Fundamental Physics with Einstein Telescope

THE RELEVANCE OF THE LOW FREQUENCY SENSITIVITY

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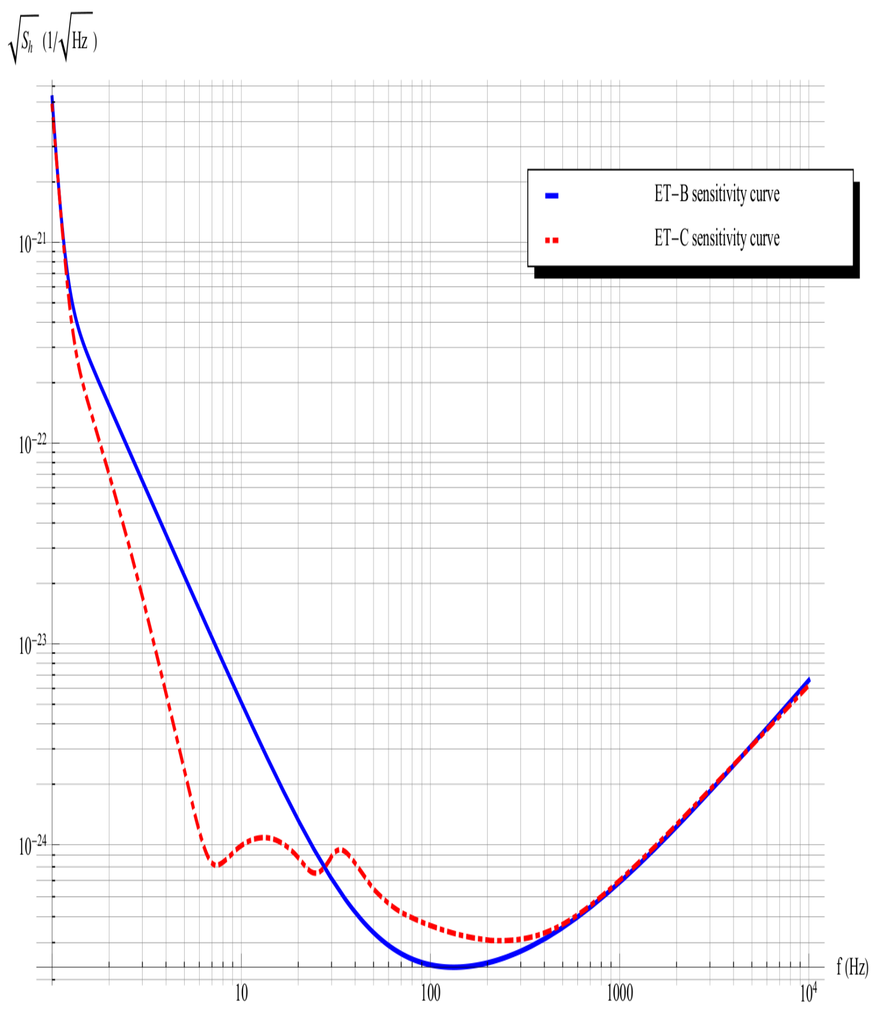
Introduction

- Interferometric detectors are bound by different noise contributions
- Cut-off frequency signifies frequency above which we trust the data to be **stationary** and **gaussian**
  - Data below cut-off frequency will be ignored in data analysis
- Research in fundamental physics will depend on detector design and its corresponding cut-off frequency

Range	Dominant Noise
<5Hz	Seismic, gravity gradient noise
5Hz-50Hz	Thermal noise (optical suspensions), radiation pressure
50Hz-300Hz	Thermal noise (suspended mirrors)
>300Hz	Shot noise (quantum noise)

Design Sensitivities



- ET-B
  - Single 3rd generation broadband detector
- ET-C: Xylophone
  - Two individual detectors
  - Cryogenic low power, low frequency
  - Room-temperature high power, high frequency

Cosmology

INFER CHARACTERISTICS OF THE UNIVERSE BY OBSERVING COALESCING BINARY NEUTRON STARS

Cosmology - Concept

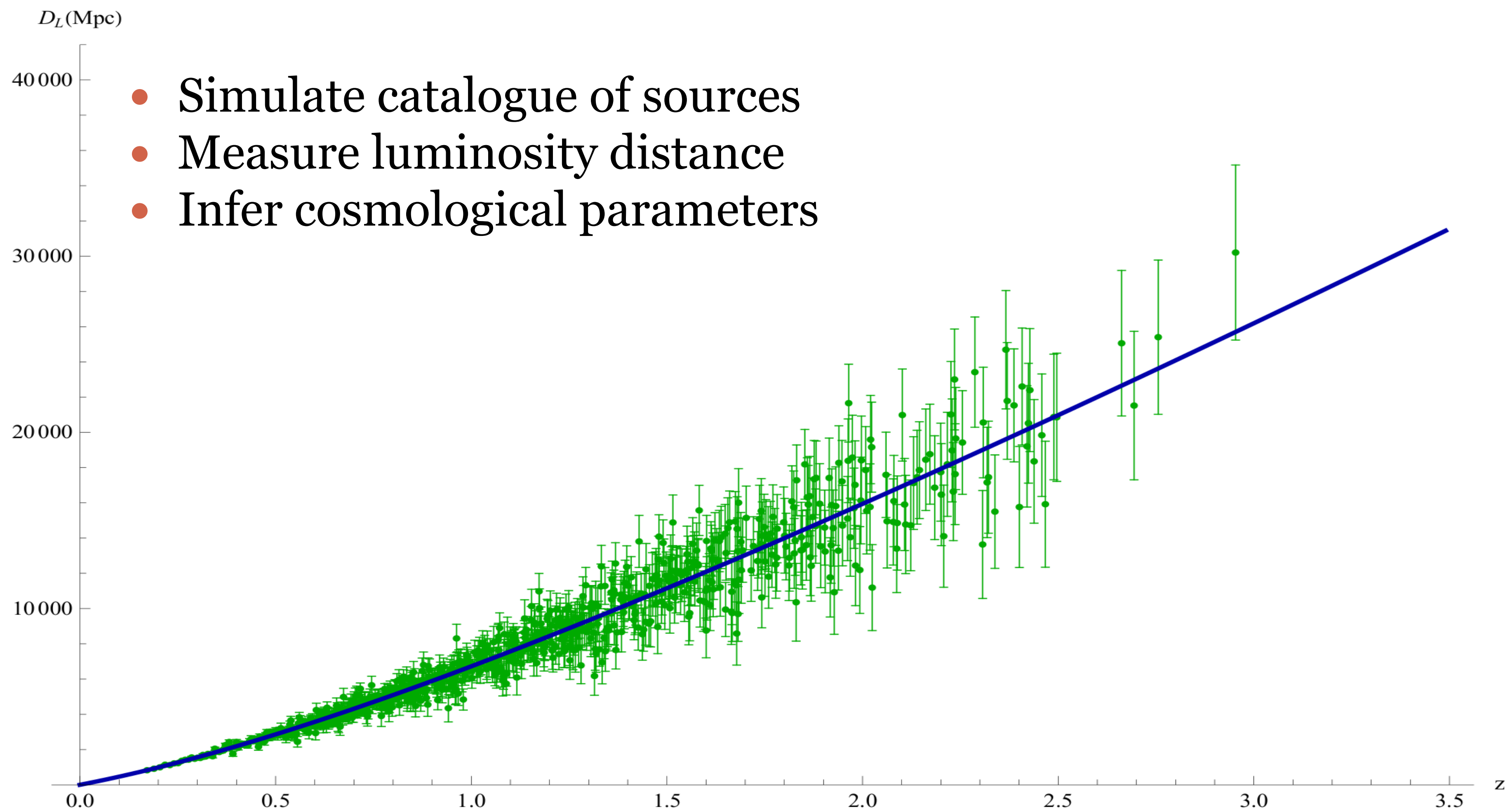
- Infer characteristics of the universe
  - GW signal carries information about the content of the universe between the source and the observer
  - Information encoded in the **luminosity distance**

$$d_L = d_L(z; \Omega_M, \Omega_\Lambda, \Omega_k, w_0, w_a)$$

- Measure set of cosmological parameters
  - Ω_i content densities
  - w_i dark energy equation of state

Measure Cosmological Parameters

- Simulate catalogue of sources
- Measure luminosity distance
- Infer cosmological parameters



Trade Study – Compare ETB/ETC

Model	Ω_M	Ω_{DE}	Ω_k	w_0	w_1
Ω_M, Ω_{DE}, Ω_k, w_0, w_1	1.215	1.005	1.049	1.050	1.057
Ω_M, Ω_{DE}, Ω_k	1.298	1.207	1.228	–	–
Ω_M, Ω_{DE}, w_0, w_1	1.104	1.096	–	1.186	1.207
Ω_M, Ω_{DE}, w_0	1.162	1.162	–	1.164	–
Ω_M, Ω_{DE}	1.178	1.178	–	–	–
w_0, w_1	–	–	–	1.158	1.183
w_0	–	–	–	1.151	-
Average relative improvement:	−15.11%				

- Ratio of **root mean square errors** (ET-C/ET-B)
- Lower cut-off frequency **10Hz**
- ETB performs on average ~15% better

Going Beyond 10Hz Cut-off

10Hz

Model	Ω_M	Ω_{DE}	Ω_k	w_0	w_1
Ω_M, Ω_{DE}, Ω_k, w_0, w_1	1.215	1.005	1.049	1.050	1.057
Ω_M, Ω_{DE}, Ω_k	1.298	1.207	1.228	–	–
Ω_M, Ω_{DE}, w_0, w_1	1.104	1.096	–	1.186	1.207
Ω_M, Ω_{DE}, w_0	1.162	1.162	–	1.164	–
Ω_M, Ω_{DE}	1.178	1.178	–	–	–
w_0, w_1	–	–	–	1.158	1.183
w_0	–	–	–	1.151	-
Average relative improvement:	−15.11%				

5Hz

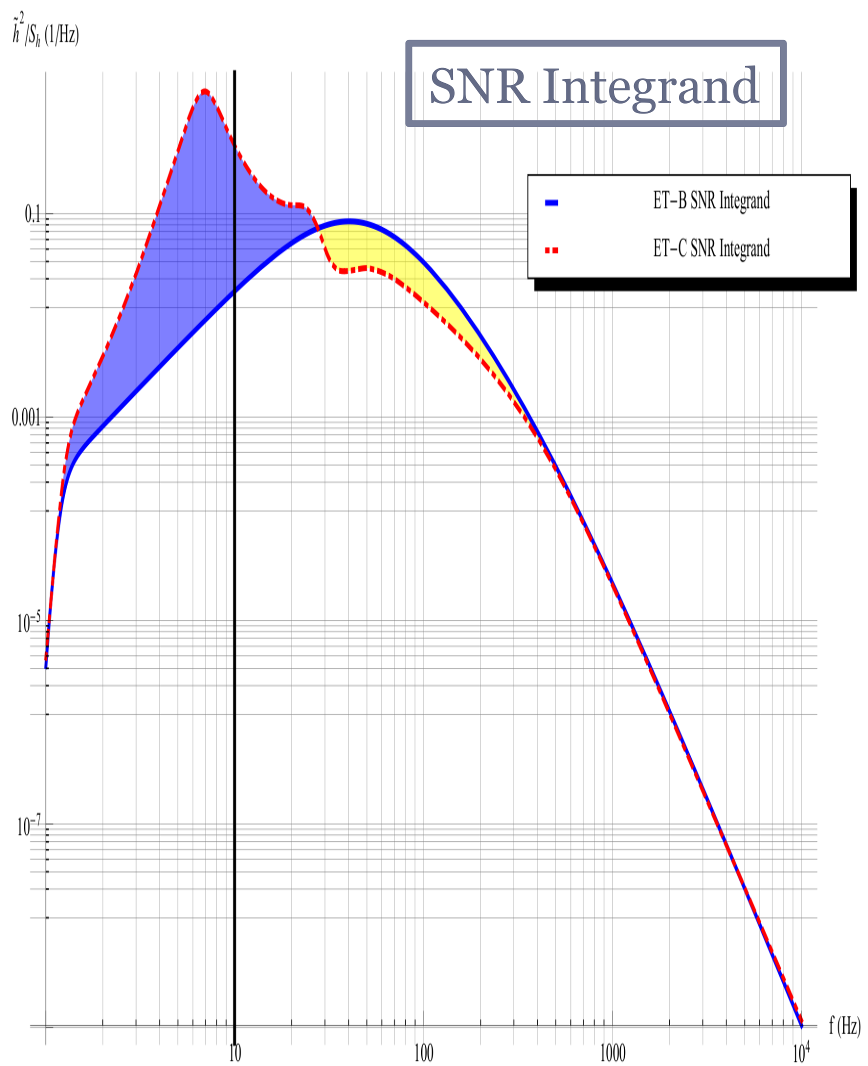
Model	Ω_M	Ω_{DE}	Ω_k	w_0	w_1
Ω_M, Ω_{DE}, Ω_k, w_0, w_1	0.813	0.992	0.967	0.923	0.989
Ω_M, Ω_{DE},Ω_k	0.777	0.826	0.810	–	–
Ω_M, Ω_{DE}, w_0, w_1	0.877	0.884	–	0.828	0.833
Ω_M, Ω_{DE}, w_0	0.815	0.815	–	0.830	–
Ω_M, Ω_{DE}	0.849	0.849	–	–	–
w_0, w_1	–	–	–	0.858	0.842
w_0	–	–	–	0.863	–
Average relative improvement:	13.75%				

1Hz

Model	Ω_M	Ω_{DE}	Ω_k	w_0	w_1
Ω_M, Ω_{DE}, Ω_k, w_0, w_1	0.805	0.993	0.967	0.914	0.969
Ω_M, Ω_{DE},Ω_k	0.765	0.816	0.799	–	–
Ω_M, Ω_{DE}, w_0, w_1	0.872	0.879	–	0.821	0.820
Ω_M, Ω_{DE}, w_0	0.804	0.804	–	0.819	–
Ω_M, Ω_{DE}	0.838	0.838	–	–	–
w_0, w_1	–	–	–	0.850	0.834
w_0	–	–	–	0.854	–
Average relative improvement:	14.66%				

- Compare parameter estimation for varying cut-off frequencies
  - 10Hz not sufficient to probe full extent of ET-C design
  - ET-C becomes beneficial when cut-off is <~5Hz
- Absolute errors improved by **~20%**

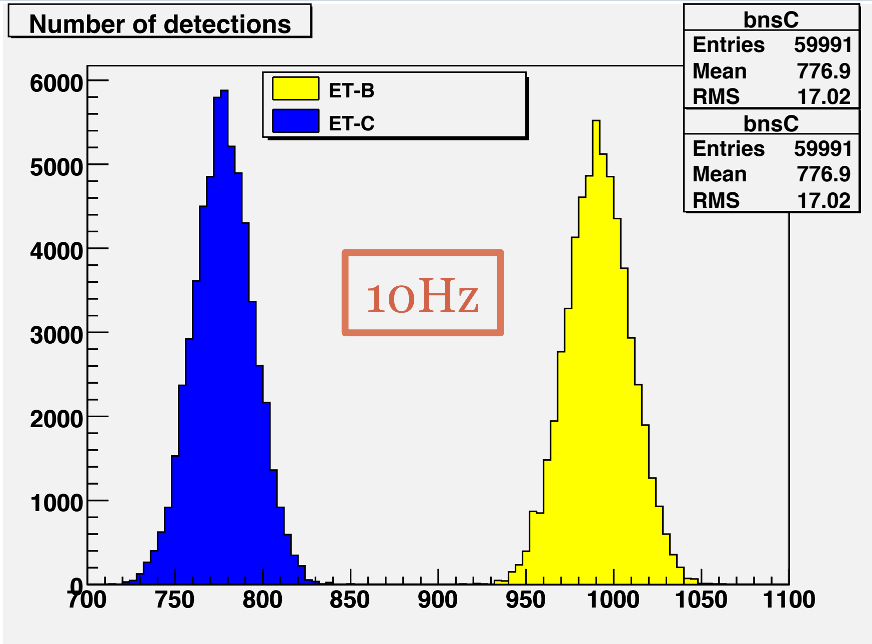
SNR Build-up



- Advantage of ETC in the low frequencies
- 10Hz lower cut-off, ETB has higher SNR due to better sensitivities at >30Hz
- Parameter estimation largely dependent on SNR

Need to push lower cut-off frequency below 10Hz to increase **parameter estimation**

Detectable Sources



Cut-off	ETB	ETC
10Hz	990.7	776.9
5Hz	996.5	1278.6
1Hz	997.3	1301.0

- SNR also influences the number of detectable sources
- SNR cut-off set to 8
- ETB has more detectable sources for 10Hz cut-off

Need to push lower cut-off frequency below 10Hz to increase **detectable sources**

Testing General Relativity

TESTING GENERAL RELATIVITY BY CHECKING CONSISTENCY BETWEEN POST-NEWTONIAN PHASE COEFFICIENTS

Post-Newtonian Formalism

- General two body problem in relativity cannot be solved analytically
- Approximate waveforms by expanding equations in powers of v/c
  - Where v is the characteristic velocity of the orbit
- General form of Post-Newtonian (PN) waveforms

$$h(t;\vec{\theta}) = \sum_{n=1}^{N_H} \sum_{k=0}^{K} A_{nk}(t) \cos(n\Phi(t) + \phi_{nk})$$

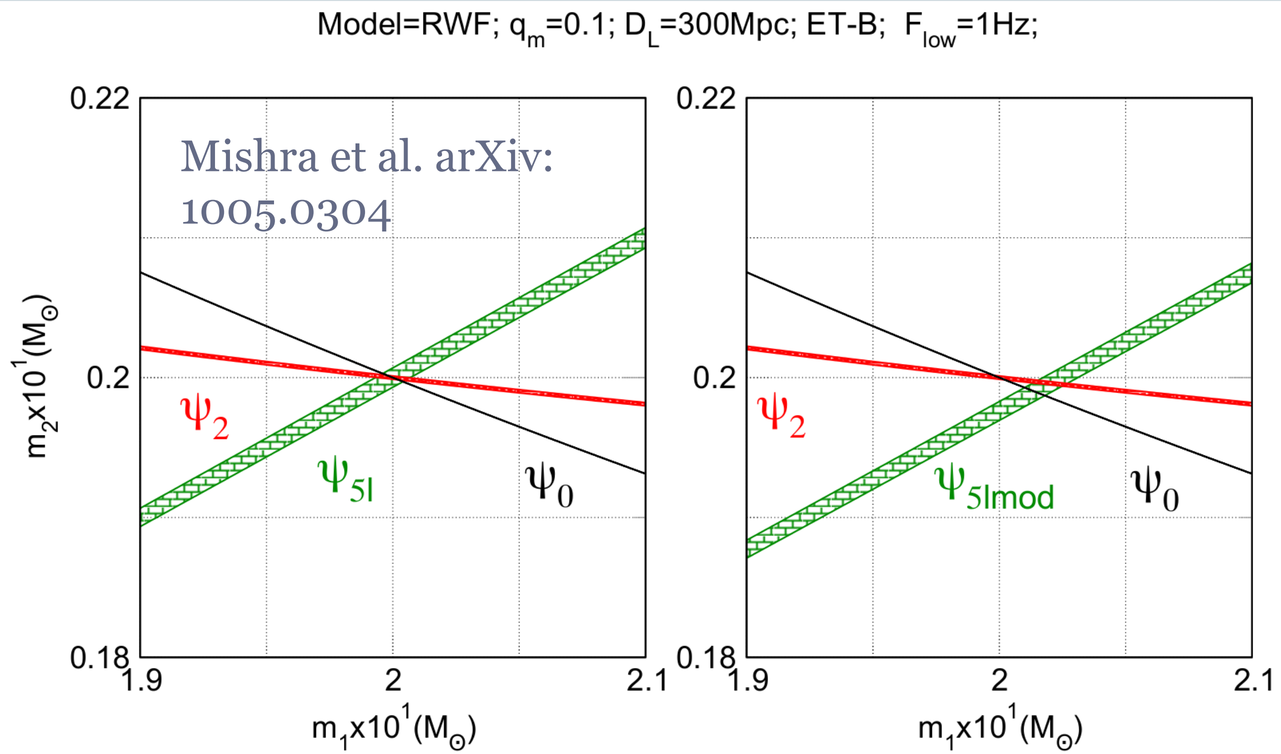
  - Phase evolution represented in time / frequency (SPA) domain

$$\Phi(t) = \phi_c - \sum_{l=0}^{L} \phi_l (t_c - t)^{(5-l)/8} \qquad \Psi(f) = 2\pi f t_c - \phi_c + \sum_{l=0}^{L} \psi_l f^{(l-5)/3}$$

Testing General Relativity - Concept

Arun et al. 2006

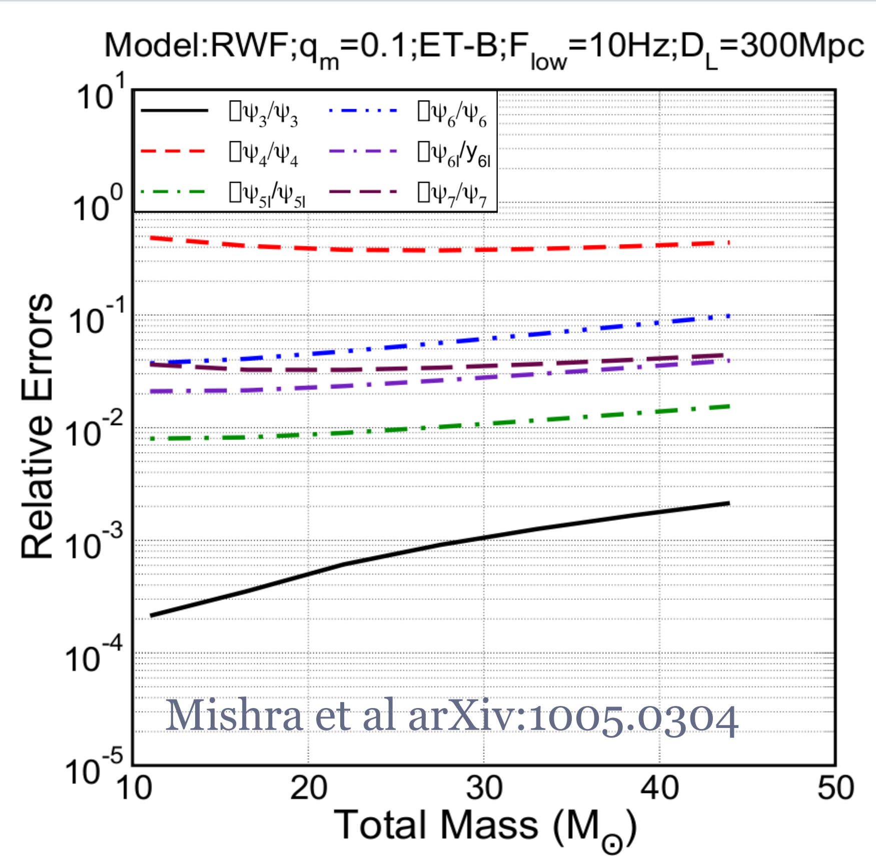
- Assume spinless systems
  - Phase coefficients only depend on component masses
  - Only two phase coefficients independent



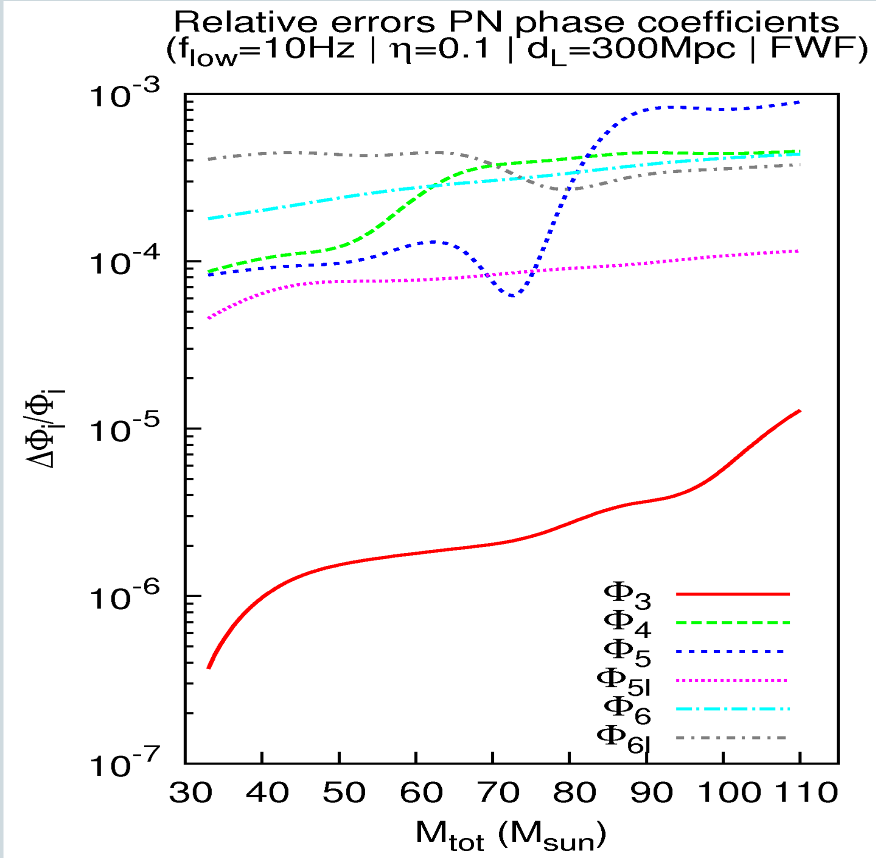
- Measure ϕ_1, ϕ_2, ϕ_T independently
- Check for **consistency** between PN phase coefficients

Relative Errors

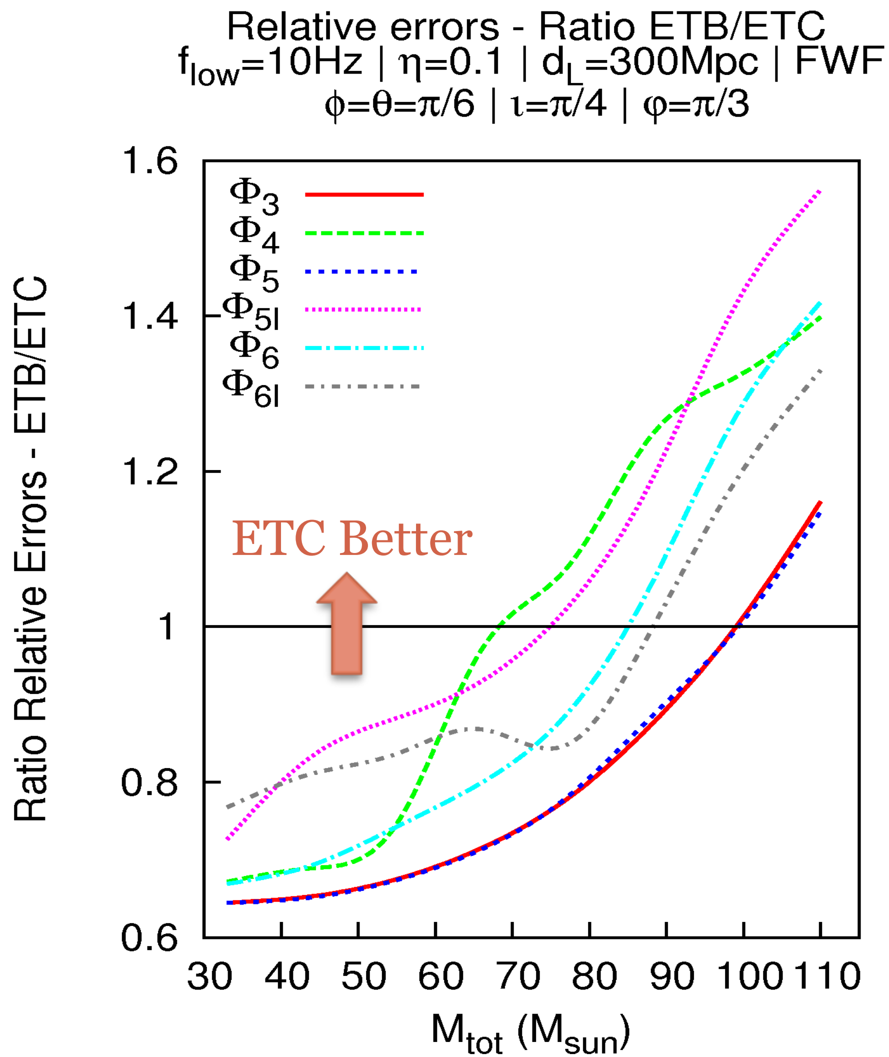
Frequency Domain



Time Domain

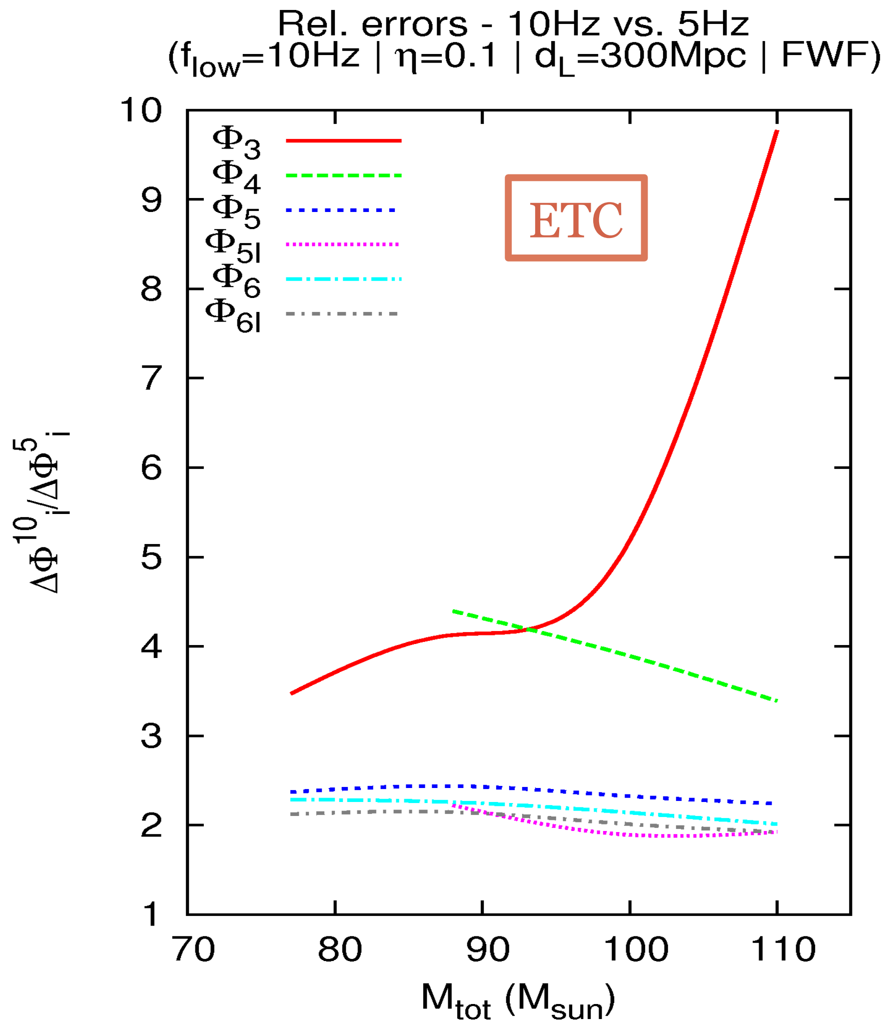


Trade Study – Compare ETB/ETC



- ETC outperforms ETB for higher masses
  - F_{lso} decreases with mass
- Difference as large as **40%**
- Transition around 70-90 total mass (solar mass)
- Best Performance depends on the **sources of interest**

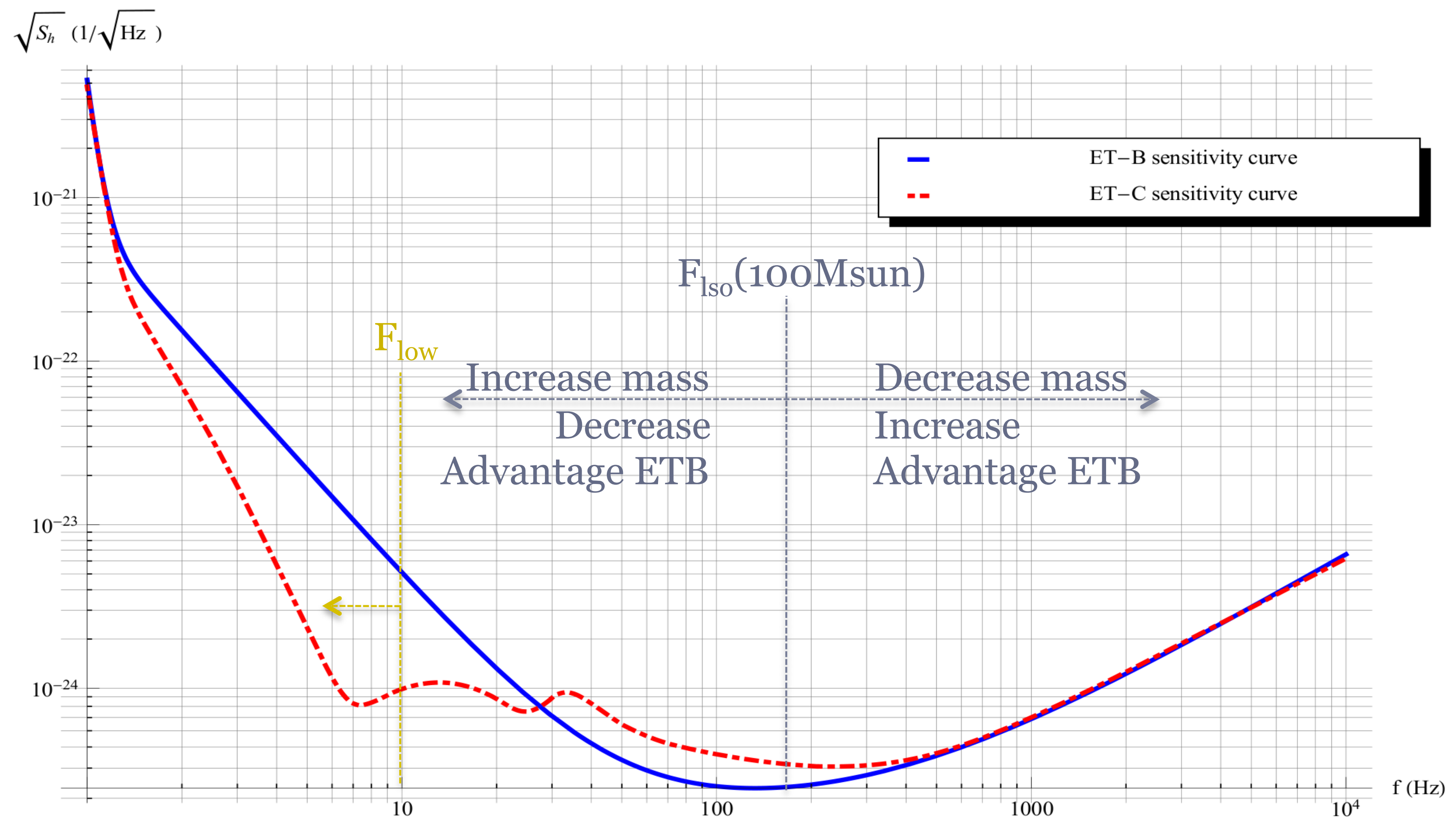
Beyond 10Hz Lower Cut-off Frequency



- Compare results for 10Hz and 5Hz
- Errors are factor ~2 smaller for 5Hz cut-off

Need to push lower cut-off frequency below 10Hz obtain better **bounds** on General Relativity

Upper Cut-off Frequency



Bounding Graviton Mass

CONSTRAINING THE MASS OF THE GRAVITON BY LOOKING PHASE SHIFTS DUE TO WAVELENGTH DEPENDENT ARRIVAL TIMES

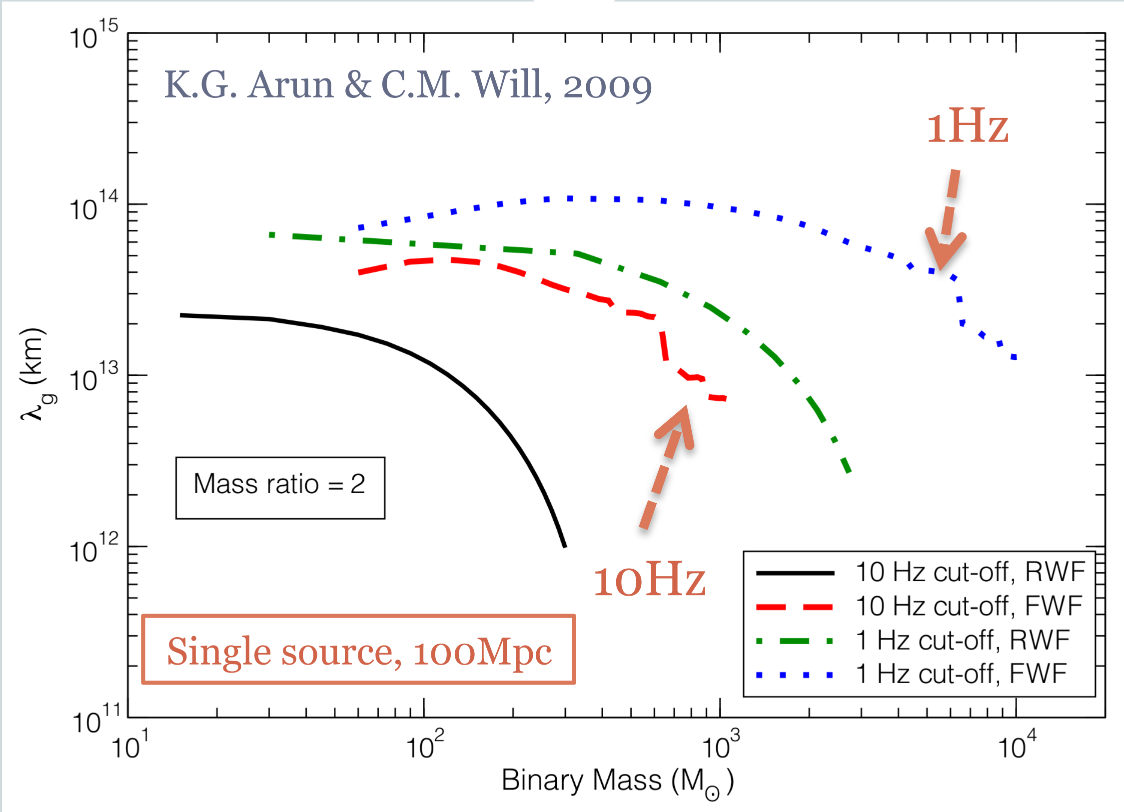
Bounding Graviton Mass - Concept

- If a mass is associated with the propagation of gravitational waves, its speed will depend on wavelength $v_g \approx 1 - \left(\lambda/\lambda_g\right)^2 \quad where\ \lambda << \lambda_g$
  - Where λ_g is the Compton wavelength of the graviton
  - A distant observer, at distance D, will see shifted time of arrival depending on wavelength $\Delta t \sim D\left(\lambda/\lambda_g\right)^2$
  - In the frequency domain, the phase of a waveform is altered with an additional shift of

$$\Delta\psi_k(f) = -\frac{\pi k^2 D}{4 f_e \lambda_g^2}$$

- Translate into a bound on λ_g

Bounds on Graviton Mass with ET



Up to an **order of magnitude** improvement on the bound of the graviton mass by having a lowering the cut-off to 1Hz

Summary

THE BENEFITS HAVING A LOWER CUT-OFF FREQUENCY BELOW 10HZ

Benefits of sub 10Hz cut-off

- Increased parameter estimation
  - Gain more knowledge per detected source
- Increased number of detectable sources
  - Acquire more confidence in detected sources and statements derived from their detections
- Increased mass range
  - Probe additional sources otherwise not detected
- Possible unknown sources
  - e.g. Intermediate Mass Black Holes

Backup Slides