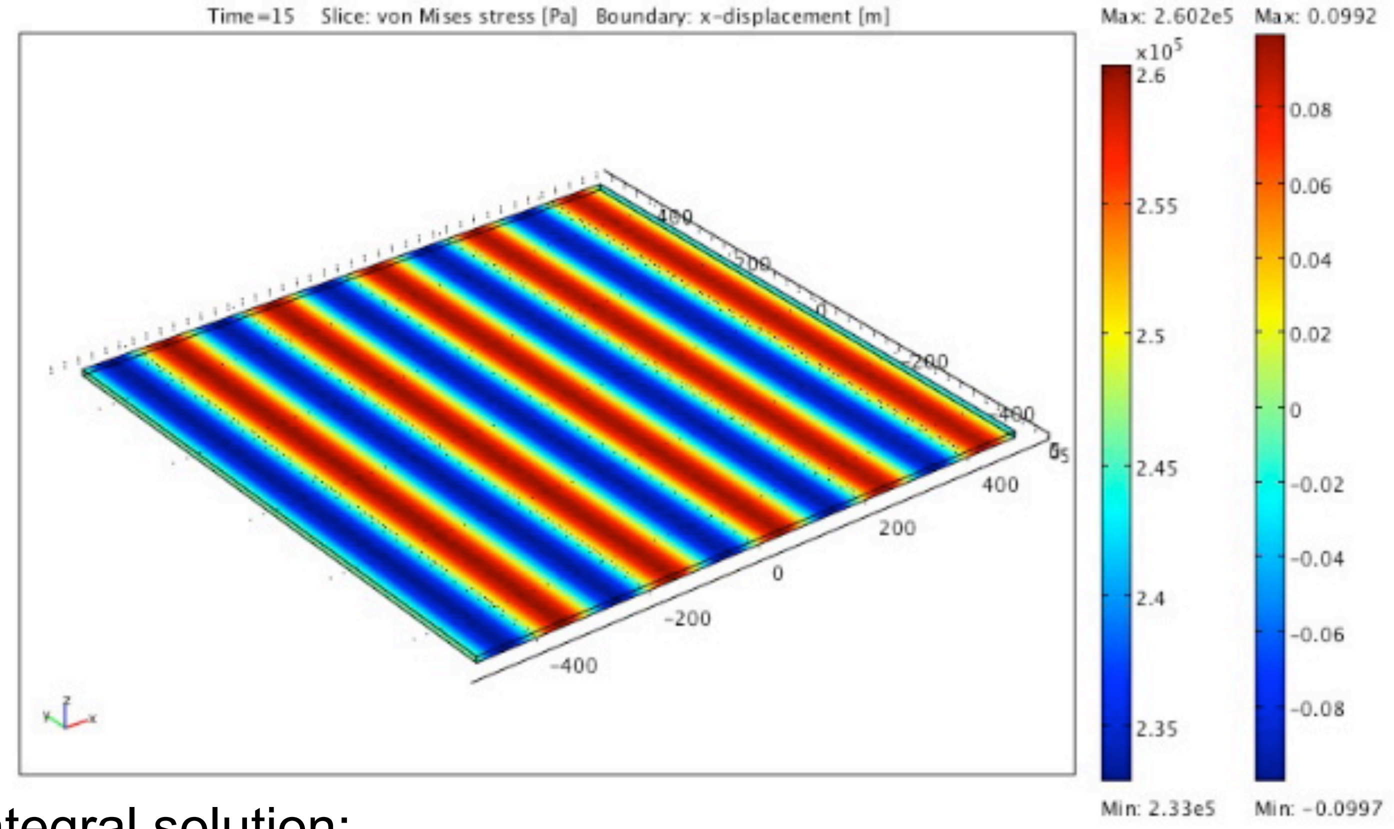
FE models are good approximations of wave equation solutions - can be compared to analytical results

- Rayleigh waves
  - *cR - Rayleigh wave speed*
  - *cP/cS - P/S-wave speed*
- Geometrical damping
  - *Body waves in medium ~1/r*
  - *Surface waves ~ 1/rt(r)*



Analytical results are used to verify FE GGN calculations

Homogenous half space as described by
Saulson (1984) and Thorne and Hughes (1998)



Plane excited on one side with
$u(t) = u_0 \cos(\omega t - kx)$
$u_0 = 0.1$ m

Many planes stacked on top of each other to create a block

Like Saulson and Thorne and Hughes an integral lower bound is used ($r_{cut\text{-}off}$) of $\lambda/4$

Ground properties:
$\rho = 1800$ kg/m^3
$v = 0.33$ cp ≈ 440 m/s

Integral solution:

$$a_{x,pwave,max} = \int_H^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \frac{G\rho u_0}{(x^2+y^2+z^2)^{3/2}} \left(1 - \frac{3x^2}{x^2+y^2+z^2}\right) \cos(-kx)dxdydz$$
$$= 2\pi G \rho u_0 e^{-2\pi\frac{H}{\lambda}}$$

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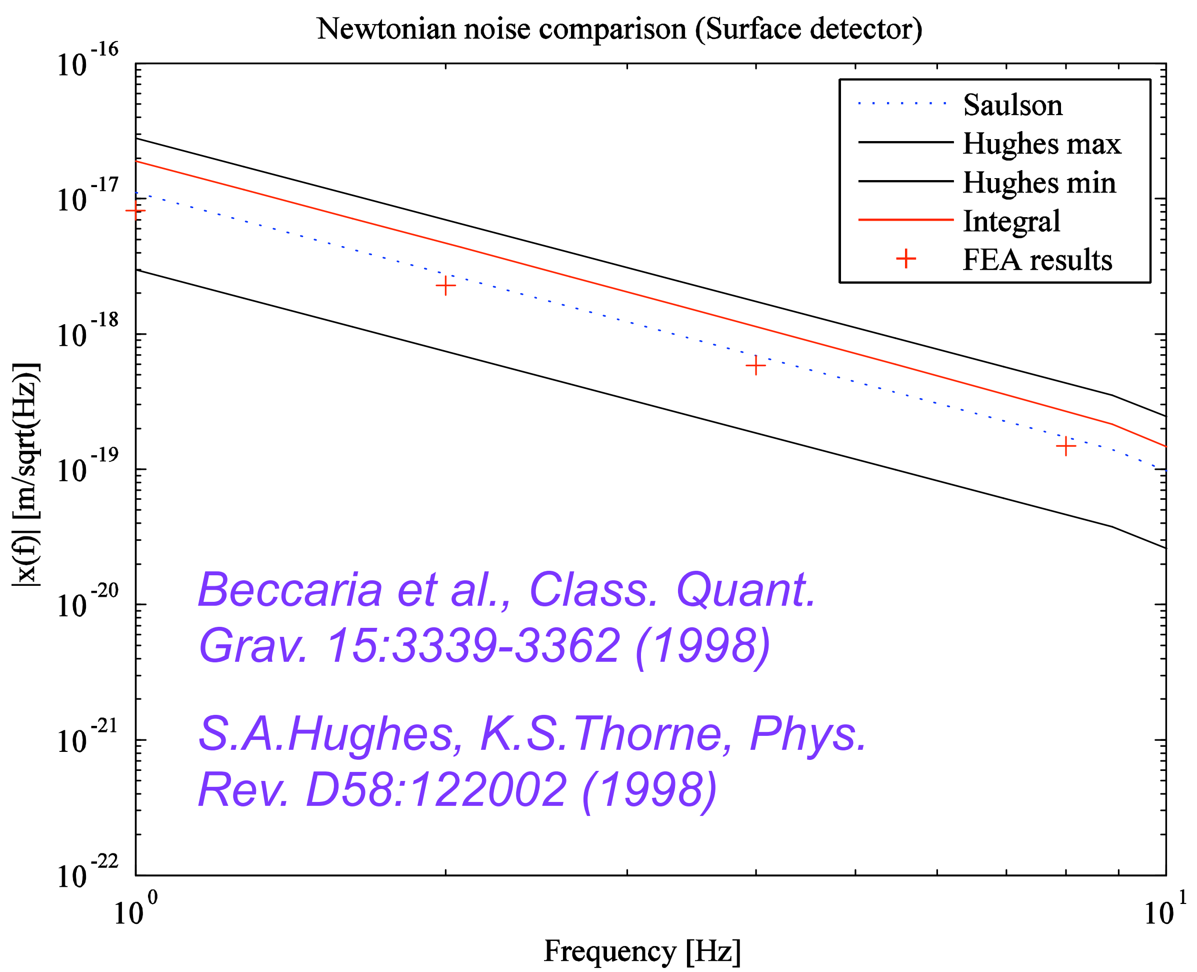
FE calculations show good correspondence with analytical results



- GGN spectra for surface detectors agree with Saulson, Cella and Hughes
  - For 1 – 10 Hz: 1 nm/rtHz
- Care should be taken regarding element size
- Deviation from integral due to integral cut-off, r

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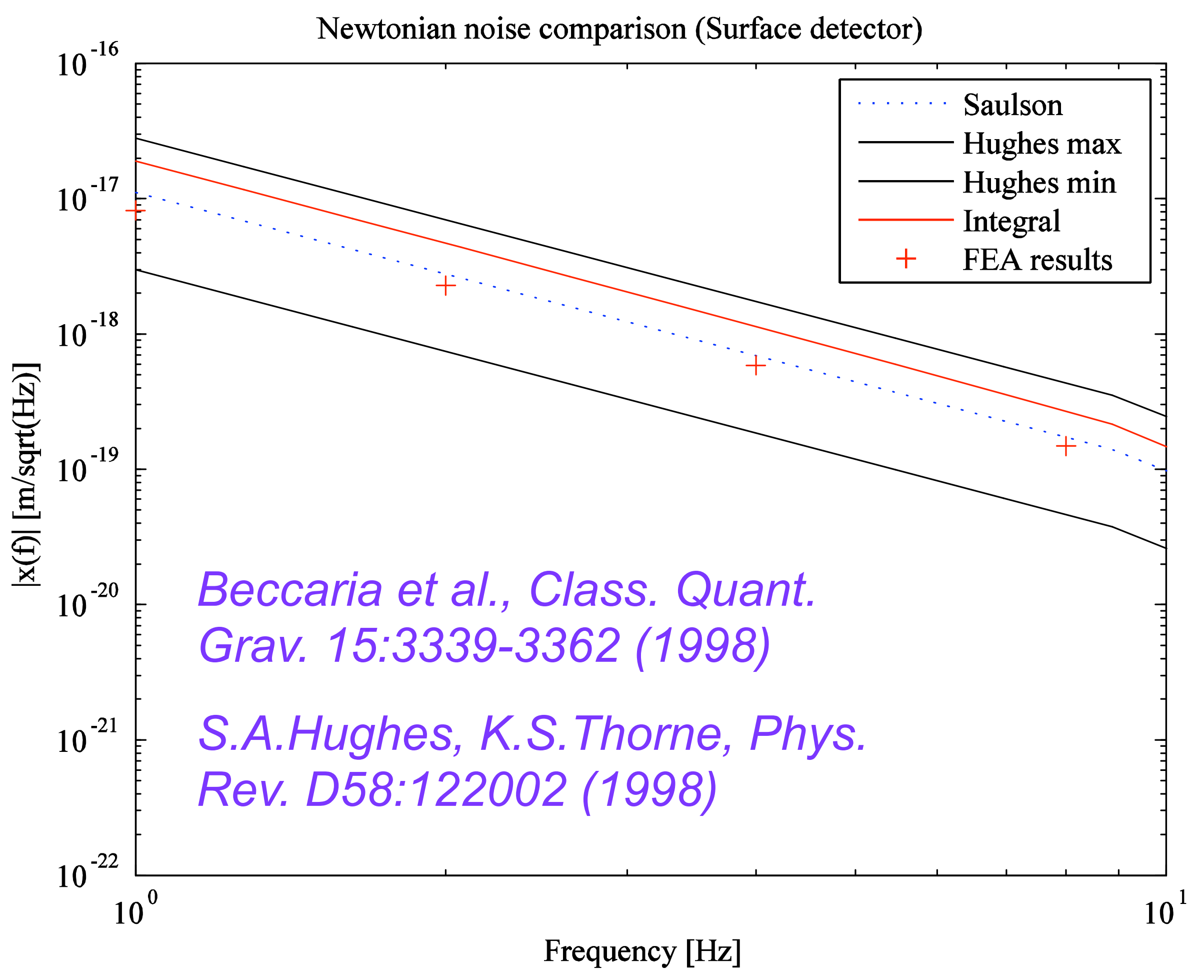

Beccaria et al., Class. Quant. Grav. 15:3339-3362 (1998)

S.A.Hughes, K.S.Thorne, Phys. Rev. D58:122002 (1998)

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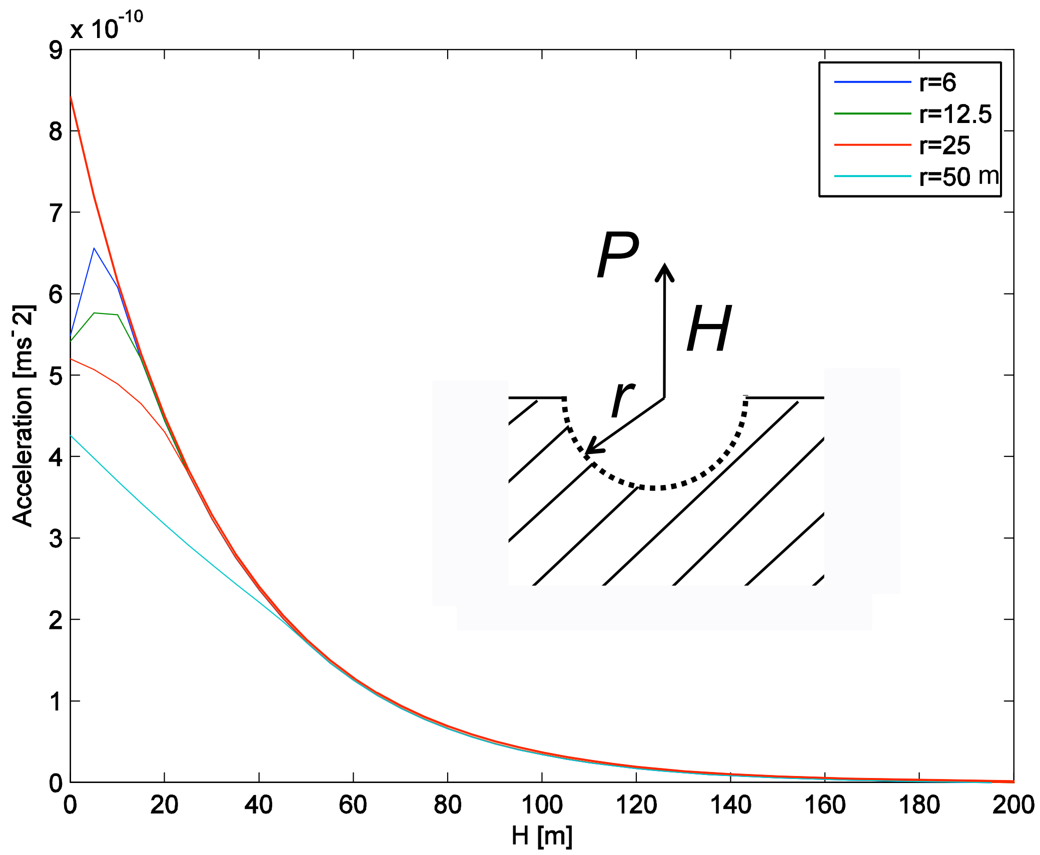


Reproduction of surface wave calculations - Cella

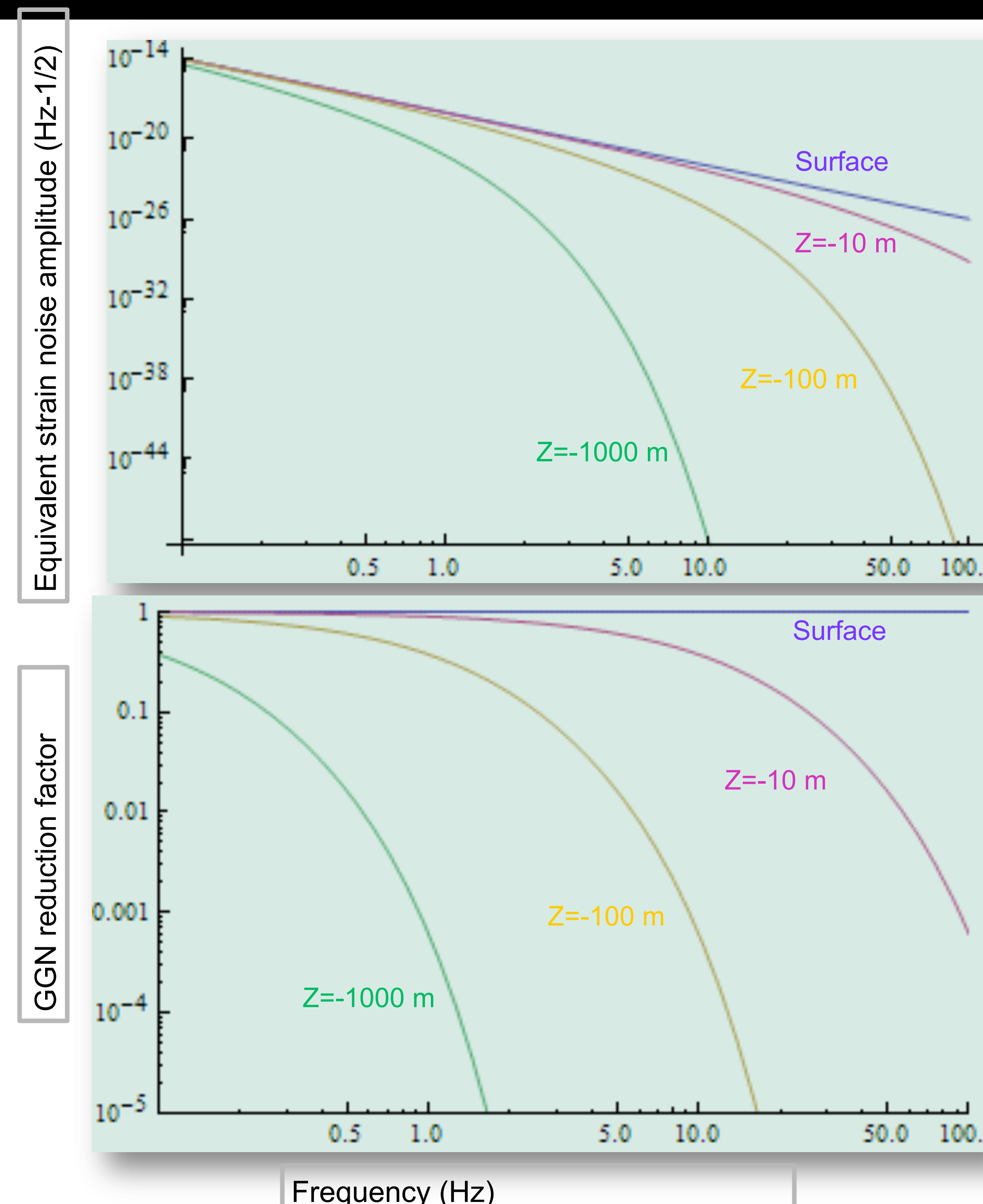
Analytical results by G. Cella
The 58th Fujihara Seminar
(May 2009)

Assumptions:

- C_P = 1000 m/s (lower is better)
- C_P/C_S = 0.5 (lower is worse)
- Surface modes and transverse mode only
- V/H ratio = ½ (lower is better)

Feasible

- Can we do better? especially in the low frequency region
- Volume waves!



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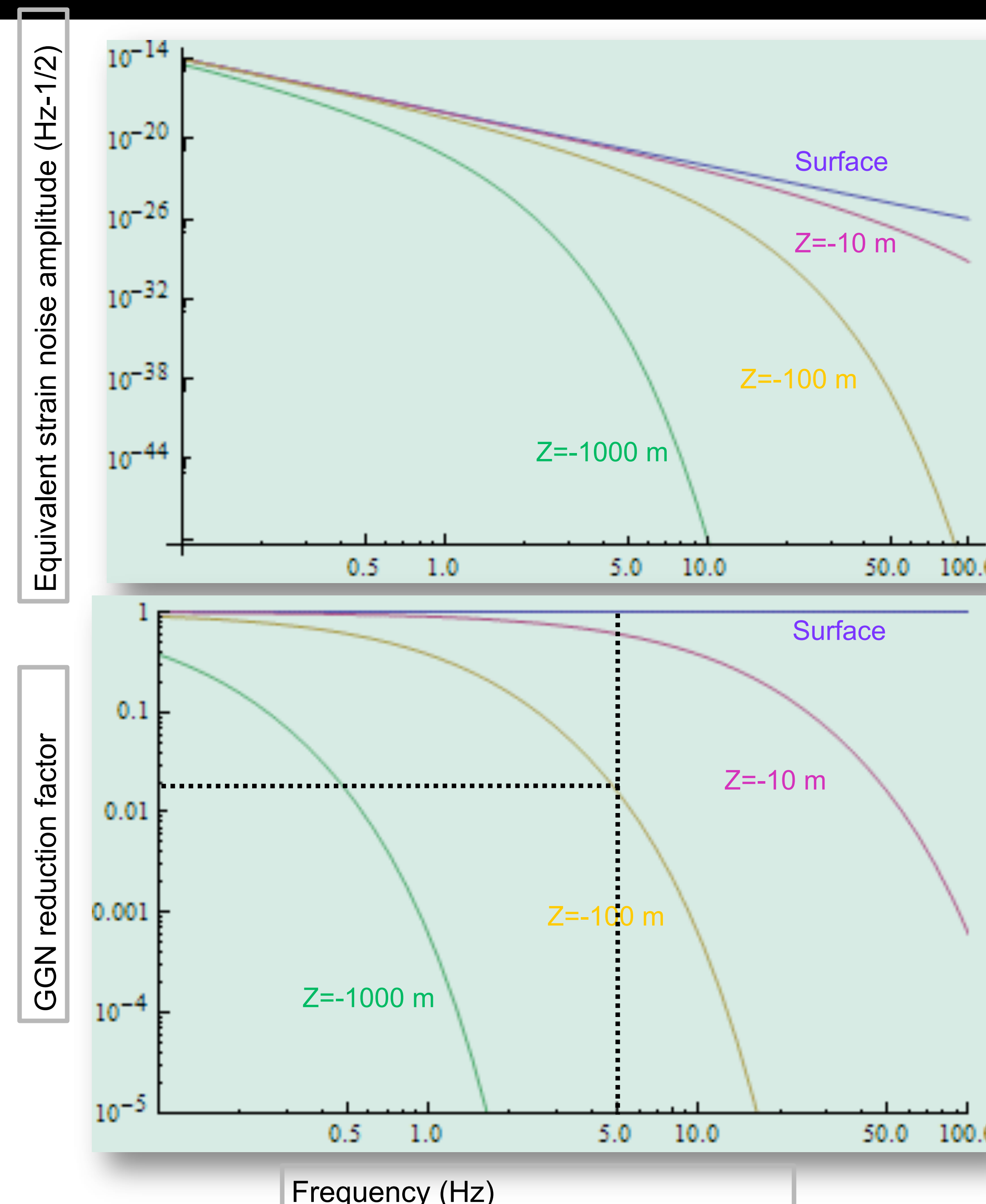


Reproduction of surface wave calculations - Cella

Double plane wave excitation of surface waves
Soil characteristics to match Cella's calculations

Material properties:

- $E = 1.22\times10^{9}$
- $v = 0.33$
- $\rho = 1800$ kg/m^{3}
- $c_p = 1000$ m
- $c_S / c_p = 0.5$

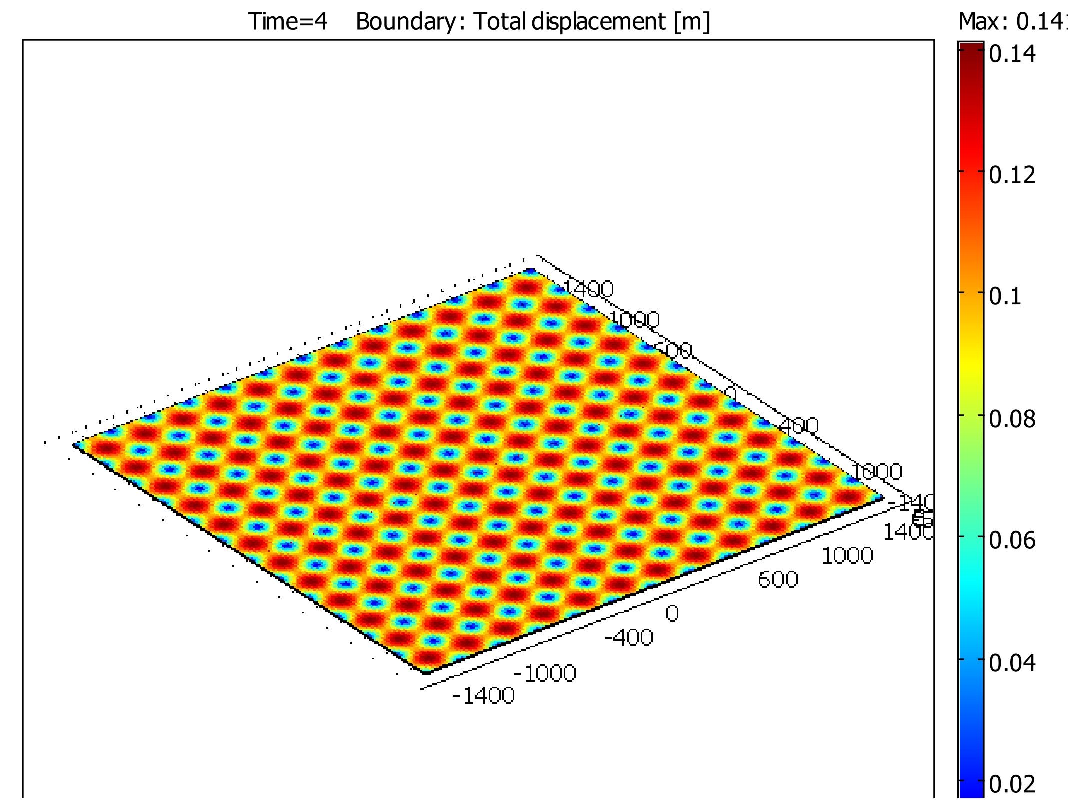
Dimensions:

- 3000x3000x10 m

Element size:

- 21.4x21.4x10 m

- Reasonable agreement with Cella
  - Limited model size
  - Deviations at higher frequencies/depths
- Geometric suppression increases with
  - Lower velocities
  - Domination by surface waves
- More realistic model
  - Surface wave amplitudes decay exponentially with depth
  - Include compression waves
  - Include incoherent sources



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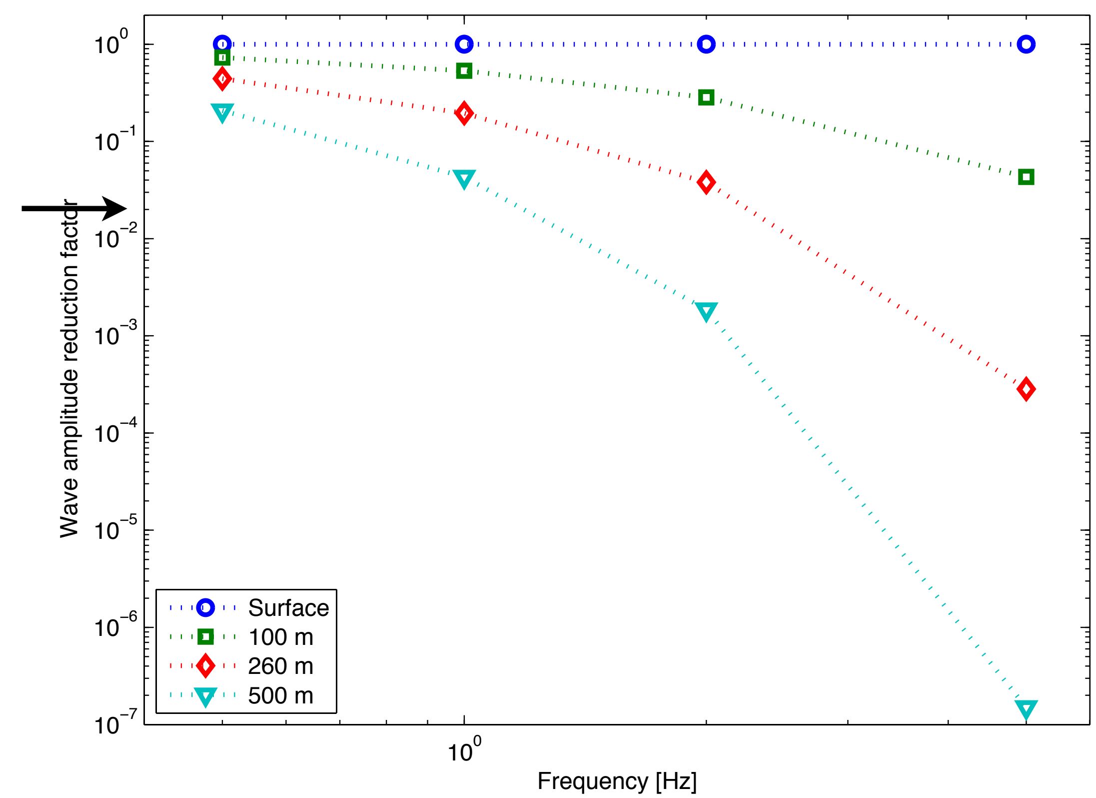

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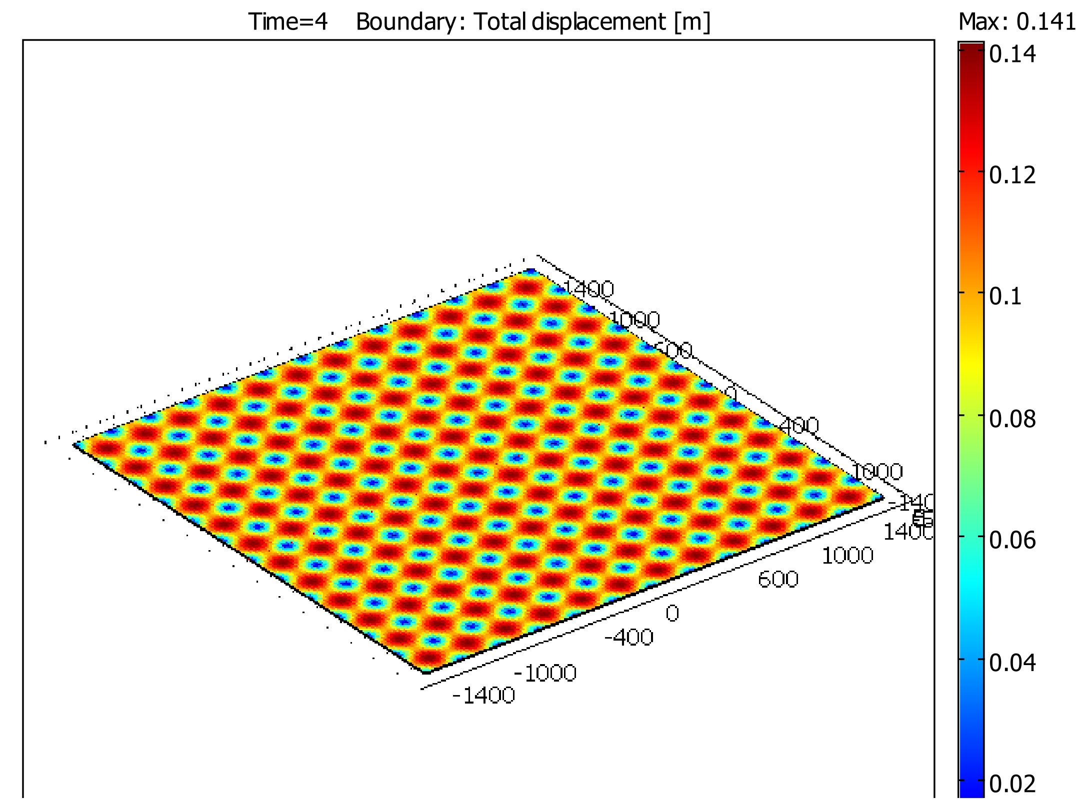

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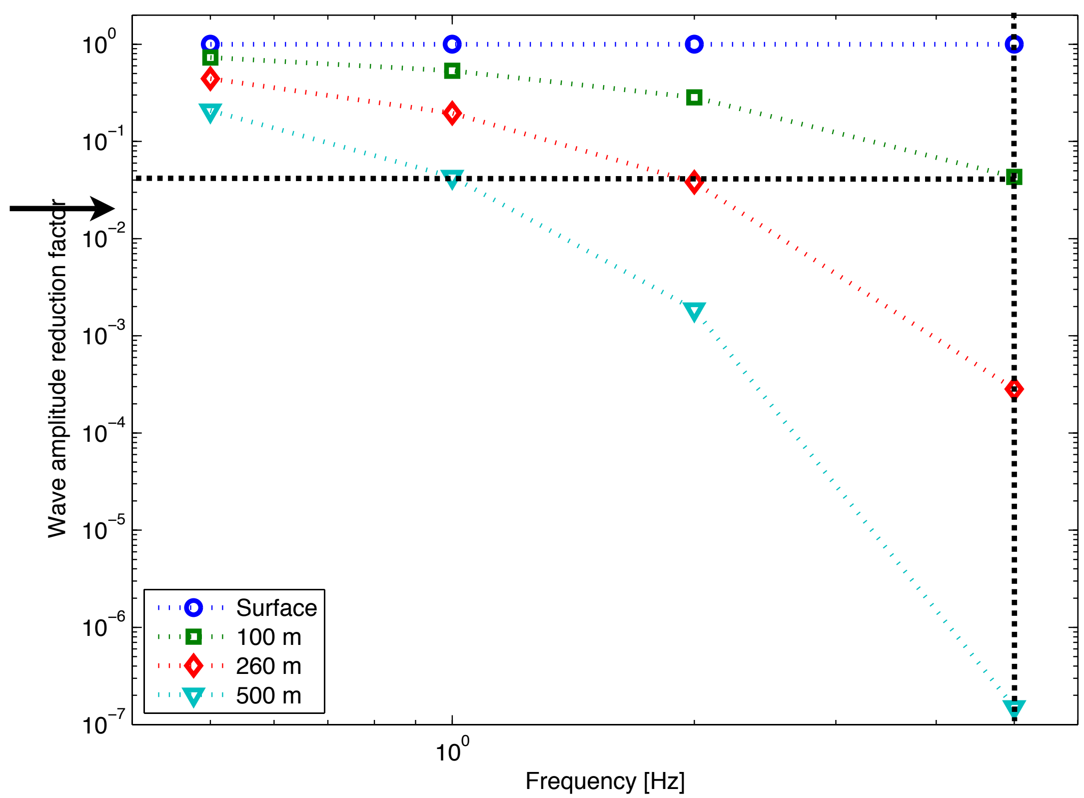

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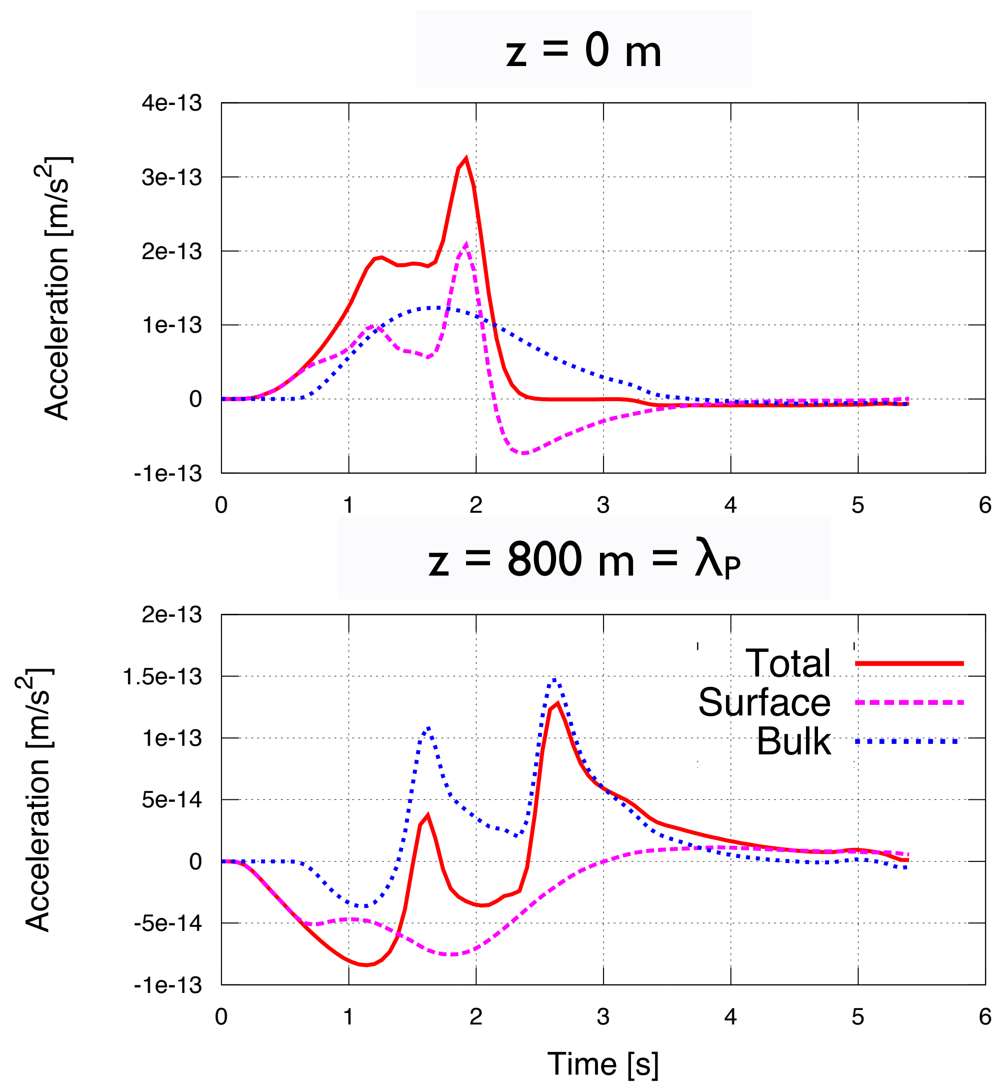
ET GGN requirements

- A reduction factor of 50 is required compared to 5 nm/rtHz
- Surface site with 0.1 nm/rtHz is unrealistic
- Underground: Surface wave averaging may provide significant reduction...
- Cultural noise (pumps / people / traffic) must be minimised
- Investigate effects of cultural noise on gravity gradients

Impulse excitations are used to simulate cultural noise events



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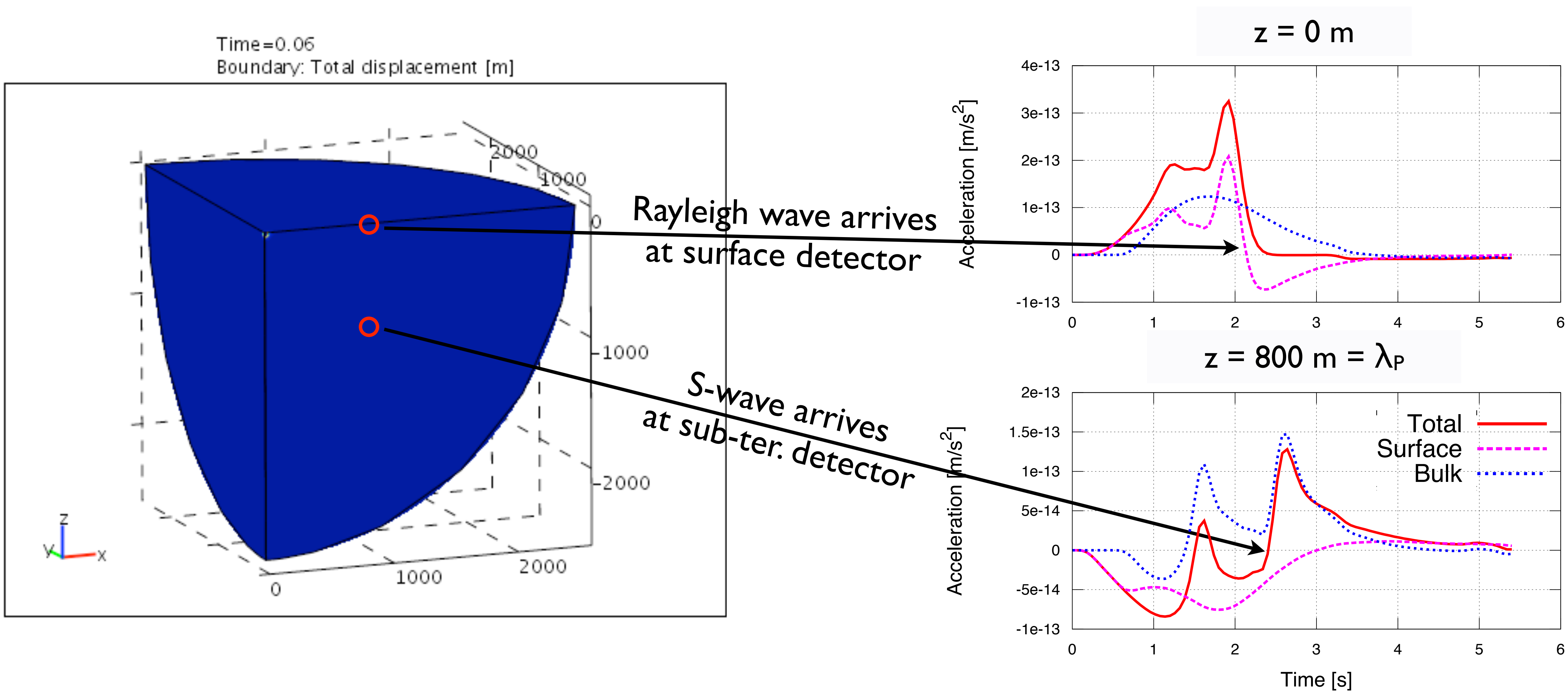

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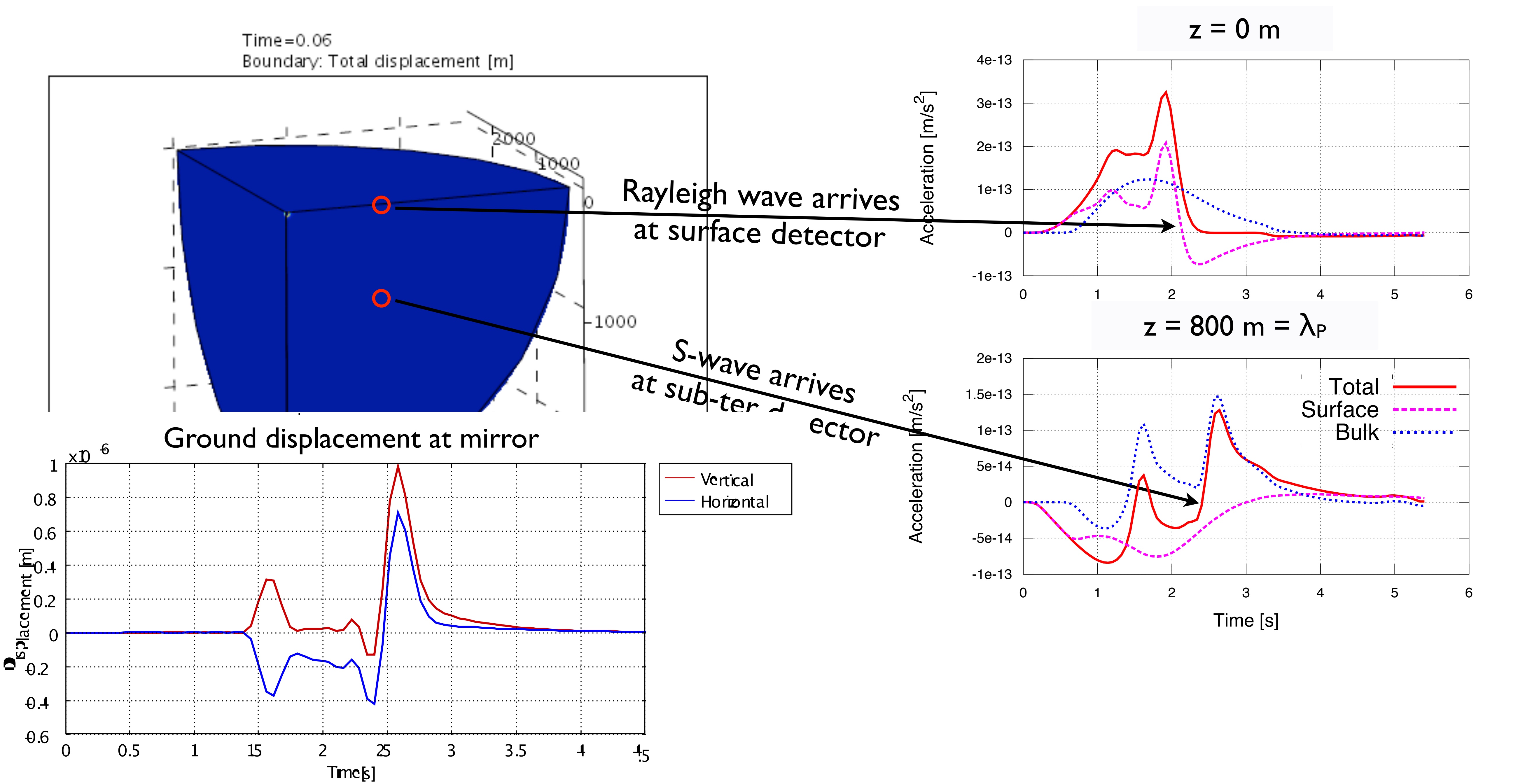

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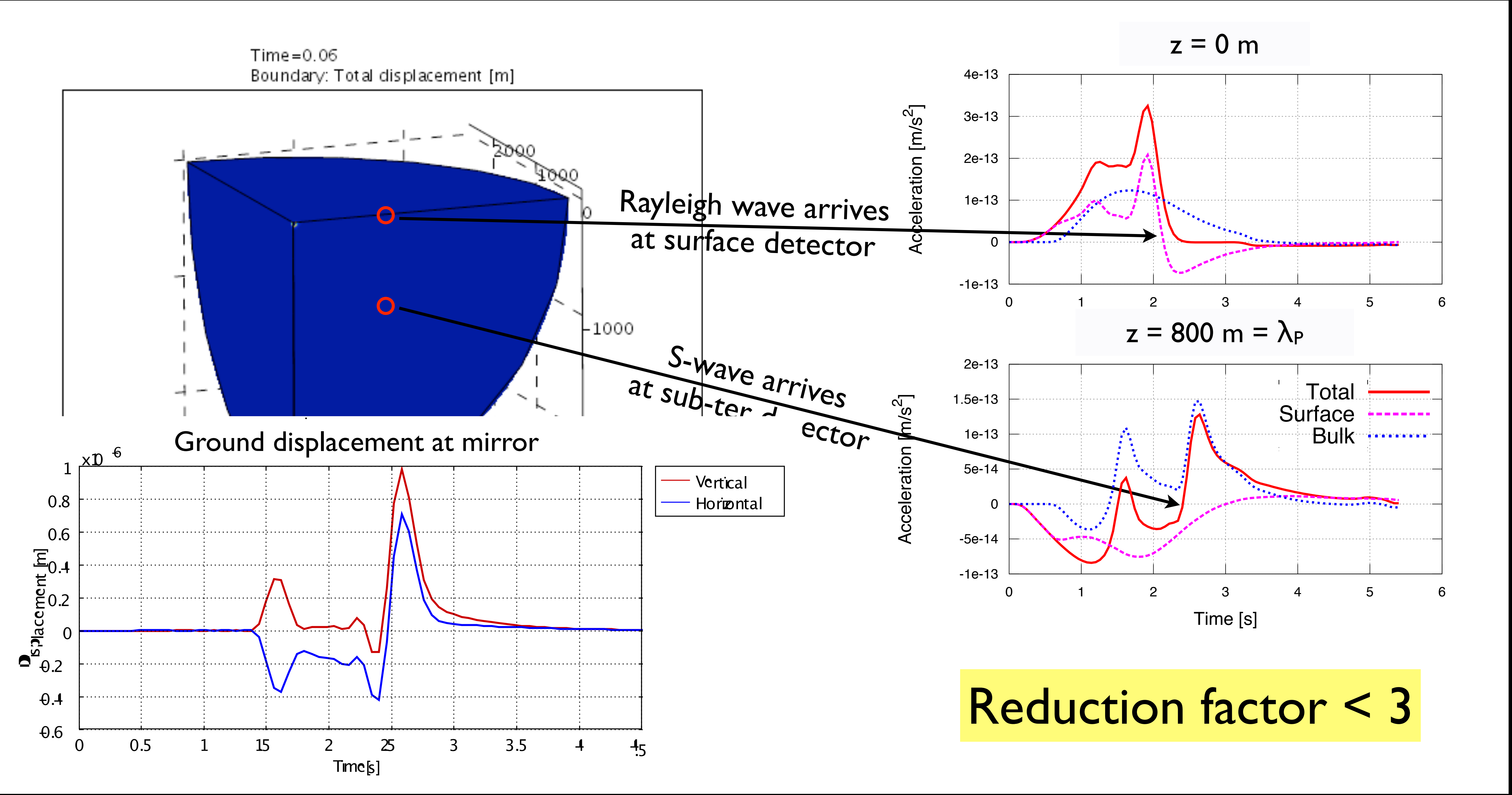

Harmonic excitation of a homogenous medium of Clay or Granite

Clay:
λ_P = 800 m
λ_S = 462 m
ρ = 2000 kg/m^3
ν = 0.25

Granite:
λ_P = 2200 m
λ_S = 1270 m
ρ = 2500 kg/m^3
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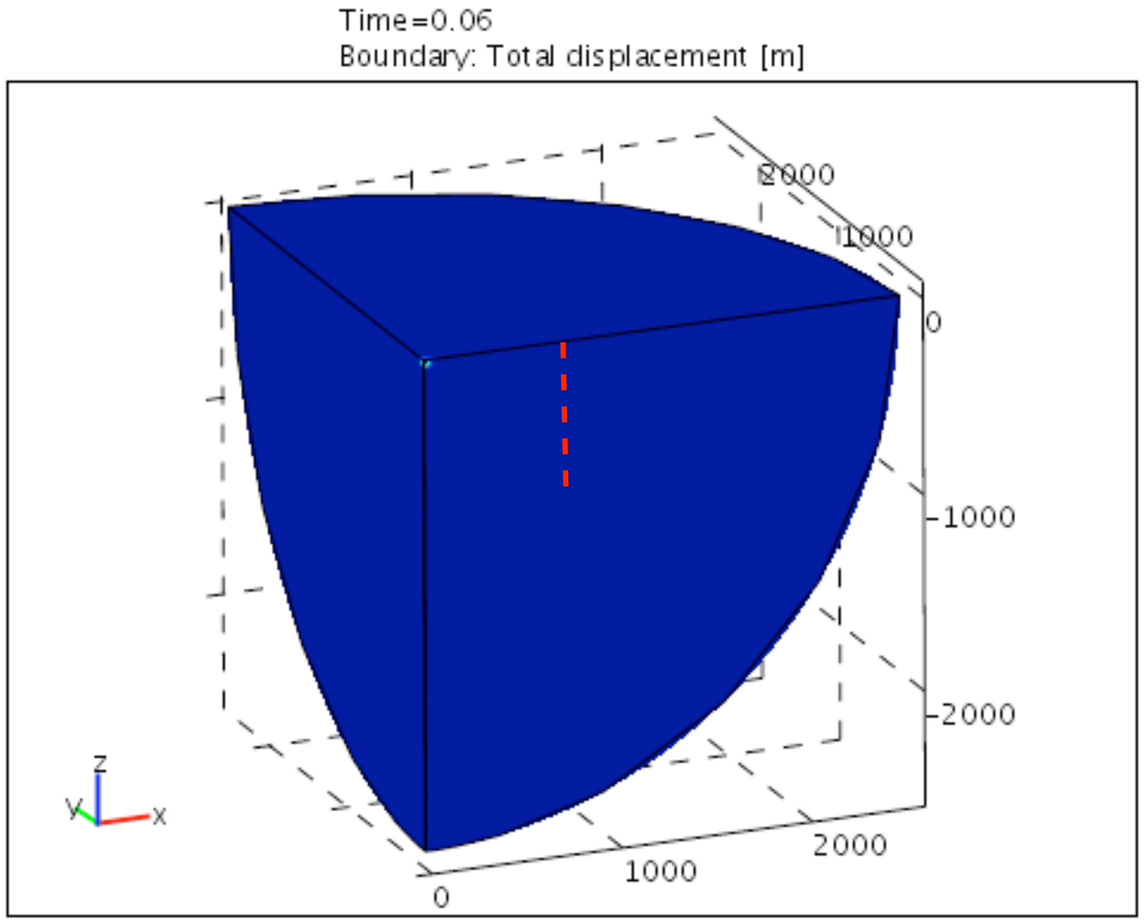

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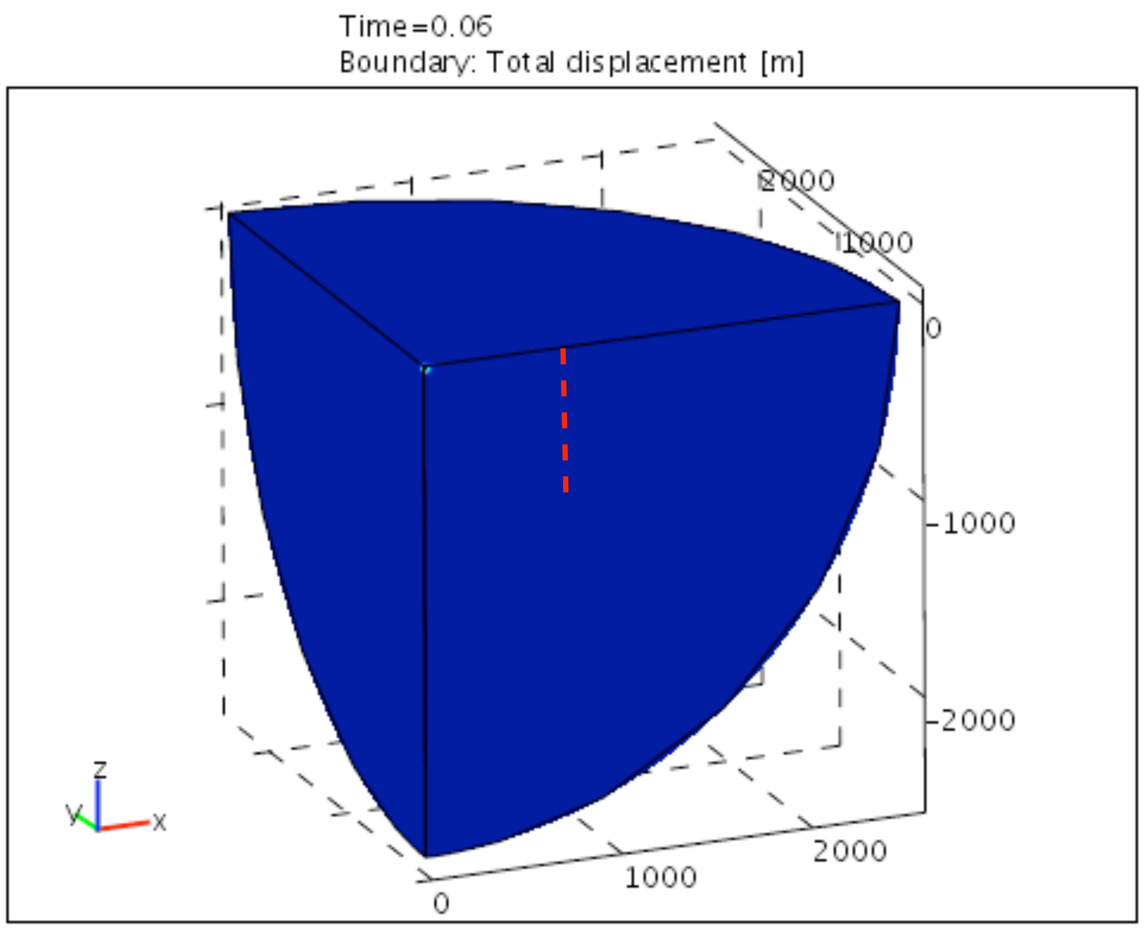

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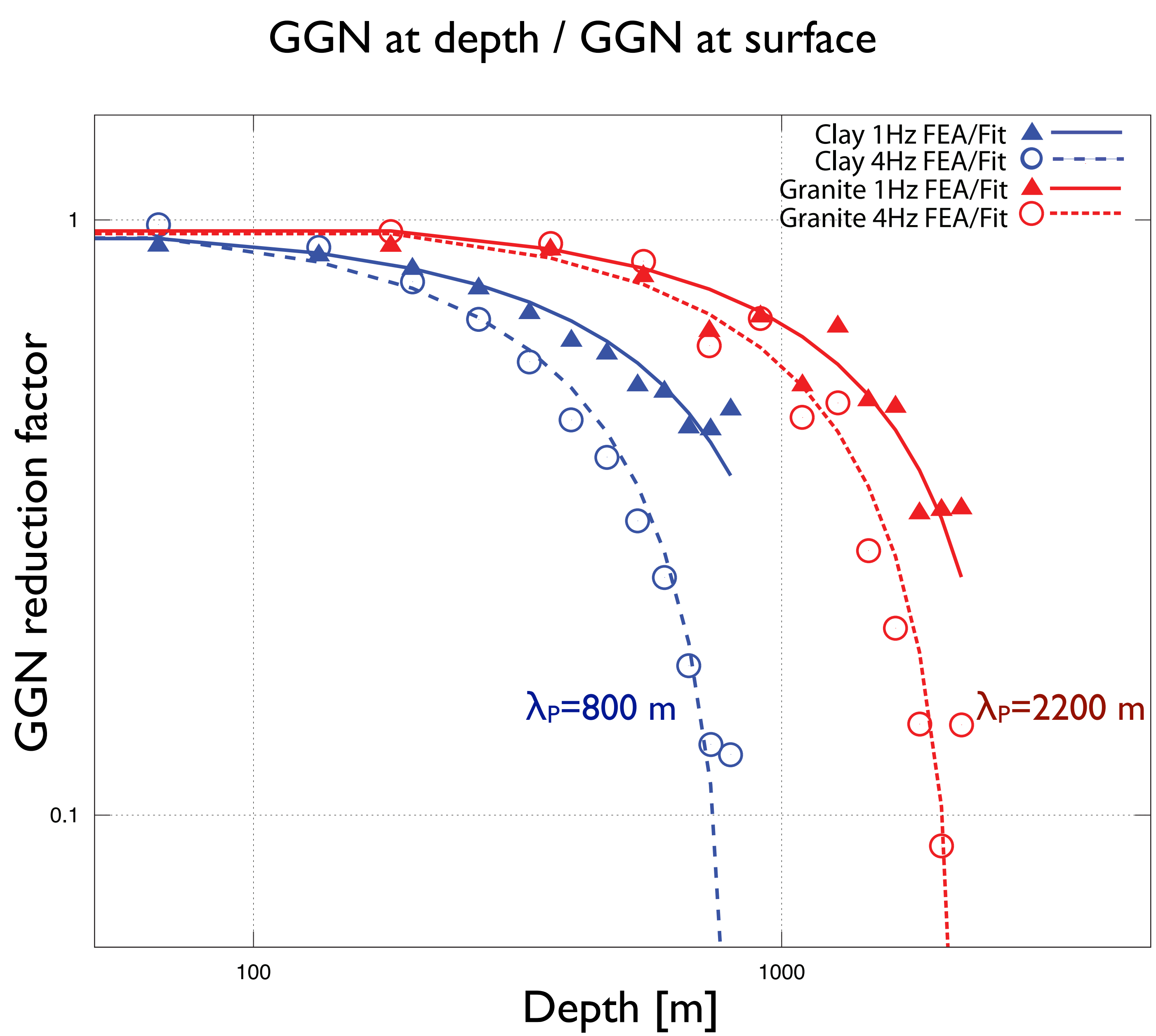

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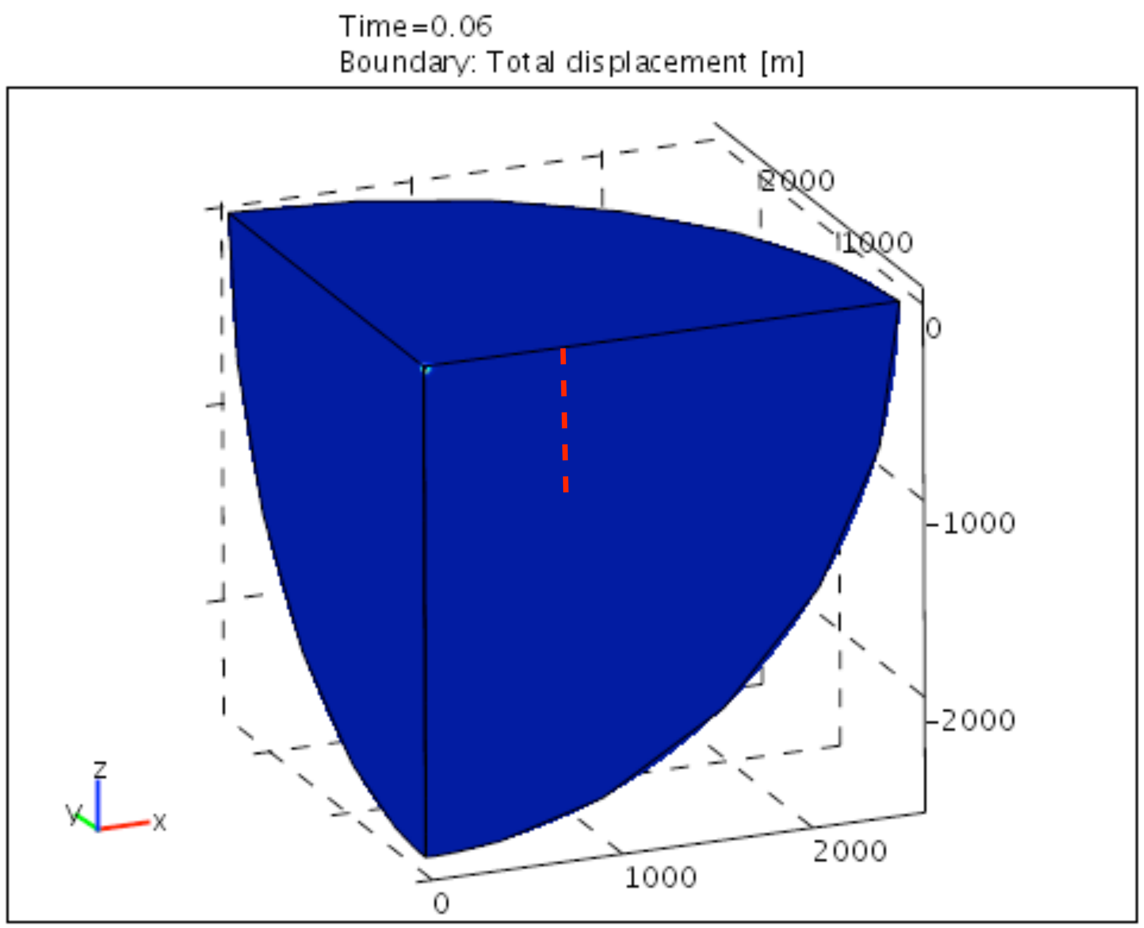

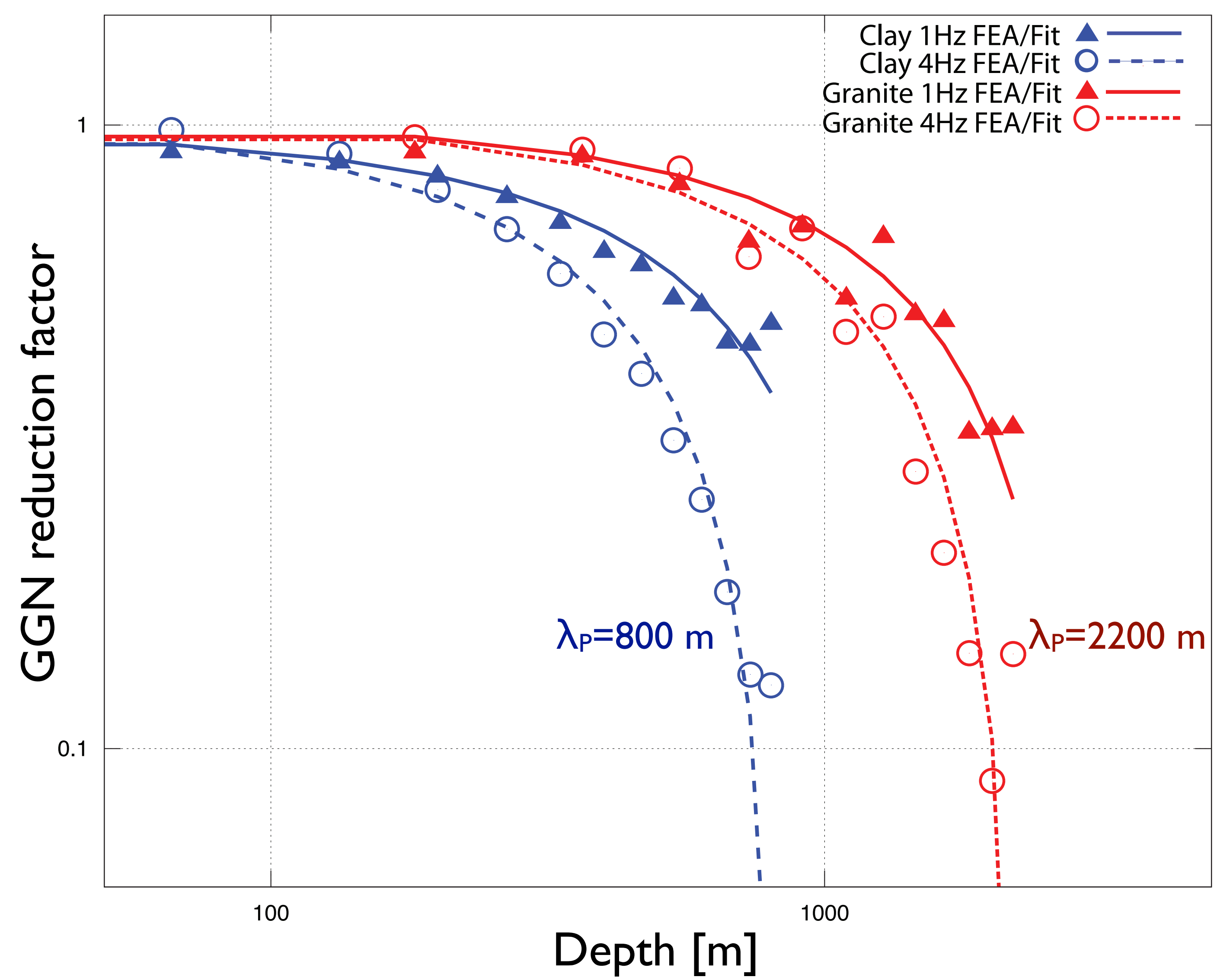

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Clay and granite show similar properties when normalised to same force and λ_P

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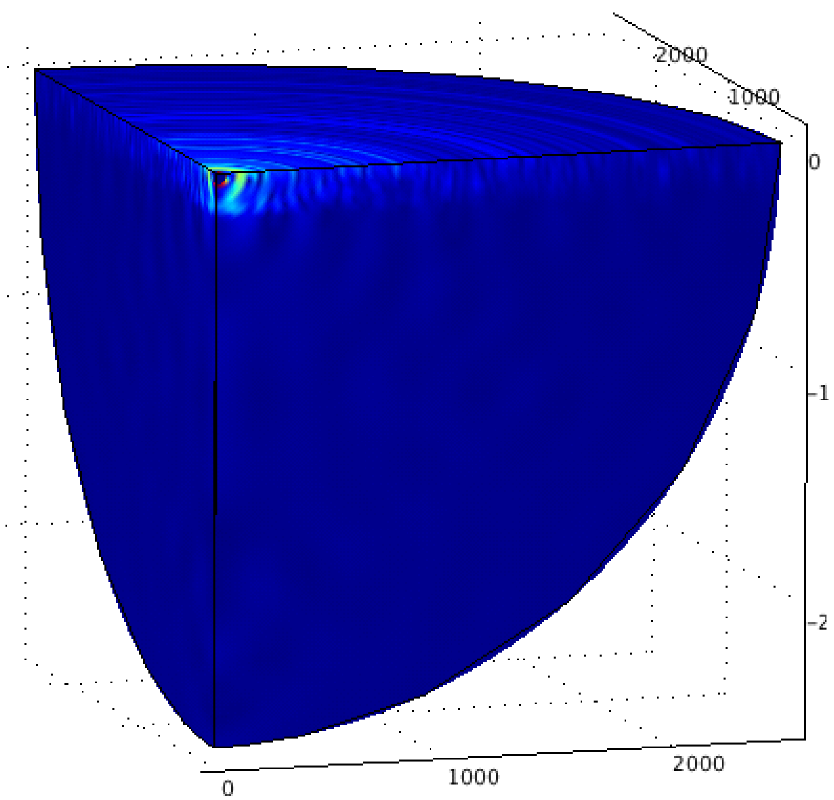
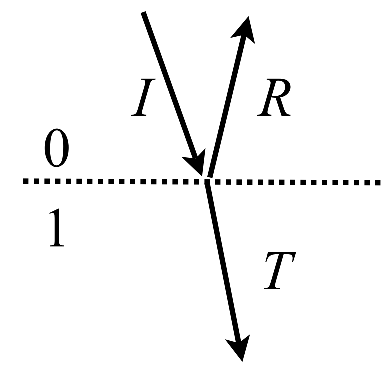
Multi layer geologies provide higher reductions factors by confining energy in top layers



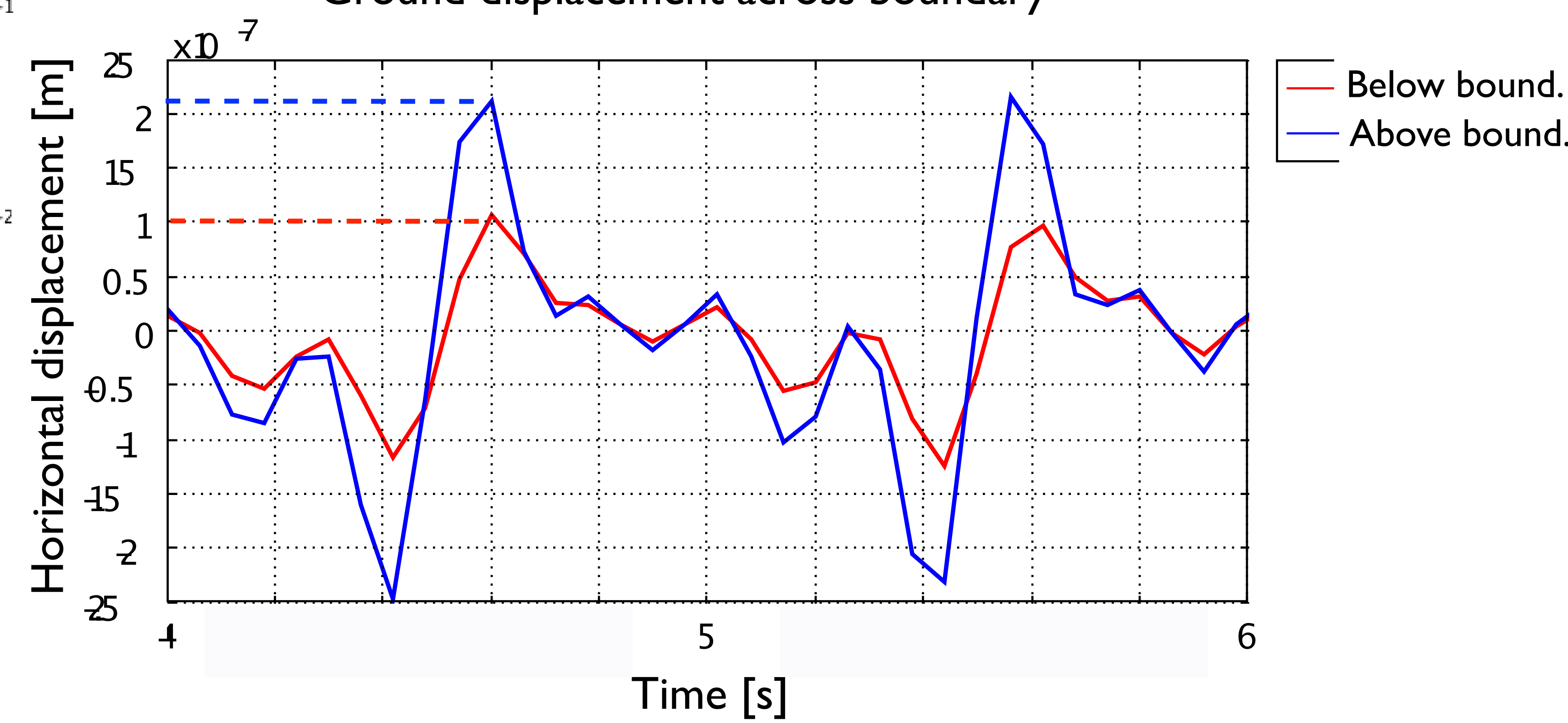
- Top layer: Clay 200 m thick
- Bottom layer: Granite



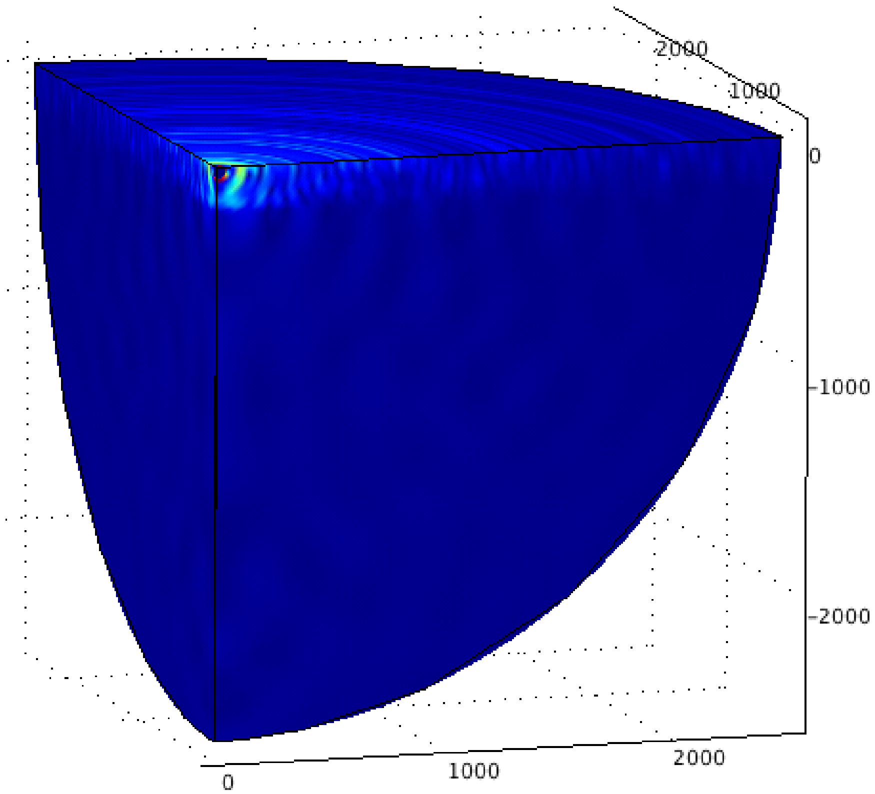
Horizontal shear wave transfer function

$$\frac{A_T}{A_I} = T_{SH} \approx \left. \frac{2}{1 + \frac{c_1 \rho_1}{c_0 \rho_0}} \right|_{\theta=0} = 0.45$$

Ground displacement across boundary



Double layer geologies provide higher reductions factors



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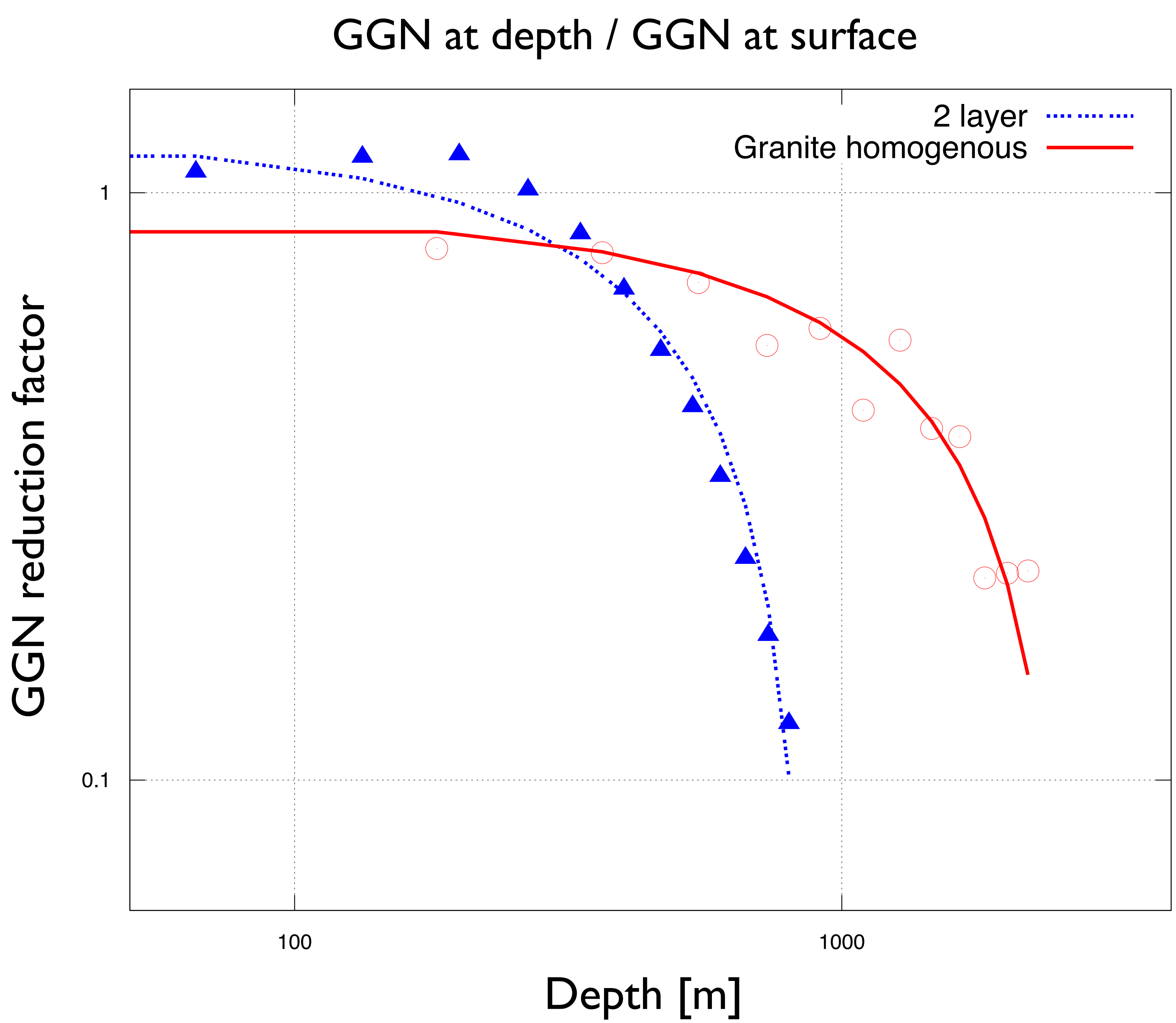

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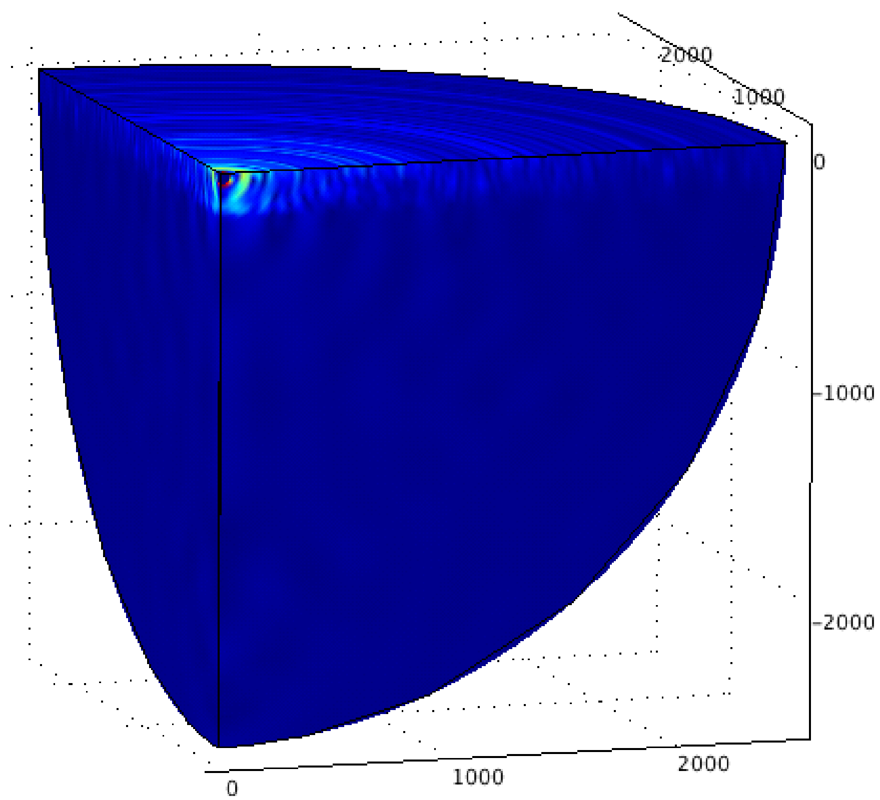
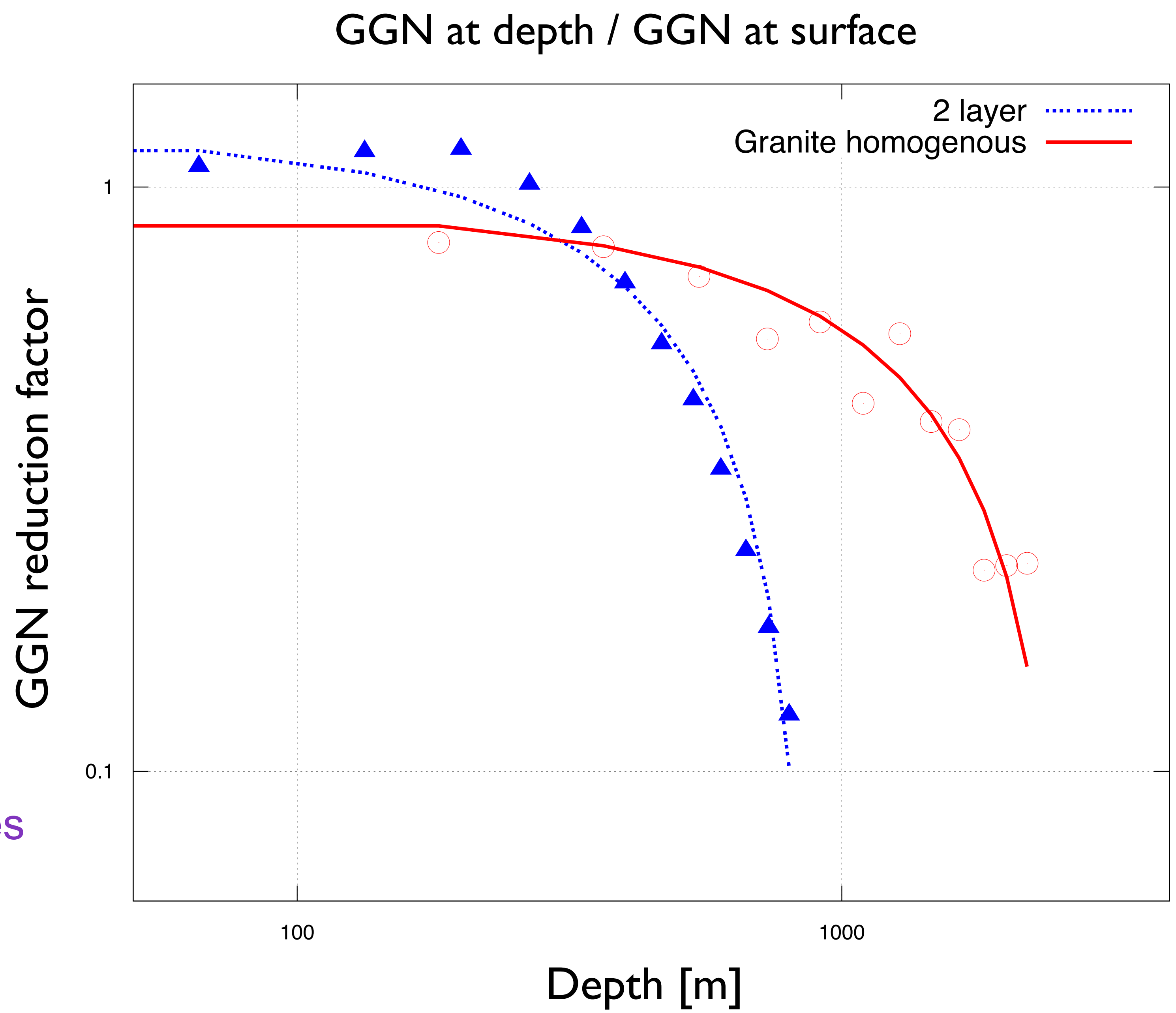


- Top layer: Clay 200 m thick
- Bottom layer: Granite
- In this example reduction factor increases to ~10 without having to dig deeper
- Other ground properties possible to increase this even further



Ongoing research

Homestake network - hard rock
Realmonte - salt
Future additional sites

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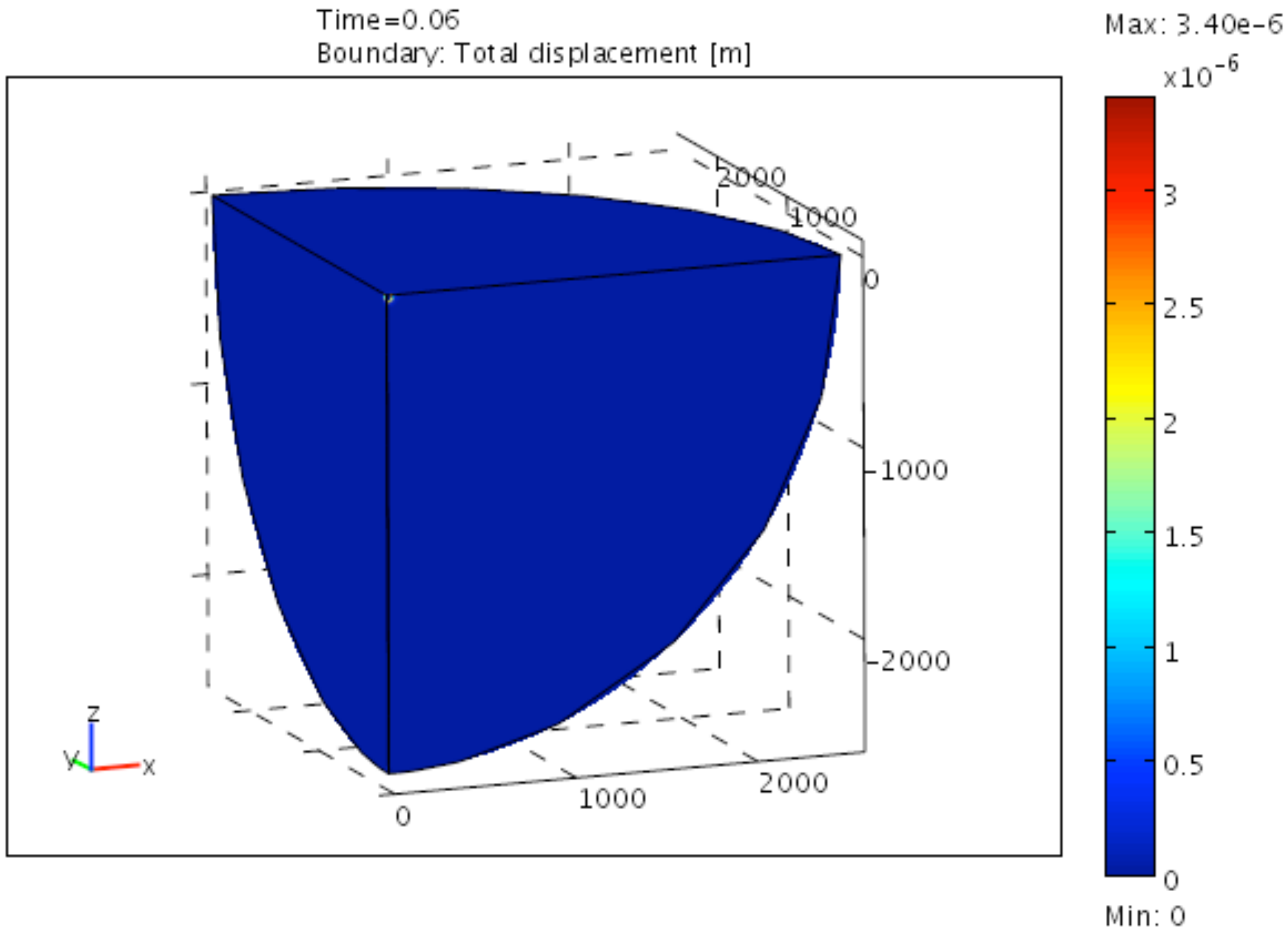
Conclusions

- ET requirements demand underground site
- FEA modeling of seismic wave fields
- Good agreement with analytical solutions
- Investigate
    - *Various geologies and excitations*
    - *GGN subtraction schemes*


Time=0.06
Boundary: Total displacement [m]
Max: 3.40e-6
x10-6
3
2.5
2
1.5
1
0.5
0
Min: 0
2000
1000
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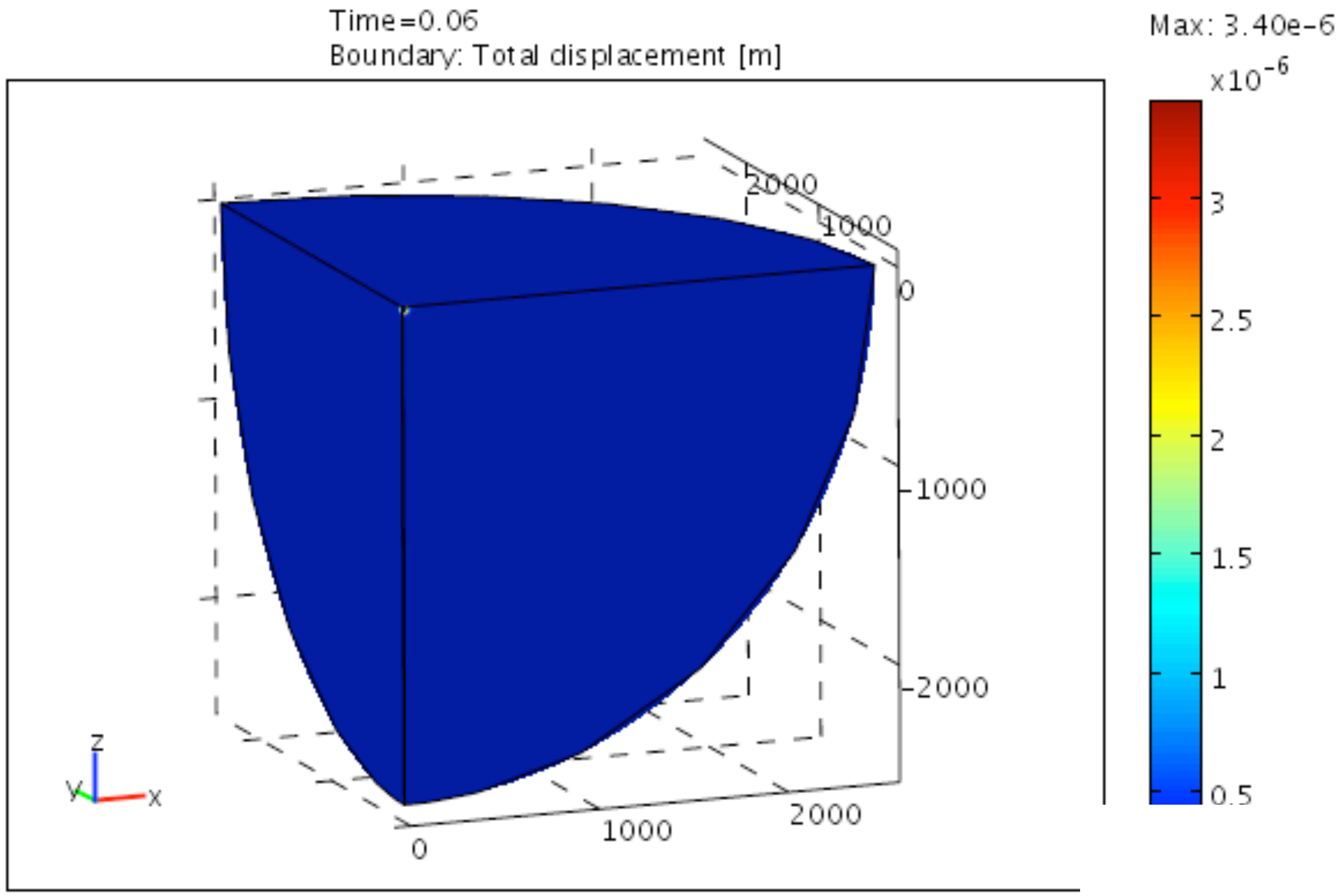
1000
2000
z
y
x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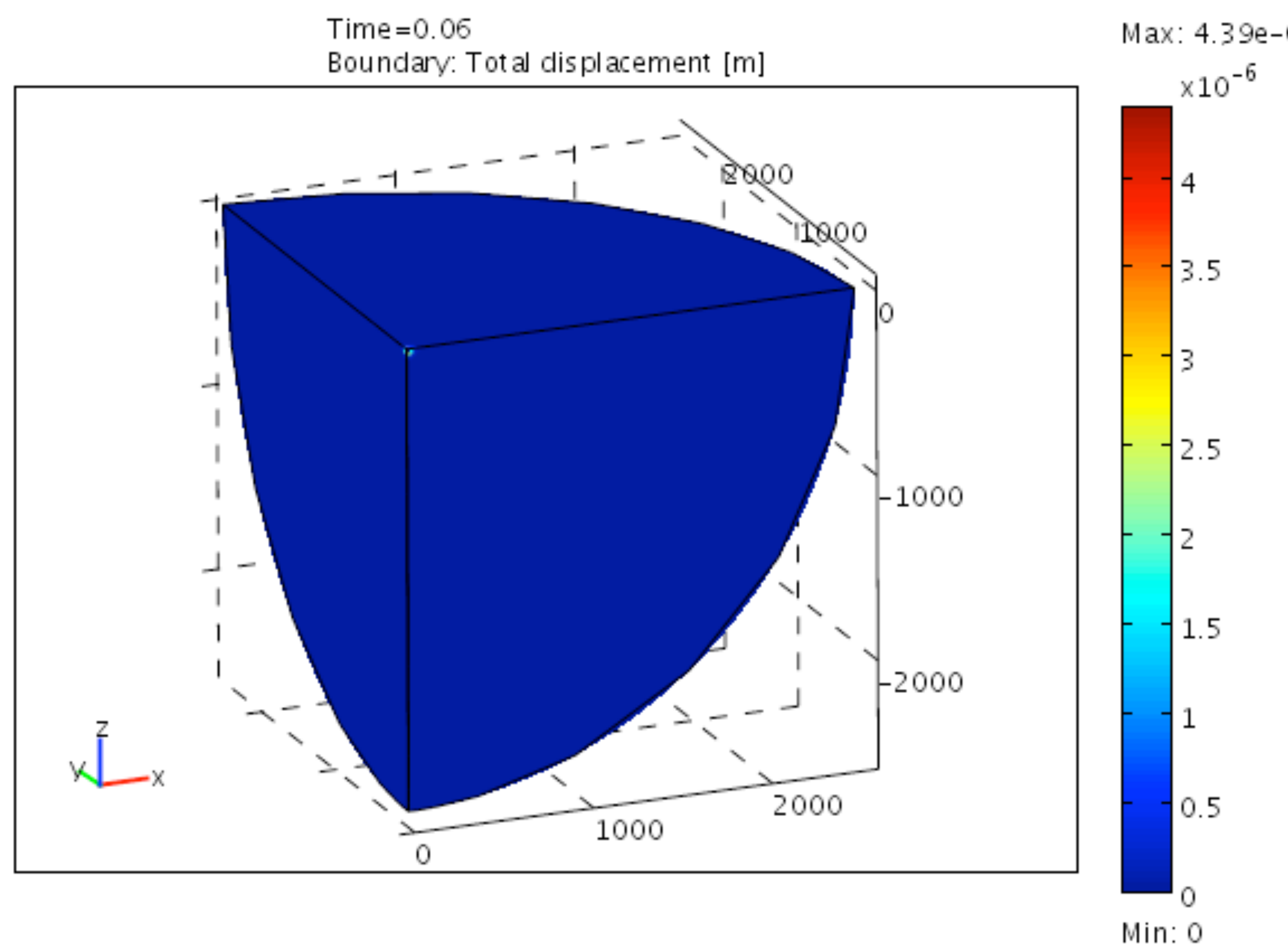
Time=0.06
Boundary: Total displacement [m]
Max: 4.39e-6
x10^-6
4
3.5
3
2.5
2
1.5
1
0.5
0
Min: 0
2000
1000
0
1000
2000
0
1000
2000
z
y
x

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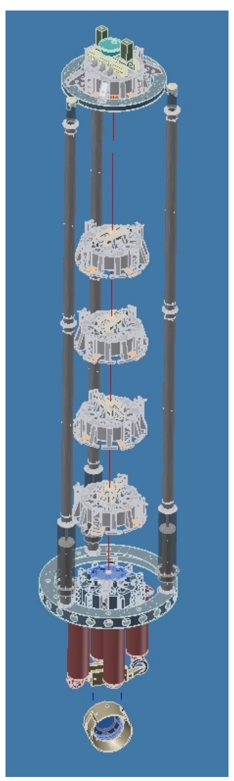
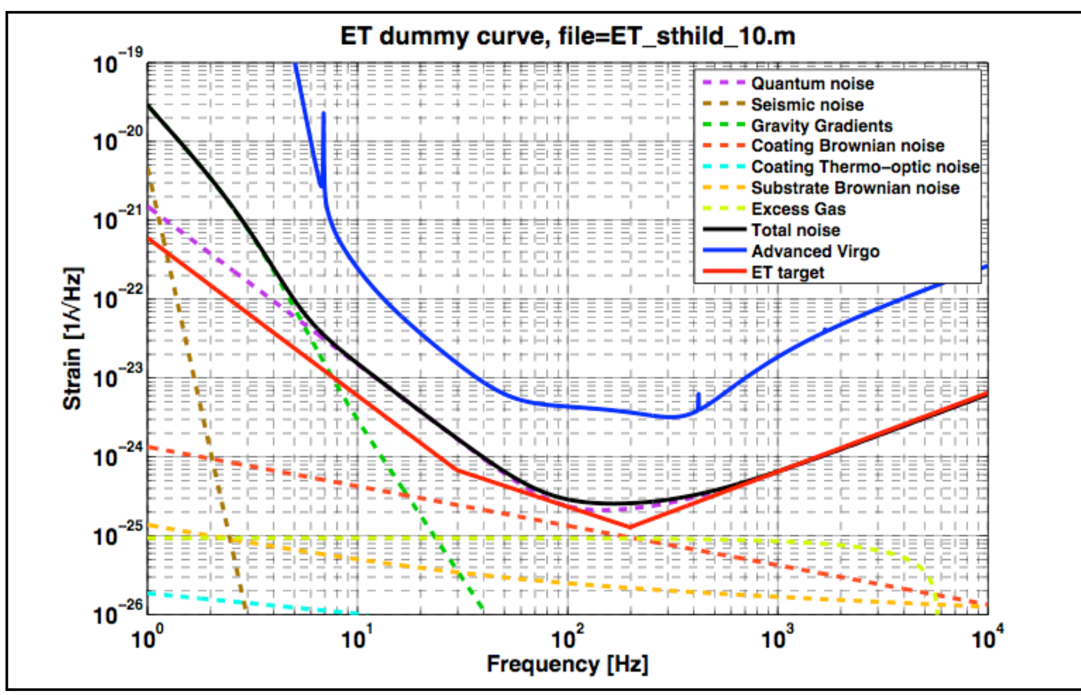
Gravity gradient has a direct coupling to the ITF mirrors

- Seismic noise suppression
  - Development of superattenuators
- Gravity gradient noise
  - Cannot be shielded
  - Network of seismometers and development of data correction algorithms





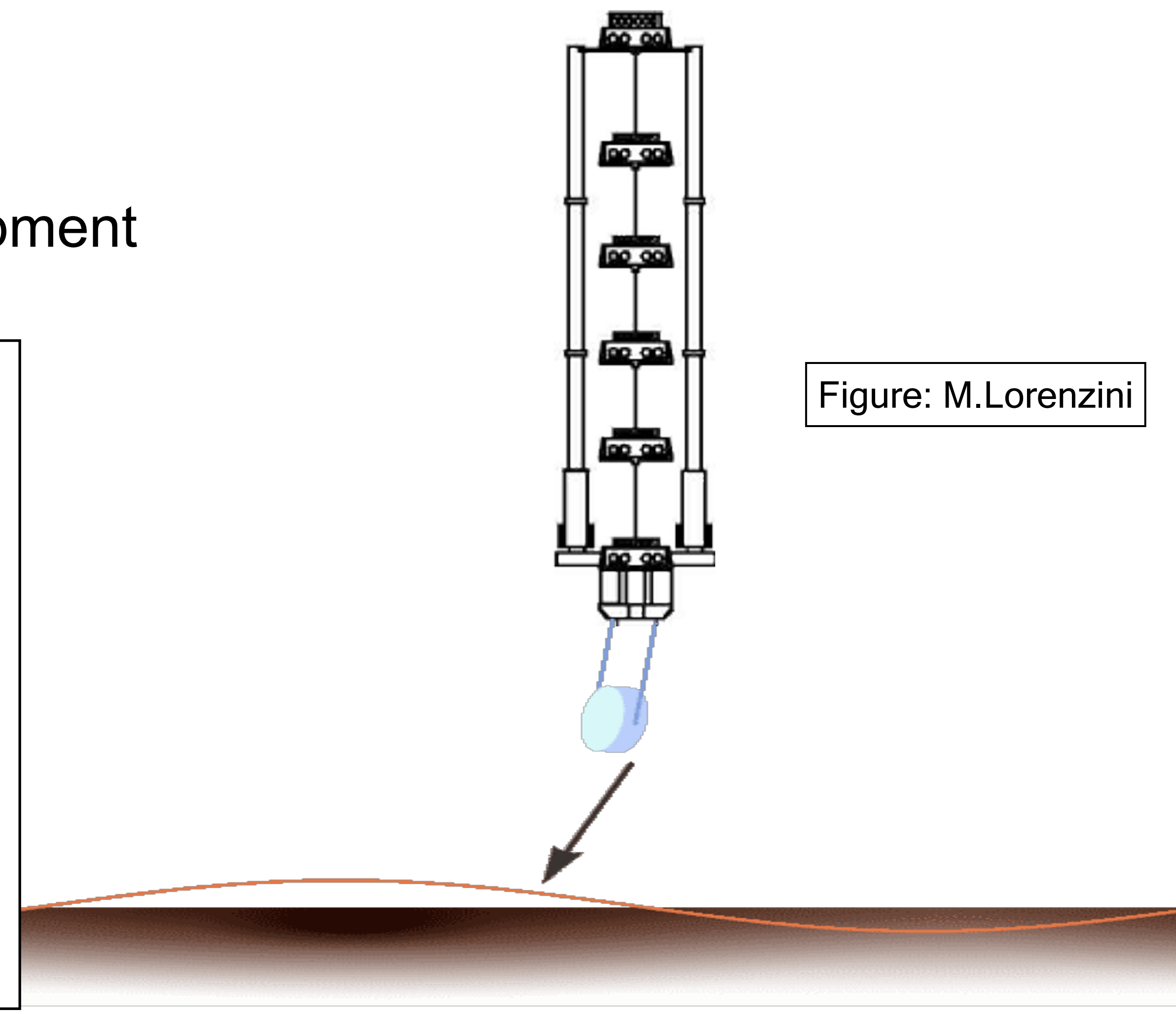
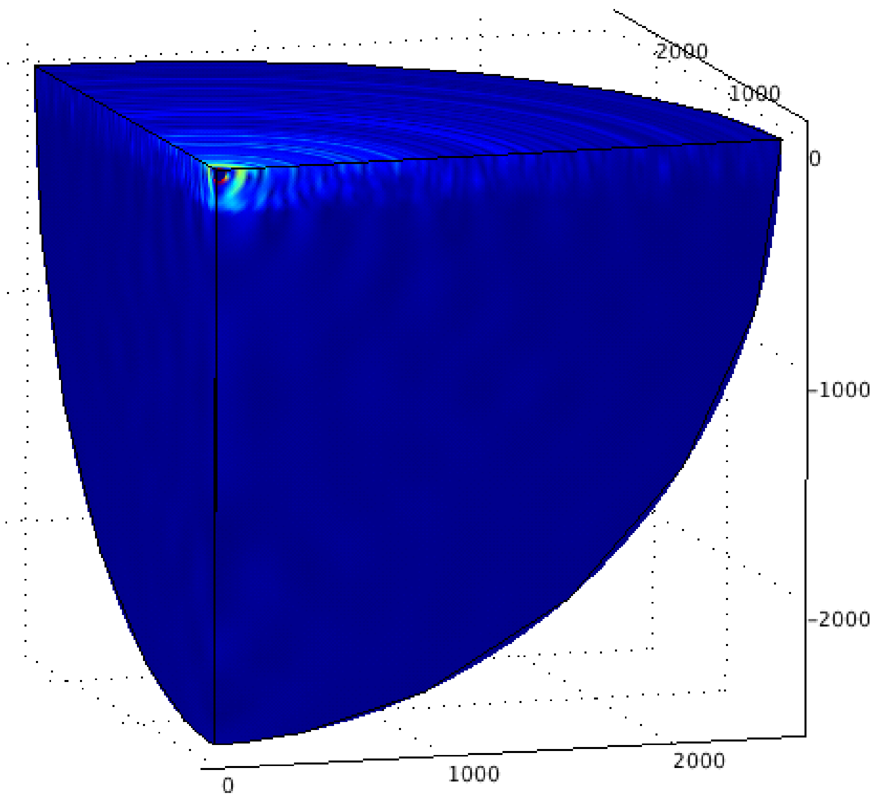


Figure: M.Lorenzini

Multi layer geologies provide higher reductions factors by confining energy in top layers



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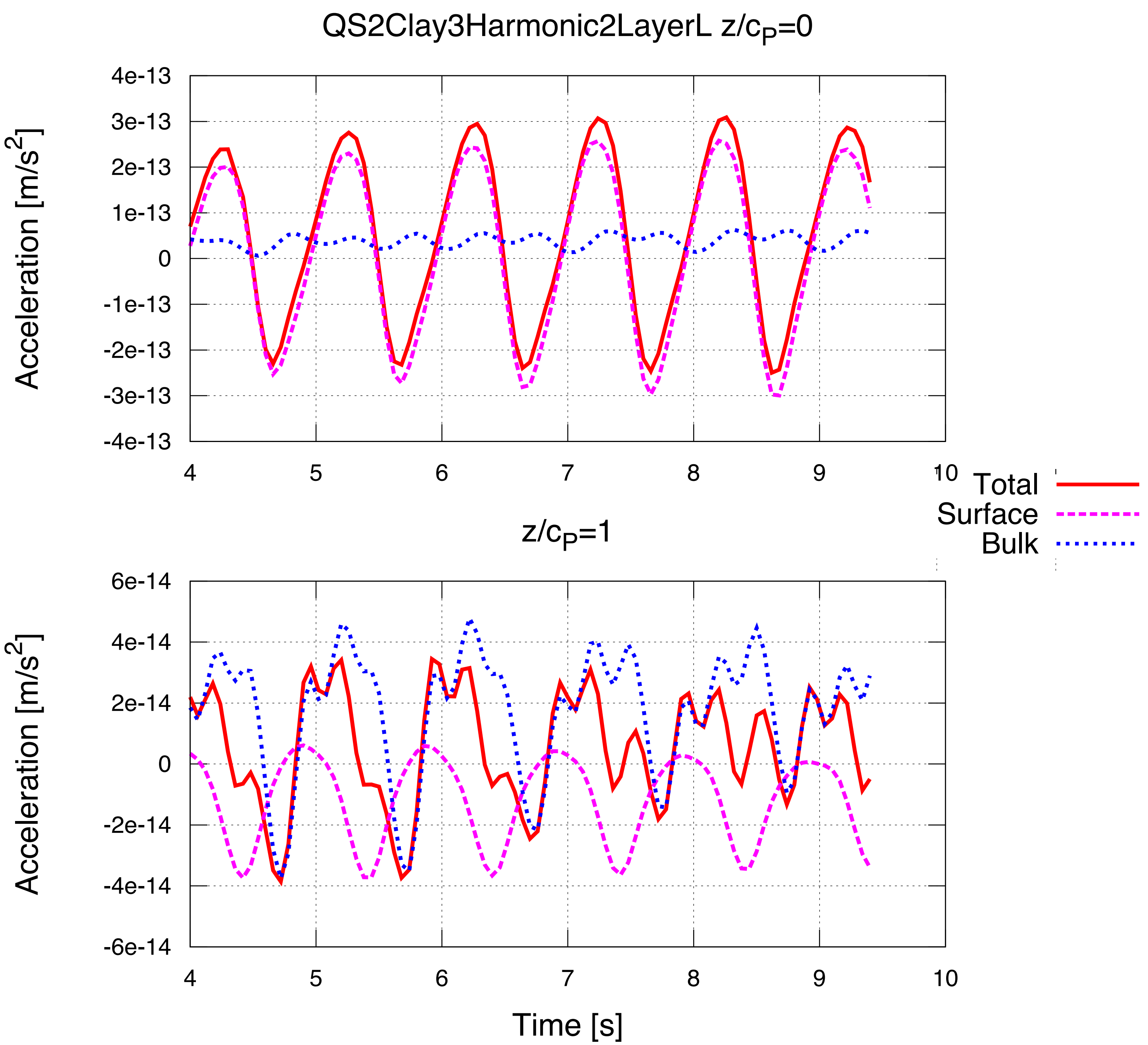

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