

A first approach to flexure design for a cryogenic test mass suspension

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Abstract

This is a companion report to a paper about a new type of suspension for cryogenic test-masses comprising rigid vertical beams with large cross sections, combined with short flexures under compressional load. Based on mechanical safety factor estimates and buckling analyses, this report considers a list of flexure dimensions to analyze. Information on the flexure spring constant and on the maximum tensile stress experienced by the suspension beam is given. The temperature evolution at different parts of the payload when cooling it down from room temperature is also reported.

1 Introduction

This is a companion report to the paper entitled *A cryogenic test-mass suspension with flexures operating in compression for third-generation gravitational-wave detectors*, which provides a first conceptual analysis of a new type of suspension for the test masses of the Einstein Telescope [1]. Its main aim is to provide complementary information that was not included in the paper and should be read in conjunction with it.

The contents are limited to the information cited in the paper. Namely, Sec. 2 provides a list of flexure dimensions to analyze according to mechanical safety factors and buckling analyses. In the Sec. 3 the cases are separated into groups according to the shape of the displacement transfer function of a single compound pendulum. Sec. 3.1 gives complementary information about the behaviour of a single flexure. Sec. 6 reports on the temperature evolution of different parts of the payload when cooling down from room temperature. Finally, Sec. 7 illustrates the shortcoming of using software like Ansys and COMSOL for the calculation of the displacement thermal noise directly from the admittance transfer function.

Future updates will include a description of the analytic rigid-body model of the compound pendulum assembly used, the design criteria for the flexure used in the upper central suspension beam of the payload, and comparison of results calculated in both Ansys and COMSOL.

Diameter (mm)	0.83	1.16	2
Safety factor	3.1	6.1	18.1
$L_{\max}$	6	13	38
L_{fail}	7	14	39

Table 1: The pendulum buckles with flexures with lengths between $L_{\max}$ and L_{fail} .

2 Flexure design

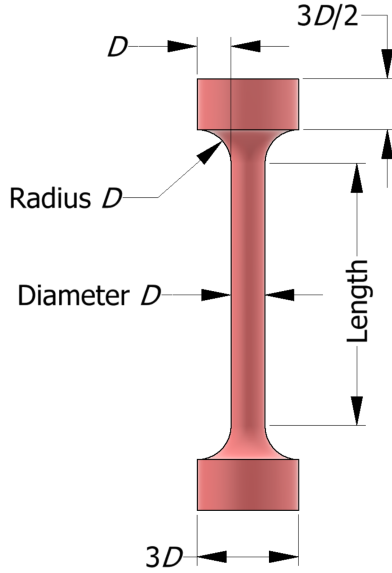


Figure 1: All dimensions other than the length are given in terms of the diameter D .

the frequency of any of the resonant modes becomes zero, condition that implies that the pendulum fails to oscillate. This happens when the flexure is unable to provide sufficient force to restore the load to its equilibrium position. This type of analysis provides an eigenvalue for each resonant mode that, when multiplied by g , produces the value of the acceleration due to gravity that makes the pendulum buckle. Therefore, the pendulum is stable when all eigenvalues are either larger than unity or negative. For each safety factor selected, Table 1 lists the longest lengths tested without buckling $L_{\max}$ and the shortest lengths tested with buckling L_{fail} . The exact value of the buckling length is between $L_{\max}$ and L_{fail} .

Table 2 shows the flexure lengths for all the cases selected. According to the buckling analysis, the longest flexures are less than 2 mm short of buckling under the weight of the load. For the

In order to sample a relatively wide span of flexure dimensions, three different safety factors and three different flexure lengths for each factor were considered. Figure 1 shows a diagram of the flexure, where the length refers to the cylindrical central section without the fillets and nailheads. The diameters were calculated from the safety factor with the formula

$$d = 2\sqrt{\frac{MgF_{\text{safe}}}{\pi S}}, \quad (1)$$

where $M = 56.6 \text{ kg}$ is the mass of the load the upper flexure supports, g is the acceleration due to gravity, $S = 3200 \text{ MPa}$ is the material strength in compression [2] and F_{safe} is the safety factor. The safety factors selected were close to 3, 6 and the value that corresponds to a diameter of 2 mm.

The maximum flexure lengths were selected according to an eigenvalue buckling analysis made in Ansys. Buckling of the pendulum happens when

Case name	Safety factor (nominal)	Element diameter (mm)	Element length (mm)
1-1	3.1	0.83	1.7
1-2	3.1	0.83	3.4
1-3	3.1	0.83	5
2-1	6.1	1.16	6
2-2	6.1	1.16	9
2-3	6.1	1.16	12
3-1	18.1	2.0	10
3-2	18.1	2.0	24
3-3	18.1	2.0	38
R-1	3.1	3.42	2000
R-2	6.1	4.8	2000

Table 2: These are the dimensions of the flexures that are analyzed. The term *element* refers to either a flexure or a suspension rod.

purposes of comparison, two suspension rods operating under tensile stress are included in the analysis. Please note that this comparison must be taken with caution for two reasons. The first one is that the rod ends have not been engineered yet. They need to be thicker at the ends where they attach to other bodies, but in this analysis they are considered uniform. The second reason is that the safety factors for the flexures describe a safer condition than those for the rods because of the differing failure mechanisms that make the compressive strength of silicon larger than its tensile strength.

3 Displacement transfer functions

Displacement transfer functions in the longitudinal direction were calculated in Ansys for the pendulum assembly shown in Fig. 2, using the anisotropic stiffness matrix of single crystal silicon [3] with a bulk loss angle $\phi_{\text{mat}} = 10^{-9}$. No other contributions to the loss angle have been considered yet. For all components, the crystal direction with Miller index [100] was set pointing upwards. For all cases, a load of 50 kg at the top of the lower flexure and a counterweight of 0.65 kg at the top of the suspension beam were used. In a pre-stressed harmonic analysis in frequency domain, an input acceleration of the bottom face of the upper flexure was set to the same value a_0 at all frequencies, and then the output acceleration a of the load on the top face of the lower flexure was calculated. The displacement transfer function T then followed from the acceleration transfer function:

$$T = \frac{a}{a_0} = \frac{-\omega^2 s}{-\omega^2 s_0} = \frac{s}{s_0}, \quad (2)$$

where ω is the angular frequency and s_0 and s are the input and output displacements respectively.

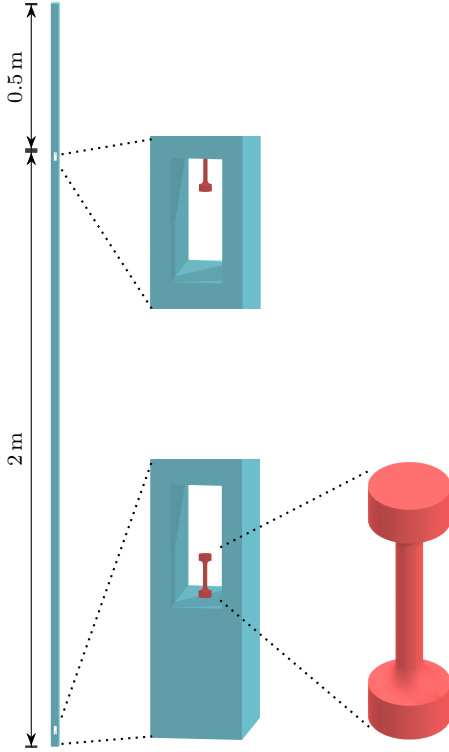


Figure 2: This is one of the suspension beams with detail views of the windows housing the flexures and one of the flexures. The length of the pendulum is 2 m.

According to the shape of their transfer functions, the cases can be divided into three groups. Fig. 3(a) shows a representative case from each group. A feature that they have in common is the pendulum mode at 356 mHz, which is almost as low as the frequency of an ideal pendulum 2 m long. The first group comprises cases **1-1**, **1-2**, **2-1** and **3-1**, whose transfer functions are shown in Fig. 3(b). In these cases, in the frequency domain, there are no rigid-body modes between the pendulum mode at 356 mHz and the first banana-shape mode at around 24.7 Hz. The second group comprises cases **1-3**, **2-2** and **3-2**, whose transfer functions are shown in Fig. 3(c). In these cases there is a mode between the pendulum and the banana mode. For example, in the case 2-2 ($\varnothing 1.16$, L 9 mm), it appears at 20.1 Hz with the banana mode at 22.4 Hz. The third group comprises cases **2-3** and **3-3**, whose transfer functions are shown in Fig. 3(d). In these cases there is a clear roll-off of the transfer function due to the presence of rigid-body modes below the the banana mode. For example, for the case 2-3 ($\varnothing 1.16$, L 12 mm), there is a rigid-body mode at 8.6 Hz. Fig. 3(e) shows a comparison between cases R1, R2 and 2-1. Above 2 Hz the magnitude for the rods in tension is lower.

The saturation plateau from 2 to 20 Hz for the case 2-1 can be modified by using the optimum value of $m_{cw} = 1.5975$ kg as shown in Fig. 3(f), and as pointed out in the paper. Under these circumstances, the magnitude of the transfer function for the rods is lower from 10 Hz onwards.

As pointed out above, buckling happens when the frequency of a resonant mode becomes zero. Therefore, having rigid-body modes, other than the pendulum mode, at lower frequencies, means that the pendulum is closer to buckling conditions. Cases that exhibit this behaviour should not be considered for a final design. These cases also have the disadvantage of introducing more resonance peaks at frequencies where astronomical observations will take place. **Therefore, all cases from the second and third groups were disregarded.**

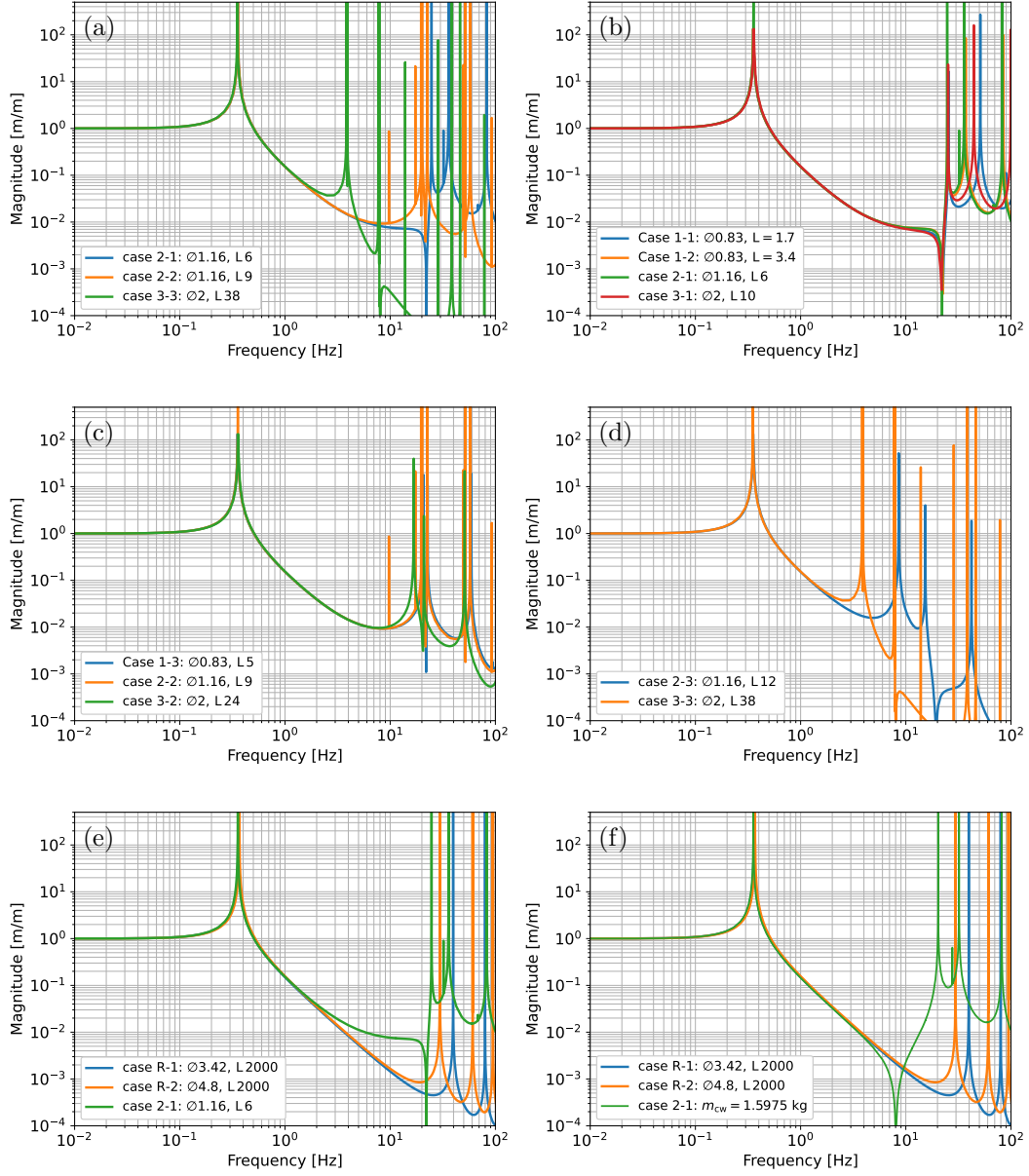


Figure 3: Panel (a) shows one transfer function from each of the three groups, and the panels (b), (c) and (d) show one of these groups each. Panels (e) and (f) compare the magnitudes for cases R-1, R-2 and 2-1 when $m_{cw} = 0.65$ kg and $m_{cw} = 1.5975$ kg respectively.

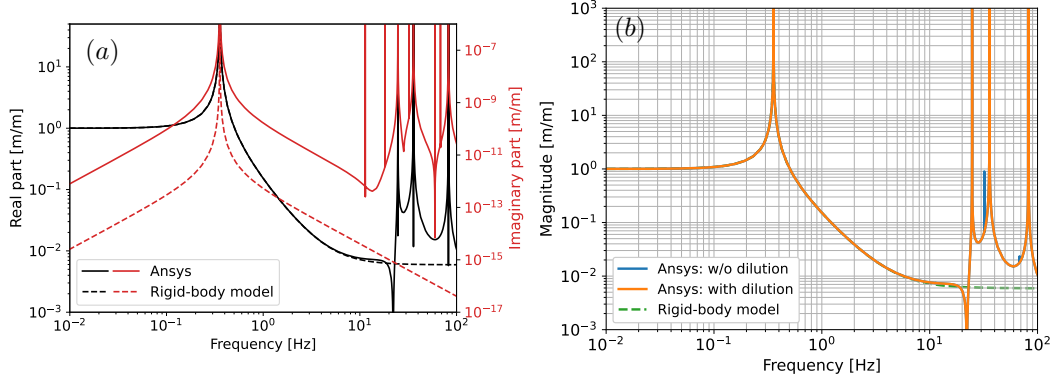


Figure 4: Panel (a) shows the real and imaginary parts of the transfer function for the case 2-1, calculated with Ansys and with the rigid-body model. The real and imaginary parts are shown in black and red lines respectively, and each one with its own scale. The peaks without foothills correspond to resonances in the transverse direction, but do not seem to represent real coupling with the longitudinal degree of freedom. When the imaginary part is corrected by hand with the dissipation dilution factor, these peaks disappear from the magnitude.

A good candidate is case 2-1, whose flexure has a central section with a diameter and length of 1.16 and 6 mm respectively. The flexure has an overall length of 11.8 mm. It has a good medium-size length, with a safety factor of 6, a small geometrical footprint and its mass is only 0.105 grams.

As described in the paper, Ansys and COMSOL are unable to account for the effect of dissipation dilution directly in the calculation of the displacement transfer functions. This is again shown in this report in Fig. 4(a), where the imaginary part is overestimated by a factor equal to the dissipation dilution factor. At most frequencies, the imaginary part is many orders of magnitude lower than the real part, introducing a negligible error in the transfer function magnitude. However, some very narrow peaks without foothills are the exception. These peaks can be seen in Fig. 3(a), (b) and (c) for cases 2-1 and 2-2. Fig. 4(b) shows the transfer function for the case 2-1 again, where these peaks appear above 30 Hz and below 70 Hz, and correspond to transverse modes. The figure also shows the corrected magnitude, calculated by dividing the imaginary part by the dissipation dilution factor $D = 313.1$. In this plot, the peaks corresponding to transverse modes have vanished. This suggests that this kind of peaks are the consequence of methodological and numerical errors rather than an indication of coupling between degrees of freedom. In real life, they will probably appear due to misalignments.

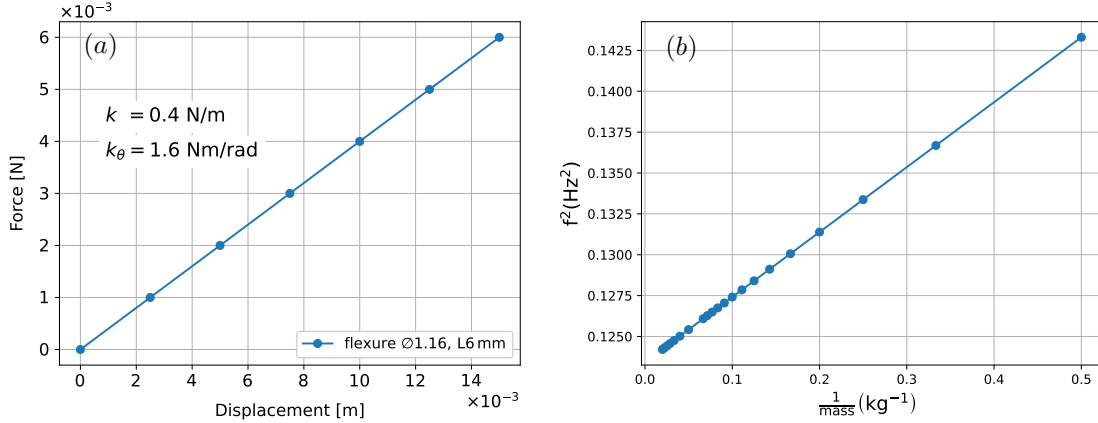


Figure 5: Panel (a) illustrates the calculation of the spring constant using Hooke's law and numerical data generated in Ansys. Panel (b) illustrates the linear relation between the square of the pendulum resonant frequency f^2 and the inverse of the mass $1/m$, condition that implies that k_θ does not depend on the compression exerted on the flexure.

3.1 Spring constant

In this section, the flexure spring constant for the case 2-1 is calculated. In order to achieve an appropriate accuracy, an idealized simplification of the single pendulum shown in Fig. 2 was used. The beam was considered infinitely rigid and massless, the counterweight and the lower flexure were removed, and a point-mass load was attached to the beam 2 m below the bottom face of the upper flexure. The point-mass load included the beam and the counterweight masses to produce the correct amount of compression on the upper flexure. This approach allows the application of the finite-element analysis to the upper flexure only, and avoid meshing and analyzing the behaviour of the material of the beam and lower flexure.

The flexure angular spring constant k_θ was calculated using Hooke's law, without considering the effect of gravity. In the Ansys simulation, transverse forces were applied on the load, the lateral displacements were calculated, and the value of the linear spring constant $k = 0.4 \text{ N/m}$ followed from a least-square linear fit. The data and the straight line fitted are shown in Fig. 5(a). The angular spring constant was then calculated from the expression $k_\theta = kl^2$, where $l = 2 \text{ m}$. The length l is far larger than the flexure length, which can therefore be neglected. The value of the angular spring constant is $k_\theta = 1.6 \text{ Nm/rad}$.

Using Ansys also, it is possible to illustrate that the angular spring constant does not depend on the compression exerted on the flexure. The equation of motion of the pendulum pivoted by the flexure is

$$\ddot{\theta} = - \left(\frac{k_\theta}{ml^2} + \frac{g}{l} \right) \theta \quad (3)$$

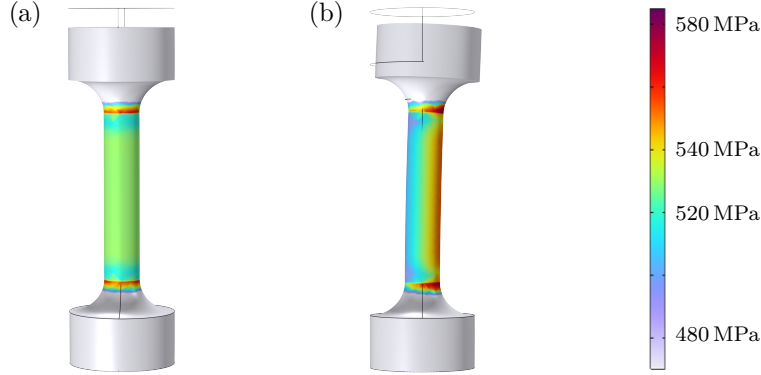


Figure 6: Panel (a) shows the stress map of the flexure with a static vertical load. There is a maximum at the fillet joining the cylindrical section with the nailhead, which Ansys evaluated to be 556.3 MPa. Panel (b) shows the map when the load has moved sideways to an end stop at ± 5 mm from the equilibrium position, condition in which the maximum stress becomes 586.0 MPa. The color scale is common to both cases. The wireframes correspond to the flexures without compression. The case 2-1 corresponds to a flexure with with $\varnothing 1.16$ mm and L 6 mm.

where θ is the angular displacement of the pendulum, m is the mass of the load and g the acceleration due to gravity. This equation implies that the frequency is

$$f = \frac{1}{2\pi} \sqrt{\frac{k_\theta}{ml^2} + \frac{g}{l}} \quad (4)$$

In the case in which k_θ does not depend on the mass m , and therefore neither on the compression exerted on the flexure, the plot f^2 versus $1/m$ is a straight line with a slope

$$\text{slope} = \frac{1}{4\pi^2} \frac{k_\theta}{l^2}. \quad (5)$$

With a pre-stress modal analysis, the resonant frequency of the pendulum mode was calculated for a set of different values of m . This data produced the plot shown in 5(b), which is clearly a straight line, indicating that k_θ does not depend on the compression exerted on the flexure.

4 Tensile stress in the suspension beam

The paper reported the stress expected in the flexures for a maximum deflection of the idealized pendulum described in Sec. 3.1. This result is reproduced in Fig. 6. For the purposes of comparison, it is convenient to provide also the maximum tensile stress that the suspension beam experiences in similar conditions.

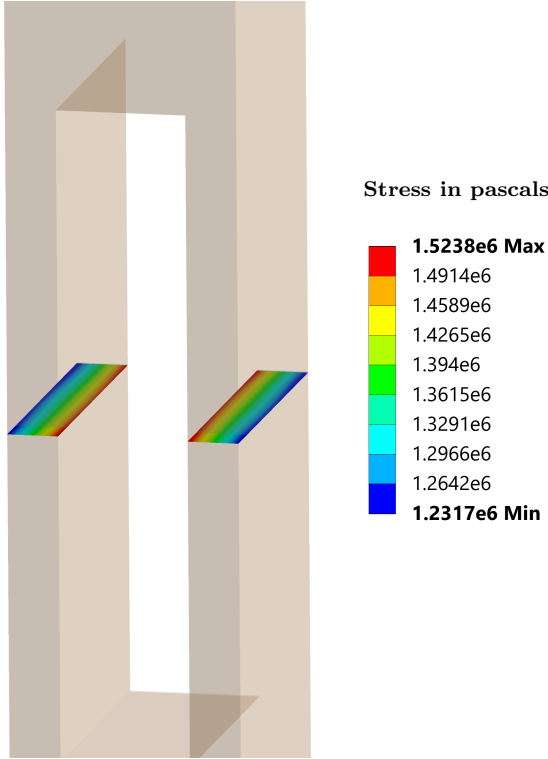


Figure 7: The maximum tensile stress in the beam is 1.5 MPa when the load of the pendulum moves away from the equilibrium position by 5 mm.

analysis of the whole suspension will be done once all the relevant mechanical properties of the upper beam and flexure are adjusted to meet suitable criteria. For example, the flexure must be soft to minimize the thermal noise, but still be rigid enough to be safe from buckling. Also, the thickness of the beam must be such that its internal resonances are as high as possible in frequency. However, in the meantime, it is necessary to design a flexure and a beam with reasonable parameters to be able to calculate the heat extraction capabilities of the suspension.

The first criteria for flexure design is that it must have a mechanical safety factor of 6. According to a mass budget calculated from the 3D-CAD model with Autodesk Inventor, the upper flexure in the upper mass supports approximately 485 kg. This implies that a diameter $d_{\text{up}} = 3.4$ mm is suitable, as it yields a safety factor of 6.1 for a material strength of 3200 MPa.

The two other criteria are used to determine the flexure length l_{up} . One is that the ratio of its cross-section area over its length equals or is larger than that of the lower flexure to ensure that its

Figure 7 shows the cross section stress map at the the walls of the upper window, which is the thinnest section of the beam, and where the maximum tensile stress is expected to occur. The load of the pendulum is at the position of maximum deflection, 5 mm away from the equilibrium position. The maximum stress is 1.5 MPa, making the beam mechanically very safe given that the tensile strength of silicon is 165 MPa or larger. When the pendulum is at the equilibrium position, the value of this stress does not differ appreciably from this value.

5 Upper flexure and upper suspension beam

In this report, as well as in the paper, for the sake of simplicity, the analysis of the mechanical performance of the suspension is limited to its lower section, which plays a more critical role than the upper one because it directly holds the mirror. The

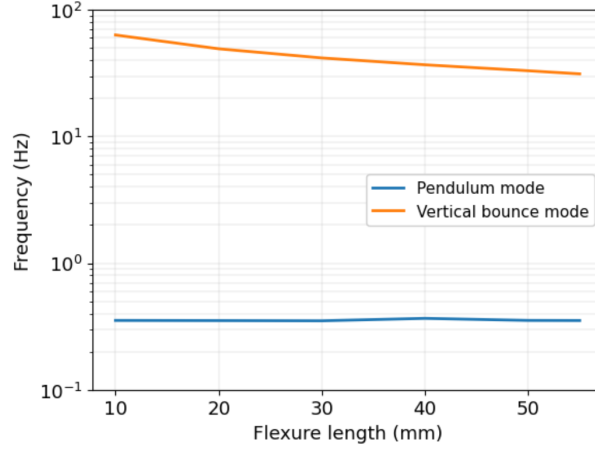


Figure 8: The pendulum and bounce mode frequencies of the 2 m pendulum were calculated in Ansys as a function of flexure length. The pendulum mode frequency remains close to 352 Hz for all lengths.

conductance is the same or higher. For the lower flexure case 2-1, the diameter is $d_{\text{low}} = 1.16$ mm and the length is $l_{\text{low}} = 6$ mm. This implies that the length of the upper flexure should be such that

$$l_{\text{up}} \leq l_{\text{low}} \left(\frac{d_{\text{up}}^2}{d_{\text{low}}^2} \right) = 51.5 \text{ mm}. \quad (6)$$

To guarantee that the flexure is soft, the other criteria is that the frequency of a 2 m pendulum pivoted by the flexure is low enough. Fig. 8 shows the pendulum and bounce modes frequencies of such a system for different flexure lengths. The suspension beam for this pendulum is massless and the load is close to 485 kg. In all cases, the pendulum resonant frequency is close to 352 mHz. Given these two restrictions, the length $l_{\text{up}} = 20$ mm was selected with the aim of keeping the system away from buckling conditions, which is a problem that will be analyzed in more detail in the future.

The dimensions of the cross-section of the upper beam were temporarily set to $80 \times 25 \text{ mm}^2$. Because this beam does not limit the conductance, the only condition it has to fulfil for now is to provide a reasonable mechanical safety factor. The walls next to the windows are the thinnest sections, with cross-section areas of $80 \times 5.5 \text{ mm}^2 = 4.4 \times 10^{-4} \text{ m}^2$. Therefore, for half the value of the load, the stress in those regions is 5.406 MPa, which gives a safety factor of 29.6 for a tensile strength of silicon of 160 MPa. As noted above, in the future the cross-section dimensions will be adjusted to have the beam internal resonances as high as possible in frequency.

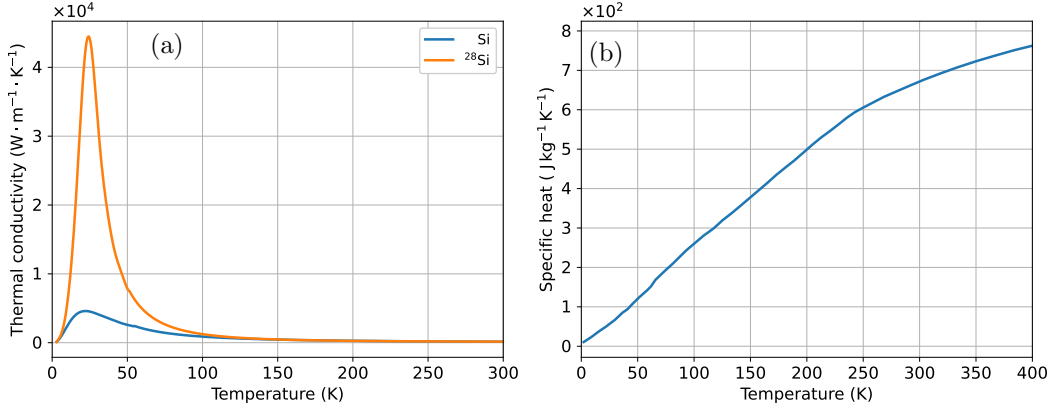


Figure 9: The temperature-dependent thermal conductivity for natural Si and ^{28}Si [4], and the specific heat capacity [5] used for both types of silicon.

6 Temperature evolution when cooling down

The temperature evolution of the cryogenic payload was also calculated in Ansys and COMSOL. These software packages solve this equation

$$k\nabla^2 T + q = \rho c \frac{\partial T}{\partial t}, \quad (7)$$

where k is the thermal conductivity, T the temperature, q the input heat power, ρ the density, c the specific heat capacity and t the time. The temperature-dependent thermal conductivity [4] and specific heat capacity [5] used are shown in Fig. 9. More recent measurements of the specific heat capacity for natural Si and ^{28}Si at cryogenic temperatures exist, but do not go above 140 K [6] and therefore cannot be used in our application, which requires values up to room temperature.

As reported in the paper, when the payload cools down from room temperature, the mirror reaches a temperature of 5 K in about 14 days when the flexures are made of ^{28}Si . Here, the temperature evolution at different places of the payload is reported, namely, at the top and bottom of the flexures. The flexures shown in Fig. 2 are simply referred to as *lower flexure* and *upper flexure*, given their location in the pendulum. The flexures shown in Fig. 10 (a) and (b) are referred to as *lower central flexure* and *upper central flexure* respectively. The first one is located inside the intermediate mass and the second one at the centre of the upper mass.

Figure 10(c) shows the temperature evolution from room temperature until it cools down completely, and Fig. 10(b) shows only the first quarter of a day to highlight the fast drop in temperature of the components above the intermediate mass. The plots also show the temperature evolution when the flexures are made of natural silicon. The fast drop in curves ⑥, ⑦ and ⑧ means that

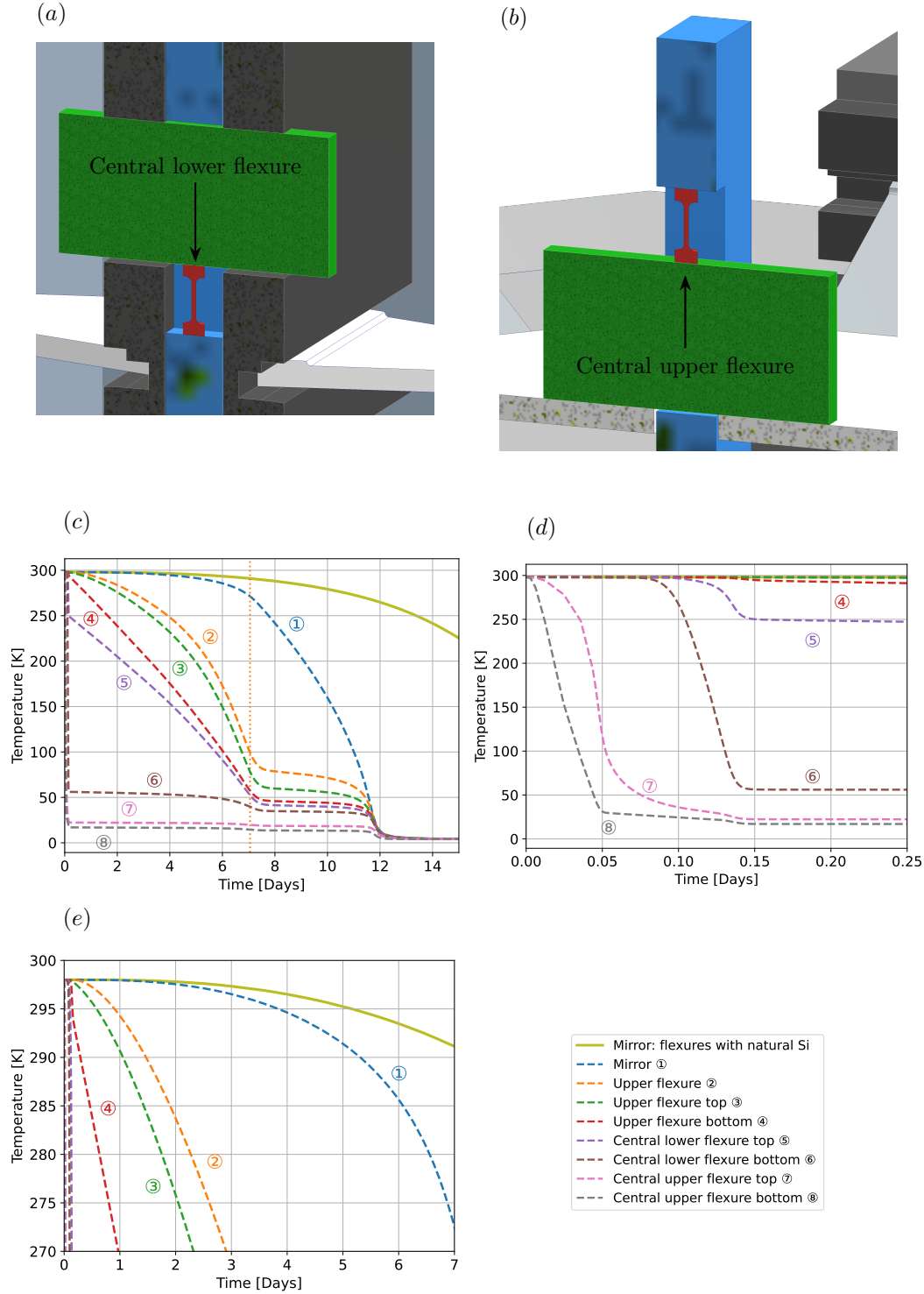


Figure 10: Panels (a) and (b) show the locations of the lower central flexure and upper central flexure respectively. Panels (c), (d) and (e) show the temperature evolution at different points of the payload during cooldown, using different time scales. Dashed lines are used for the case in which the flexures are made of ^{28}Si , and a continuous line when they are made of natural silicon. The legend applies to all three panels.

Component	Lower flexure	Upper flexure	Lower suspension beam	Central lower flexure	Central upper flexure	Central beam
Temperature drop (K)	0.35	0.43	0.2	0.63	1.36	0.73

Table 3: For the case 2-1, the table lists the temperature drop across the suspension beams and flexures, when the latter are made of ^{28}Si .

the upper mass and the central suspension beam cool down relatively fast. They reach below 60 K in 3.6 hours. This happens for different reasons. The first one is because the central upper flexure has a cross-sectional area 8.6 times larger than that of one of the flexures used in the lower beams. It has a diameter of 3.4 mm and a length of 20 mm. Another reason is because it is made of ^{28}Si , and as soon as it reaches 100 K, the material experiences an increase in conductivity, as indicated in Fig. 9(a). Another contribution is the large cross-sectional area of the central beam, feature that implies a large conductance. A modelling element that contributes to this result is the value of the contact thermal resistance between the heat link and the upper mass used in the simulation. This value was considered zero, but a realistic value still has to be estimated.

A consequence of the upper central flexure quickly reaching a temperature where its conductance increases significantly, is that the mirror cools faster even during the first few days, as shown in Fig. 10(e). Once all the flexures have reached 100 K, after approximately 7 days, time that is indicated with a dotted vertical line in Fig. 10(c), the mirror cools even faster than when the flexures are made of natural silicon.

As reported in the paper, once the payload is cold at 4 K and the Fabry-Perot cavity is locked, the mirror warms up to a steady temperature of 8.2 K in 33 hours when the flexures are made of ^{28}Si [1]. Table 3 lists the temperature drops across suspension beams and flexures. The differences across the flexures are smaller when they are closer to the mirror, where the heat source is located, because at these temperatures the thermal conductivity increases with temperature.

The temperature evolution of the payload was not considered for the suspension rods in tension because we do not know the complete payload configuration for those cases.

7 Suspension thermal noise

This section deals with the shortcoming of using Ansys to calculate the admittance transfer function directly from a pre-stress harmonic analysis. As will be seen from the plots, Ansys overestimates the real part of the admittance by a factor equal to the dissipation dilution factor.

The paper reports the calculation of the suspension thermal noise using the rigid-body model instead. In the pendulum, dissipation dilution happens when the sideways stiffness in the longitu-

dinal and transverse directions is produced by the tension in the vertical direction produced by the force of gravity [7]. This relation is introduced in the equations of motion by expressing the vertical displacements of bodies, directly affected by gravity, in terms of their sideways displacements. Namely, the vertical displacements are written to second order in the angular displacement of the beam, parameter that determines the sideways displacements together with the lever arm length of each body. Geometrically, this constraint indicates that the motion of objects happens along arcs. See Fig. 1 in the paper for the definition of the longitudinal, transverse and vertical directions.

Software packages like Ansys and COMSOL linearize the equations of motion to cope with a very large number of degrees of freedom [7]. This strategy has the side effect of neglecting the non-linear terms that indicate that in a pendulum the motion of objects happens along arcs rather than in straight lines.

For the case 2-1, Fig. 11(a) shows the real part of the admittance transfer function V/F calculated in Ansys for the pendulum shown in Fig. 2. A least-square fit of this mathematical model

$$\text{Re}\left(\frac{V}{F}\right) = \frac{\omega\omega_0^2\phi_{\text{pen}}}{M\left[\omega_0^4\phi_{\text{pen}}^2 + (\omega_0^2 - \omega^2)^2\right]}, \quad (8)$$

to this numerical data yields the value of the pendulum loss angle $\phi'_{\text{pen}} = 10^{-9}$, which is supposed to be diluted, but instead equals the material loss angle $\phi_{\text{mat}} = 10^{-9}$. In the model, $M = k/\omega_0^2$, $\omega_0 = 2\pi f_0$, $f_0 = 356.30$ mHz and ω is the angular frequency [8, eq. 1.33]. Fig. 11(b) shows the admittance of the same pendulum calculated with the rigid-body model, which upon a similar fit yields the correct diluted loss angle $\phi_{\text{pen}} = 3.18 \times 10^{-12}$. Both admittances are shown in Fig. 11(c) together with a corrected Ansys estimate, calculated by dividing the result by the dilution factor $D = 313.1$, operation that makes the admittance coincide with the rigid-body model estimate.

The conversion of the suspension thermal noise from a displacement spectral density ($\text{m}/\sqrt{\text{Hz}}$) to a strain spectral density ($1/\sqrt{\text{Hz}}$) in a gravitational-wave detector sensitivity, is obtained by dividing by the arm length of 10 km and adding the contributions of four independent mirror suspensions in quadrature, operations that yield a conversion factor of $2 \times 10^{-4} \text{ m}^{-1}$.

8 Conclusions

One of the most relevant pieces of information reported here is the criteria for selecting the optimum dimensions of a flexure. A buckling analysis was used to determine an upper limit to the length of flexures for different mechanical safety factors. Then, the presence of rigid-body modes at lower frequencies, other than the pendulum mode, was used to rule out some of the cases on the grounds of avoiding more resonances in the frequency range of astronomical observations, and for keeping the suspension well away from buckling conditions.

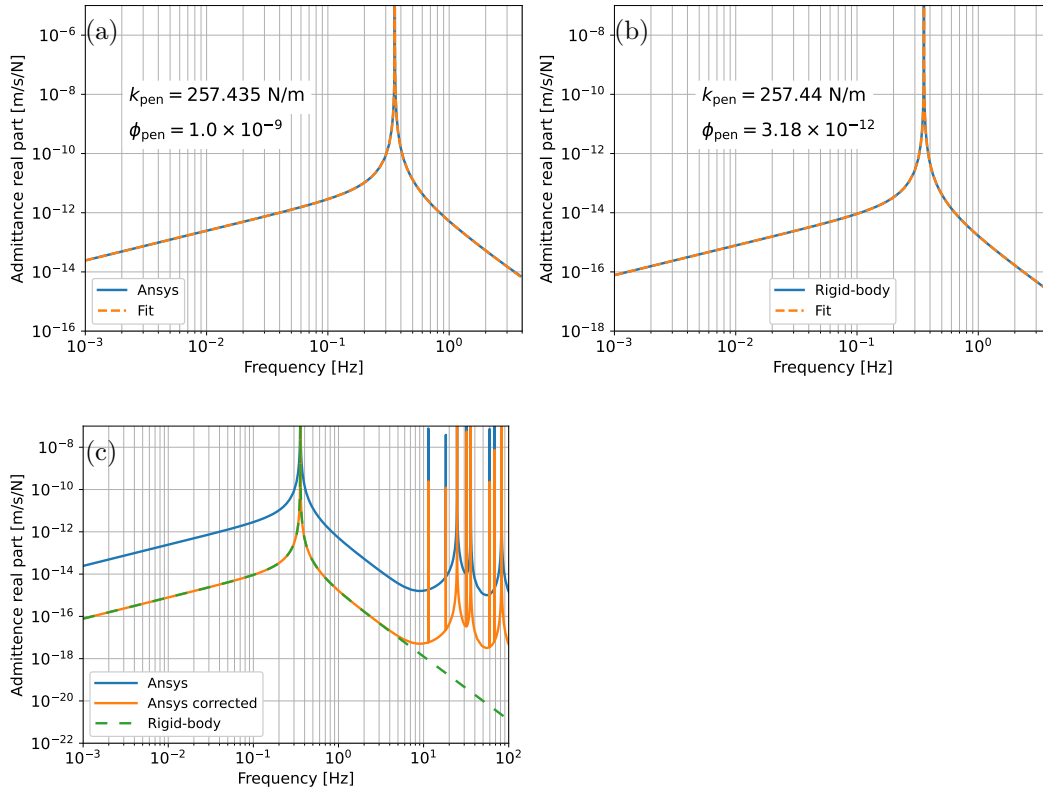


Figure 11: Panel (a) shows the real part of the admittance calculated with a pre-stress harmonic analysis in Ansys. This calculation overestimates the loss angle of the pendulum, which should be smaller due to dissipation dilution. Panel (b) shows the same quantity calculated with the rigid-body model, which accounts correctly for dissipation dilution. Panel (c) shows both estimates again, together with a corrected Ansys estimate.

A good candidate is case 2-1, with a mechanical safety factor of 6.1. This flexure has an overall length of 11.8 mm, and a central section of 1.16 mm in diameter and 6 mm in length. It has a small geometrical footprint and a mass of only 0.105 grams, which would allow the use ^{28}Si at an affordable price.

The maximum tensile stress experienced by the beam was calculated to be 1.5 MPa, occurring at the upper window walls, as shown in the stress map in Fig. 7. This makes the beam mechanically very safe as the tensile strength of silicon is 165 MPa or higher.

Figure 10 in Sec. 6 shows the temperature evolution at different places of the payload while it cools down from room temperature to 4 K. When the flexures are made of ^{28}Si , the mirror cools down faster than when they are made of natural silicon. The mirror cools much faster after all the flexures have reached 100 K, when the thermal conductivity of ^{28}Si becomes higher than that of natural silicon. The mirror reaches 5 K in about 14 days.

In the case of the warming up of the mirror from the initial temperature of 4 K when the interferometer is locked, the mirror reaches a steady temperature of 8.2 K in 33 hours [1]. Table 3 shows the temperature differences across different components. The flexures closer to the mirror, where the heat source is located, experience smaller temperature differences because they are hotter, and a higher temperature produces a larger thermal conductivity of the material.

Finally, this reports also shows the admittance transfer function calculated in Ansys, in which the real part is overestimated by a factor equal to the dissipation dilution factor. Because of this, in the paper, the analytical rigid-body model was used instead to calculate the suspension thermal noise.

References

- [1] Fabián Peña Arellano et al. *A cryogenic test-mass suspension with flexures operating in compression for third-generation gravitational-wave detectors*. Paper for a peer-reviewed scientific journal, ET-0037A-25. Available from <https://apps.et-gw.eu/tds/q1/?c=17764>. 2025.
- [2] *A Background to Silicon and its Applications*. <https://www.azom.com/properties.aspx?ArticleID=599>. Accesed on 04-09-2024.
- [3] Matthew A. Hopcroft, William D. Nix, and Thomas W. Kenny. “What is the Young’s Modulus of Silicon?” In: *Journal of Microelectromechanical Systems* 19.2 (2010), pp. 229–238. DOI: 10.1109/JMEMS.2009.2039697.
- [4] Alexander V. Inyushkin et al. “Ultrahigh thermal conductivity of isotopically enriched silicon”. In: *Journal of Applied Physics* 123.9 (Mar. 2018), p. 095112. ISSN: 0021-8979. DOI: 10.1063/1.5017778. eprint: https://pubs.aip.org/aip/jap/article-pdf/doi/10.1063/1.5017778/19756584/095112_1_online.pdf. URL: <https://doi.org/10.1063/1.5017778>.

- [5] A.S. Okhotin et al. *Thermophysical Properties of Semiconductors*. Cited by [9]. Moscow: Atom Publishing House, 1972.
- [6] Quanli Hu et al. “Low-temperature specific heat of isotopically enriched silicon single crystals”. In: *Physics Letters A* 299.5 (2002), pp. 656–659. ISSN: 0375-9601. DOI: [https://doi.org/10.1016/S0375-9601\(02\)00679-5](https://doi.org/10.1016/S0375-9601(02)00679-5). URL: <https://www.sciencedirect.com/science/article/pii/S0375960102006795>.
- [7] Mark Barton. *Dissipation Dilution*. Technical Report LIGO-T070101-00. Available from <https://dcc.ligo.org/LIGO-T070101/public>. 2008.
- [8] J.V. van Heijningen. “Turn up the bass!: Low-frequency performance improvement of seismic attenuation systems and vibration sensors for next generation gravitational wave detectors”. English. PhD-Thesis - Research and graduation internal. Vrije Universiteit Amsterdam, 2018. URL: <https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/DocDB/ShowDocument?docid=7985>.
- [9] *Silicon thermal properties*. <https://www.ioffe.ru/SVA/NSM/Semicond/Si/thermal.html>. Accesed on 27-09-2024.