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Study of silicon blade springs for a novel cryogenic test-mass suspension system for the Einstein Telescope Gravitational Wave Observatory

Technical Report

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Abstract

This report complements a broader analysis of a novel cryogenic suspension system incorporating blade springs. These blades introduce vertical compliance in a test-mass suspension system with flexures and suspension beams under compression and tension respectively, compensating for manufacturing variations, as well as incorporating a means of providing vertical thermal noise attenuation.

Designed with an isosceles triangular shape for uniform bending, the blades attach at an angle, with a pre-bent tip so that the area on which the load is supported lays horizontal. Through FEM simulations using Ansys, we identified a configuration in which we can base our future studies, with a 10.5 mm thickness, a 13.63 m curvature radius, and a base tilt of 12.453 mrad, ensuring a maximum surface tensional stress of 52.69 MPa—maintaining a safety factor close to 3.

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1 Introduction

This report complements a paper on a novel suspension system for cryogenic test masses, featuring rigid vertical suspension beams with large cross-sections and short flexures under compressional load, for the Einstein Telescope [1]. In this technical report, we describe our initial the design considerations of blade springs, a key component of this system.

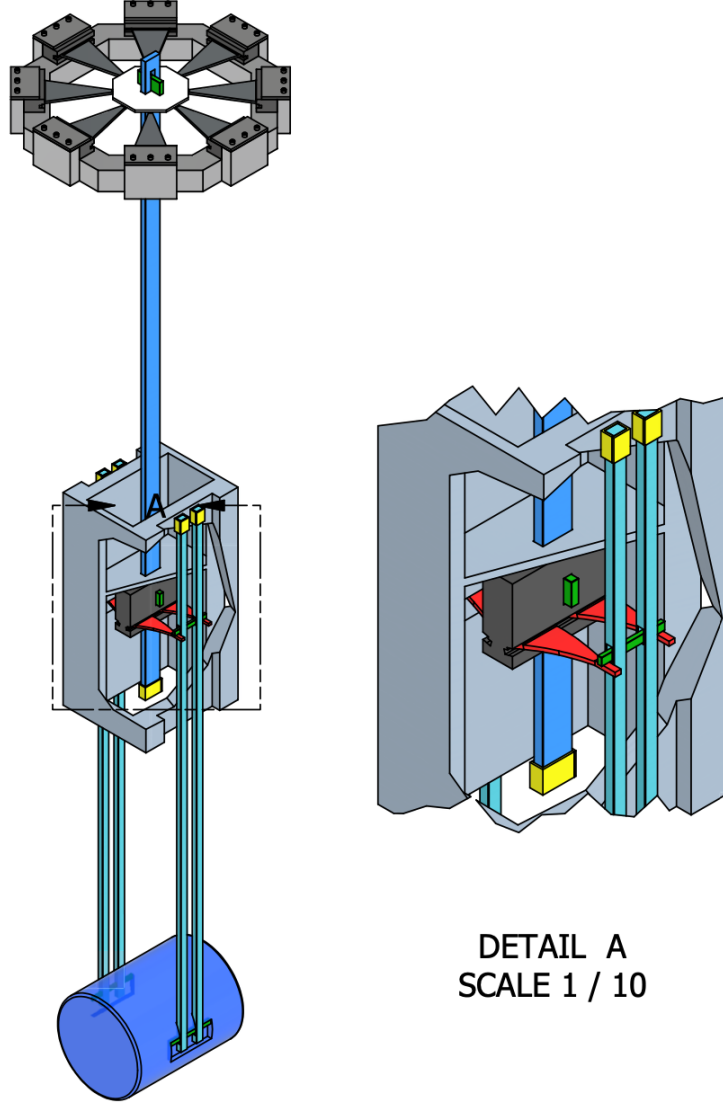


Figure 1: Full assembly of the test mass suspension system, in which the blade's bending is exaggerated.

Our present design, in its lower stage, incorporates four beams made of silicon, a material of very low elastic compliance, supporting a 200 kg test mass. Even with the tightest machining tolerances, our design goals require a mechanism for vertical compliance.

Additionally, ensuring an even load distribution across the four beams is crucial

to minimize stress imbalances.

To address these issues, cantilever springs, extending from the center of the intermediate mass, were incorporated into the design. These springs have the added benefit of incorporating a means to introduce vertical noise attenuation, which will be explored in future studies

2 Blade spring design

The blade springs are designed with an isosceles triangular shape, optimal for uniform bending and stress distribution [2, p. 90]. The wider end attaches to a holder at a slight upward angle so that the narrow tip of the blade is horizontal when the blade bends in response to the load. The tip is equipped with a straight extension to provide a mounting point for the cross-beam connecting to the suspension beam, as well as a means of incorporating components for any active control mechanism. The components and configuration are illustrated in Figure 1, in which the blade springs are colored red and the cross-beam, green. To minimize bending at the base and the tip, these sections are thicker on the upper side and a fillet is foreseen to generate a smooth transition of the stress. The design is easy to fabricate through standard silicon grinding techniques.

The ANSYS Finite Element Method simulation software was used to simulate the stress distribution and find the thinnest, and therefore softest, configuration that maintains a safety factor of 3. A maximum tensile strength value of 165 MPa [3] was chosen in this first iteration of the design, which is just one of several found in literature. Keeping this tensile strength value in mind, our aim is to keep the maximum stress under 55 MPa.

The actual component tensile strength can vary enormously as a function of polishing, crystal orientation and surface conditioning. Extensive testing will be necessary to achieve the maximum and consistent tensile stress possible.

2.1 Finite-element method

The finite element method divides structure into smaller, simpler parts called finite elements. A system of equations is generated based on the physical laws governing each element. The collective behavior of these elements provides an approximation of the entire structure's response to loads, boundary conditions, and material properties. In this study, we utilize the finite element method to analyze the behavior of the blade spring. We utilize the software Ansys to perform this finite element analysis.

2.2 Material choices

As part of the proposed suspension system, silicon is used for the blade composition. Specifically, an anisotropic silicon material assignment is applied in Ansys to the geometry. As an initial approach, we arbitrarily selected the crystal orientation [100], in the Miller index system, vertically oriented, that is, perpendicular to the

plane formed by the length and width of the blade. A density of 2330 kg/m^3 [3] is considered as well as the following stiffness matrix [4]:

$$\begin{pmatrix} 1.657 \times 10^{11} & 6.39 \times 10^{10} & 6.39 \times 10^{10} & 0 & 0 & 0 \\ 6.39 \times 10^{10} & 1.657 \times 10^{11} & 6.39 \times 10^{10} & 0 & 0 & 0 \\ 6.39 \times 10^{10} & 6.39 \times 10^{10} & 1.657 \times 10^{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.9 \times 10^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.9 \times 10^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 7.9 \times 10^{10} \end{pmatrix}$$

We also considered a bulk loss angle of $\phi = 1 \times 10^{-9}$.

2.3 Boundary conditions, static structural and modal analyses.

The static structural analysis incorporates the following boundary conditions.

- A load of 50 kg, applied to the tip at approximately the cross point, where the two lines of the triangle would converge, this is highlighted in red on Figure 2.
- The base of the blade, highlighted in yellow on Figure 2 is constrained from linear movements in the X, Y and Z directions, as well as rotations in X and Y.
- Standard earth gravity of 9.81 m/s^2 is considered for the whole analysis.

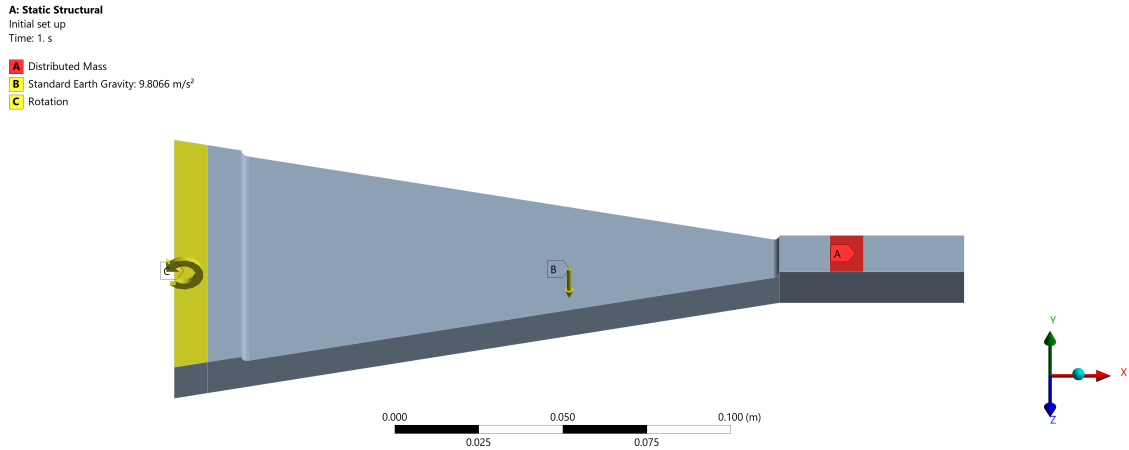


Figure 2: Static structural conditions.

We also consider that the tip of the blade must lay flat after being loaded with the 50 kg weight to support the cross-beam without forcing the flexure to be tilted in the equilibrium position, remaining horizontal. We calculate the bending angle due to the load, using the two displacement probes shown in Figure 3, and the angle is calculated via trigonometric relations. This same angle is then applied to the base of the blade in the opposite direction, achieving our desired result.

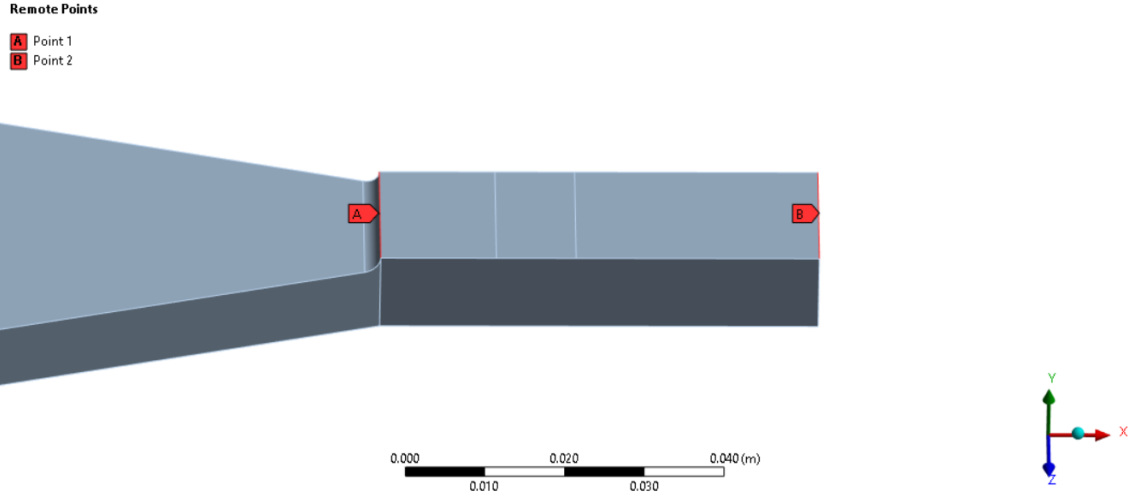


Figure 3: Displacement probes.

2.4 Loading area.

Yet another element to consider is the position of the loading area, this is the contact region between the tip of the blade and the cross-beam, highlighted in red in Figure 4. By moving this load application area closer to the base of the blade, the stress within the blade material becomes more evenly distributed. This reduces the maximum stress in the material, helping to meet the desired safety factor.

We start considering different points of the loading area, establishing the origin as the start of the rectangular region of the tip of the blade spring. This is illustrated in Figure 4.

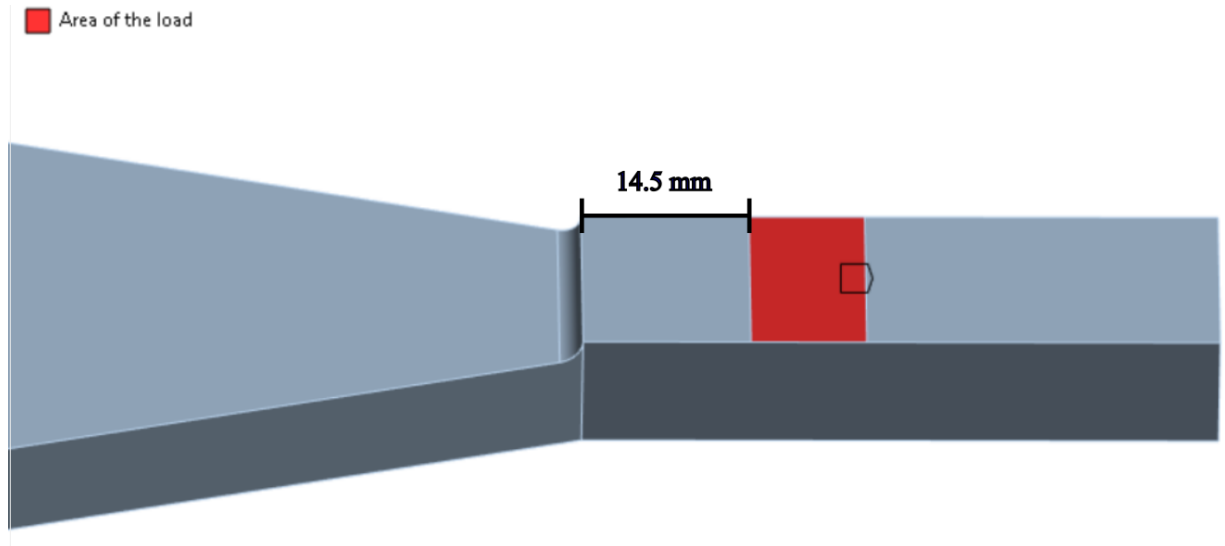


Figure 4: Loading area, taking the start of the rectangular region as the origin.

It is worth noting that the rotation angle applied to the base of the blade had to be recalculated with each change in loading area, as the stress distribution shifts.

This adjustment ensured that the loading tip remained horizontal, in line with our design considerations.

With these considerations, we calculated the maximum stress experienced by the top face of the blade spring, excluding the not yet optimized fillet region. The following results were obtained:

Start of the loading area (mm)	Max Stress (MPa)
22.5	55.40
18.5	53.88
14.5	52.55
10.5	51.80
6.5	50.57
2.5	49.37

Table 1: Stress values for different positions of the loading area, as illustrated in Figure 4. For a spring blade of 10.5 mm, as an example, and excluding the fillet region.

Subsequently, a plot of these results is presented:

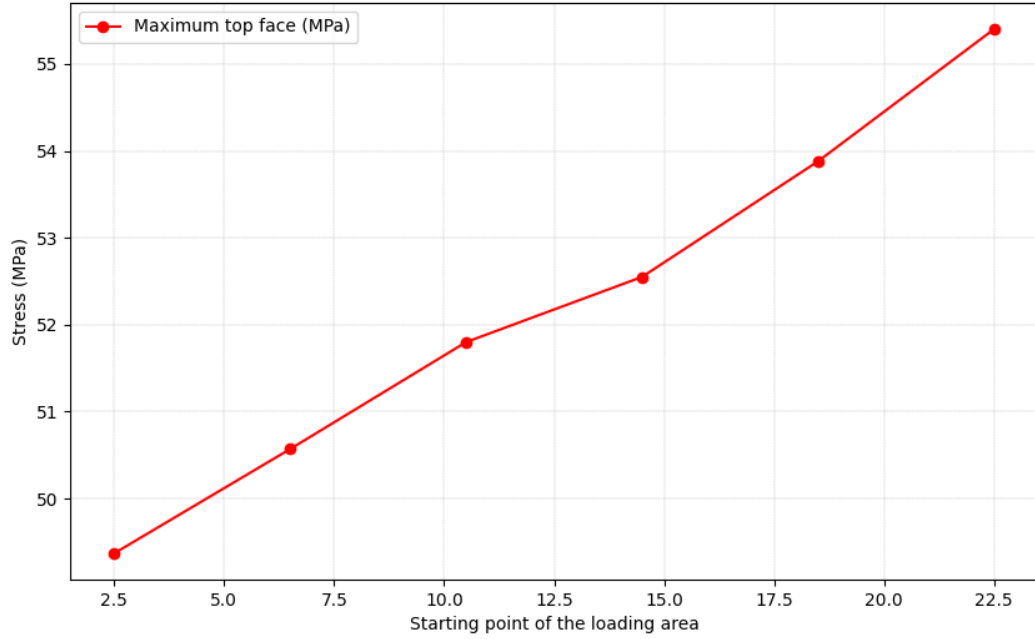


Figure 5: Maximum stress as we decrease the starting point of the loading area (excluding fillet region).

We also include some images of the stress distribution for some of these varying positions of the loading area, the following two are for the case of 10.5 mm thickness, as an example.

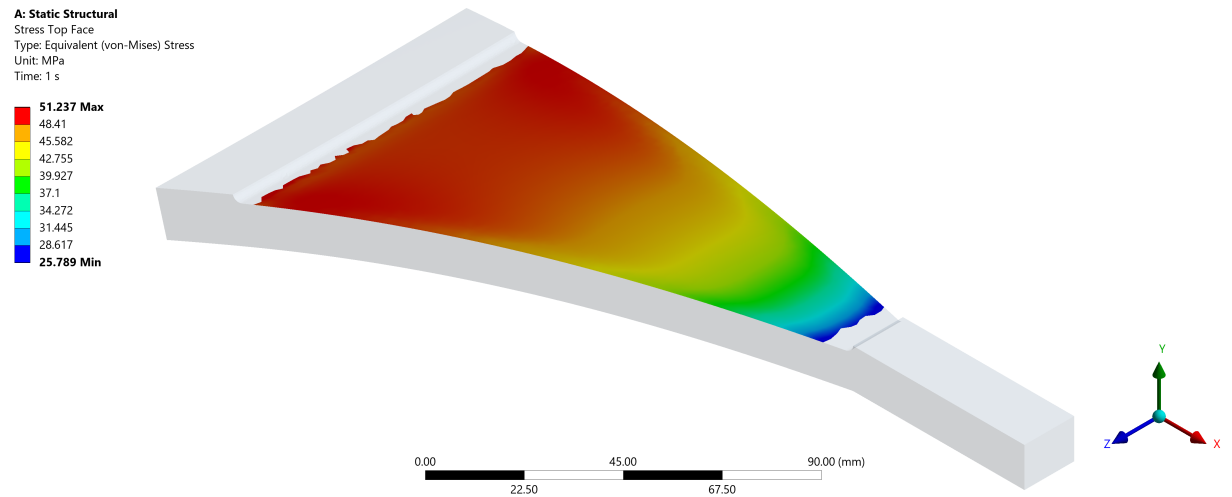


Figure 6: Stress distribution of the 10.5 mm case, with the loading region placed at 2.5 mm from the start of the rectangular loading region.

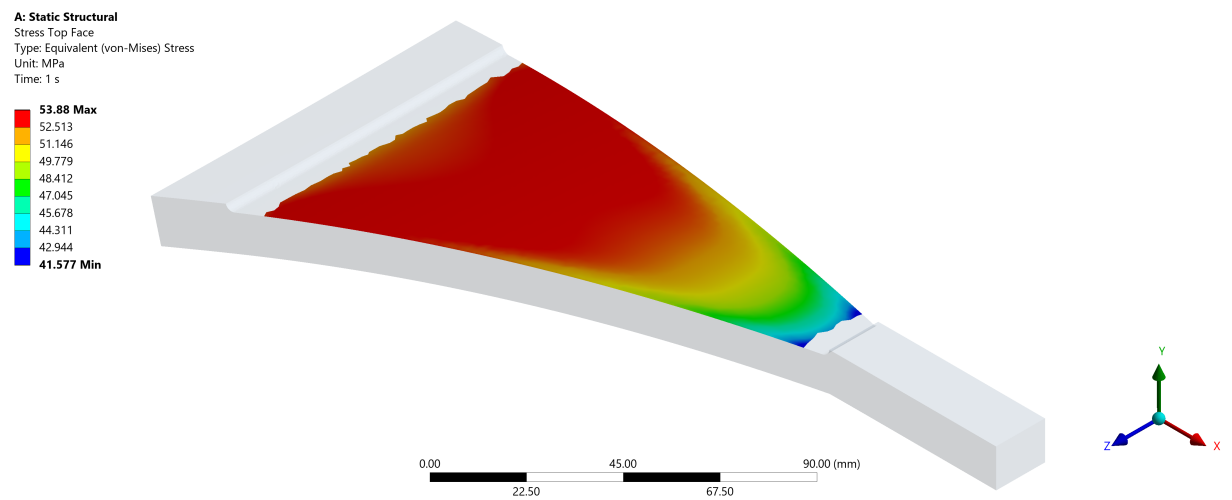


Figure 7: Stress distribution of the 10.5 mm case, with the loading region placed at 18.5 mm from the start of the rectangular loading region.

In Figures 6 and 7 we can note that as the loading area gets closer to the triangular central section of the blade, the stress distribution becomes more uniform, and thus, the maximum stress is lowered, this is yet another parameter to consider when calculating the safety factor of any case to perform future calculations on.

For the rest of this document, we have arbitrarily chosen the loading area for our calculations to start at 15 mm from the tip of the blade spring, as described in this section.

2.5 Stress

Considering the fact that silicon exhibits a far greater tolerance to stress when being compressed, compared to when it's subjected to tension, the top face of the blade

where tensile stress happens is our main area of focus. We also exclude the fillet region as its shape has not been optimized.

With this, the following table was obtained:

Blade thickness(mm)	Maximum stress of the top face (MPa)
4	401.9
5	249.9
6	171.1
7	125.0
8	95.3
9	75.3
10	61.0
11	50.4
12	43.7
13	36.2
14	31.2
15	28.0
16	24.4
17	21.5
18	20.2
19	18.1
20	15.8

Table 2: Maximum tensile stress values as the thickness of the blade increases.

With these data, and keeping in mind our maximum tensile strength value of 165 MPa, we can start ruling out some of the cases that exceed our safety factor limit of 3, as seen in Figure 8.

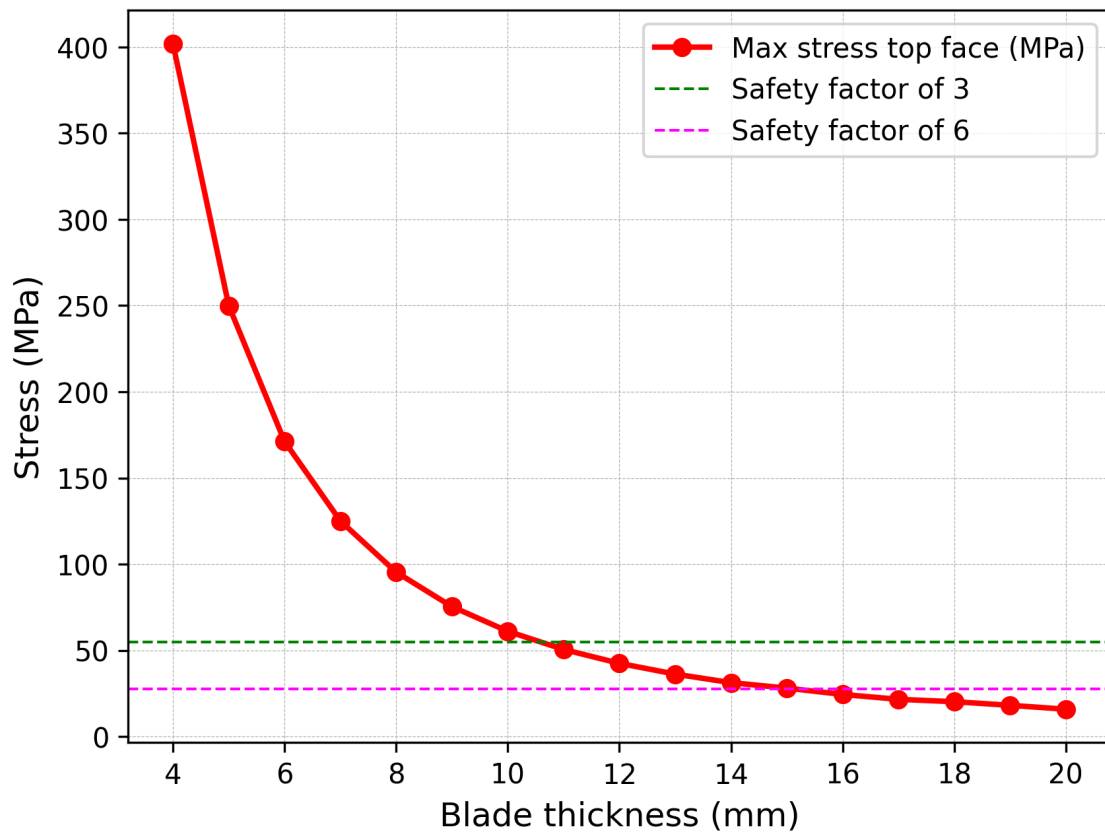


Figure 8: Stress vs blade thickness.

With this in mind, a conservative case to make future investigations on is the one with thickness of 10.5 mm, with a maximum stress of 52.69 MPa (excluding the fillet region) achieving a safety factor of 3.1.

Figure 9 and Figure 10, display the stress distribution of the spring blade with thickness 10.5 mm.

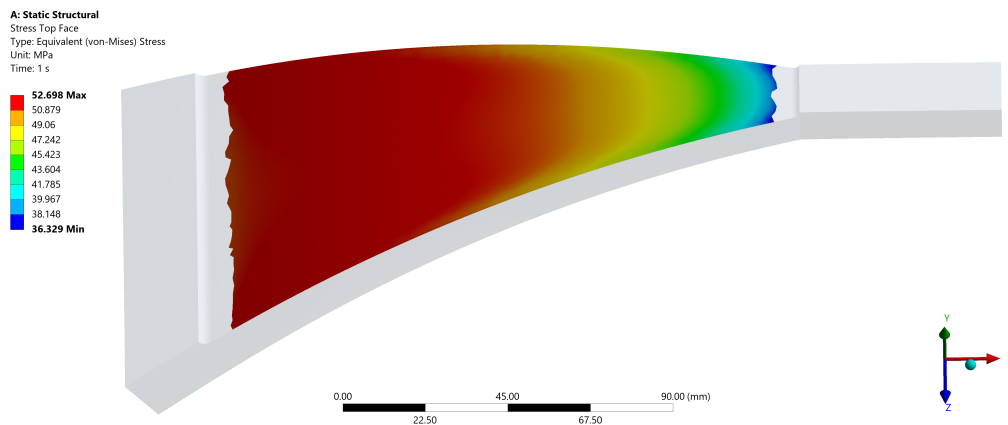


Figure 9: Stress distribution of the top face of the 10.5 mm case. In which the deformation shown here is exaggerated by a factor of 76.

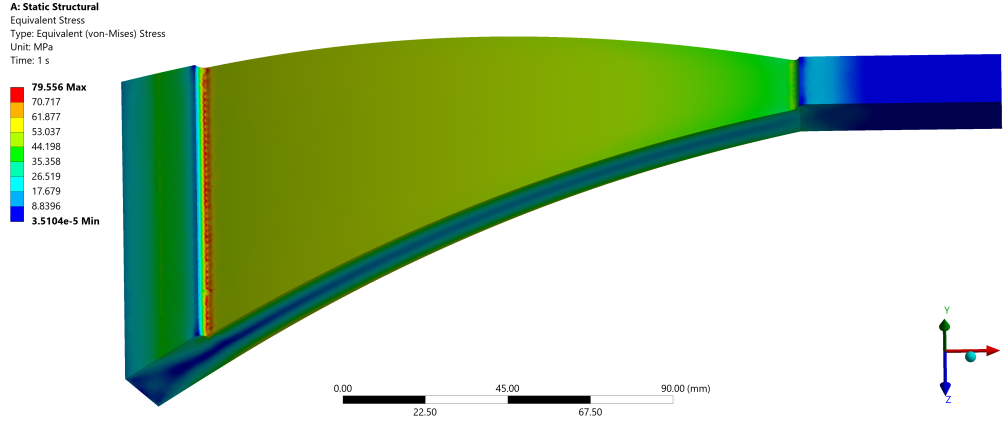


Figure 10: Stress distribution for the 10.5 mm case. Noting the peak stress in the fillet region is above our desired safety factor limit of 3, an optimized non-cylindrical fillet may further reduce the peak stress.

2.6 Spring constant

In order to obtain the spring constant of the blade, we subject the blade spring to a force in the vertical direction and calculate the displacement. With the force displacement pairs calculated by Ansys, the spring constant value k is extracted through Hooke's law:

$$F = -kx \quad (1)$$

In Ansys this was calculated by imposing a 300 N force in the loading area of the blade, in the vertical direction, and evaluating the given displacement due to the force. The results for our various cases can be found in Table 3. We've performed checks to ensure that these calculations were within the linear regime for our various cases, these checks can be found in: Appendix A.1.

Blade thickness (mm)	Spring Constant (kN/m)
4	20
5	30
6	60
7	100
8	150
9	210
10	290
11	380
12	490
13	620
14	780
15	950
16	1150
17	1370
18	1620
19	1900
20	2210

Table 3: Spring constant value as the thickness of the blade increases.

Figure 12 shows the relationship between the thickness of the blade and the value of the spring constant of the system.

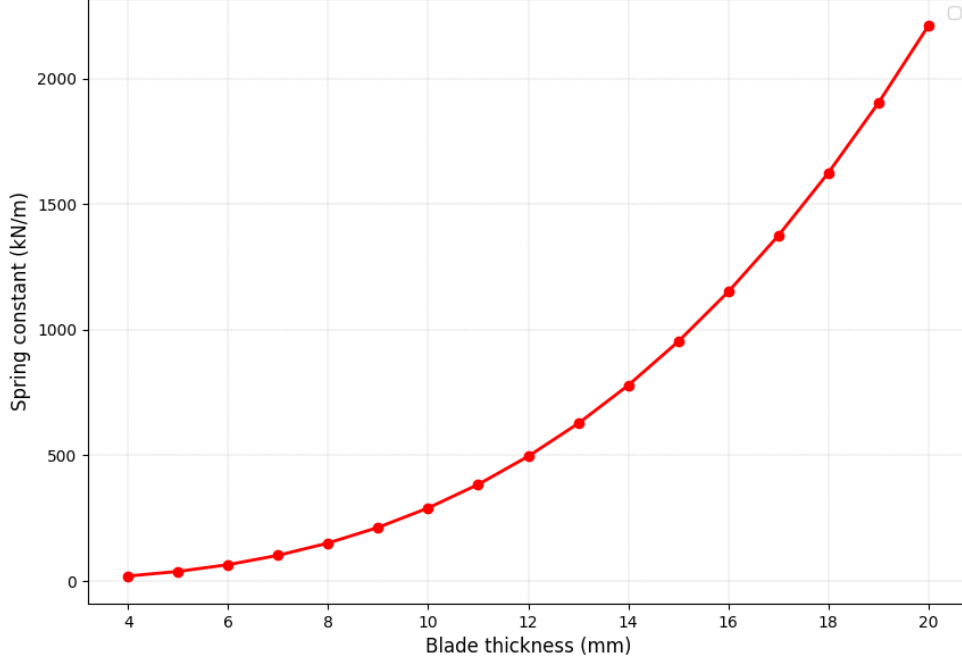


Figure 11: Spring constant value as a function of the thickness of the blade.

2.7 Modal analysis

The normal modes of a system are obtained by solving the eigenvalue problem derived from the system's equations of motion for free vibrations [5], taking into account the boundary conditions imposed earlier. The equation of motion for our simplified system can be written as:

$$M\ddot{x} + Kx = F(t), \quad (2)$$

where M is the mass matrix, K is the stiffness matrix, x is the displacement vector, $\ddot{x}$ is the acceleration vector, and $F(t)$ is the external force vector. In free vibration, $F(t) = 0$.

Ansys typically solves the undamped eigenvalue problem to obtain the natural frequencies and mode shapes

$$M\ddot{x} + Kx = 0, \quad (3)$$

in which a harmonic solution of the following form is assumed:

$$x = \Phi e^{i\omega t}, \quad (4)$$

where Φ is the mode shape given by the eigenvector and ω is the natural frequency, through this, Ansys solves:

$$(K - \omega^2 M)\Phi = 0. \quad (5)$$

Through the described pre-stressed modal analysis, the following results were obtained:

Blade thickness (mm)	First normal mode frequency (Hz)	Second normal mode frequency (Hz)
4	3.091	32.85
5	4.278	39.60
6	5.591	43.20
7	7.013	46.34
8	8.530	49.32
9	10.14	52.16
10	11.83	54.86
11	13.60	57.43
12	15.4476	59.8841
13	17.3657	62.2320
14	19.3516	64.4813
15	21.4022	66.6401
16	23.5147	68.7166
17	25.6865	70.7162
18	27.9149	72.6453
19	30.1979	74.5092
20	32.5329	76.3117

Table 4: Normal modes as the thickness of the blade increases.

The interest of our study encompasses the two first normal modes, but the first one takes special importance, as it is the one that determines the threshold of the $1/f^2$ mechanical attenuation, as will be confirmed in the displacement transfer function calculation. The higher modes are so high in frequency that they fall outside our area of interest.

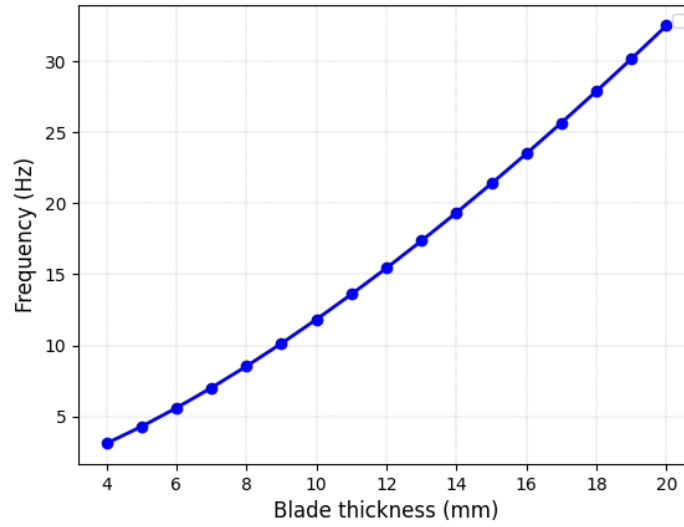


Figure 12: Frequency of the first normal mode as the thickness is increased.

By plotting the resonant frequency as a function of the blade thickness, in Figure 12, we can see the relationship these two have. The spring blade of course becomes more and more stiff as its thickness increases.

The shape of these normal modes is as follows:

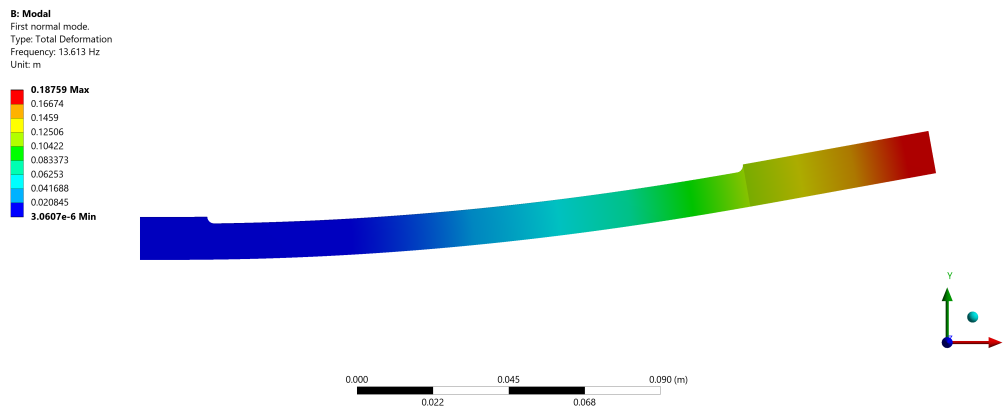


Figure 13: Shape of the first normal mode, for the 10.5 mm case, as an example. Where the scale denotes deformation.

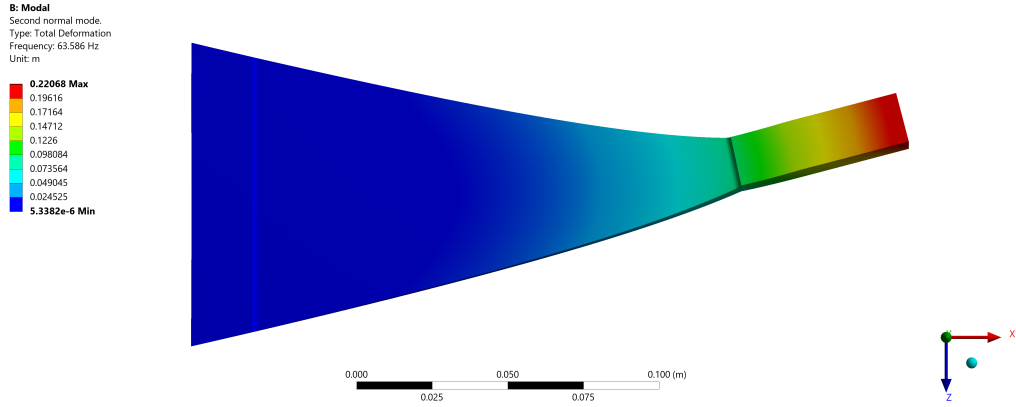


Figure 14: Shape of the second normal mode, for the 10.5 mm case, as an example. Where the scale denotes deformation.

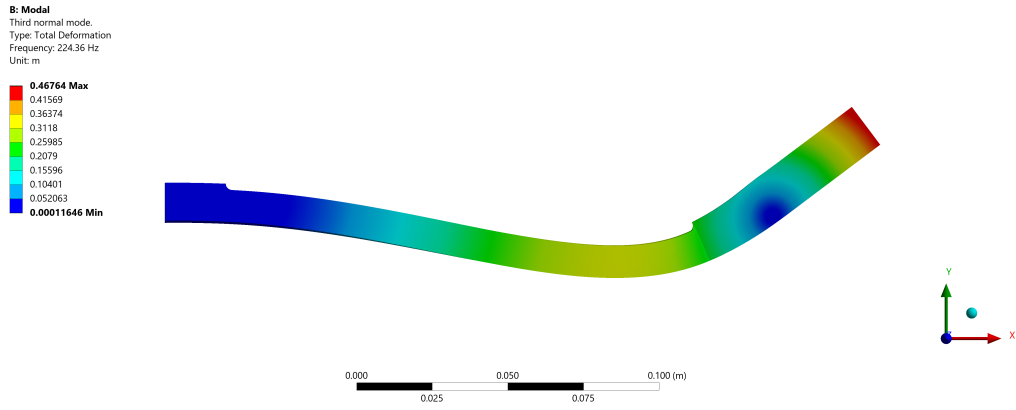


Figure 15: Shape of the third normal mode, for the 10.5 mm case, as an example. Where the scale denotes deformation.

2.8 Displacement transfer function.

The displacement transfer function quantifies, in the frequency domain, the displacement at the tip of the blade for a given displacement of its base. This is illustrated in Figure 16.



Figure 16: Conditions for the transfer function simulation, in which a harmonic excitation, a , is applied to the base of the blade and its effect at the end of the blade, a_0 is obtained. (deformation is exaggerated here).

Passive vibration isolation is achieved by leveraging the frequency dependence of the transfer function, which follows a $1/f^2$ decay after the first resonant frequency. This behavior means that beyond this first resonance, the system naturally attenuates displacement transmissions, with higher frequencies experiencing a large amplitude attenuation. To maximize this effect, the aim of the design is to set the frequency of the first resonant mode as low as possible, extending the frequency range of the $1/f^2$ drop, and thus, enhancing the isolation effect.

The transfer functions were obtained using Ansys through a pre-stressed harmonic analysis performed in the frequency domain.

The displacement transfer function T is derived from the acceleration transfer function, where we initially subject our geometry to an acceleration a_0 at the base of the blade and probe the acceleration a at the end of it, as shown in Figure 16. Through the relationship between acceleration and displacement, we can, thus, obtain the actual displacement transfer function:

$$T = \frac{a}{a_0} = \frac{-\omega^2 s}{-\omega^2 s_0} = \frac{x}{x_0} \quad (6)$$

where ω denotes the angular frequency, x_0 the given displacement at the wider end, and x the resulting displacement at the narrower end, in which the load is placed.

We present the calculated displacement transfer function for the 10.5 mm case. From which the first resonant frequency resulting from the first normal mode is observed at the expected frequency of 13.61 Hz, followed by the known $1/f^2$ decay.

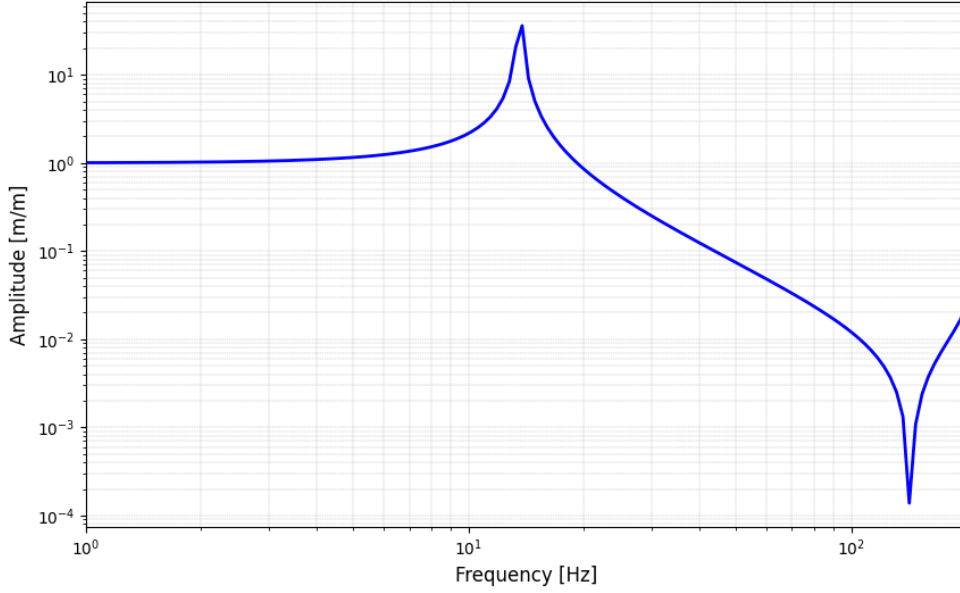


Figure 17: Displacement transfer function of the 10.5 mm case.

From this, one can argue that the introduction of an active anti-spring external to the blade could be used in order to lower the frequency of the first resonance and generate useful mechanical attenuation. Our geometry configuration allows for the introduction of an electrostatic actuator with optical sensing to achieve this effect.

2.9 Radius of curvature

For this calculation, we capture multiple points along one of the edges of the blade in the XY plane, as illustrated in Figure 18. We then determine the radius of curvature by constructing a system of linear equations and solving it using the least squares method.

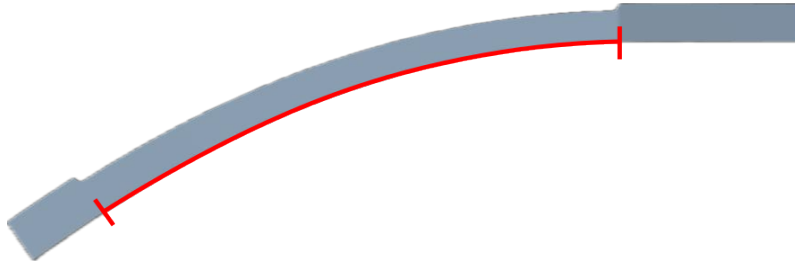


Figure 18: Highlight of the boundary in which the x , y coordinates were captured (deformation is exaggerated here).

This is accomplished using Python's `numpy.linalg.lstsq` function, which finds the best-fitting solution to an overdetermined system. The following Python code implements this computation, using `numpy` to form and solve the linear system and

extract the circle parameters. The calculation is performed for the 10.5 mm case, which achieves the desired safety factor of 3.1.

```

1 x_coords = data['Location X']
2 y_coords = data['Location Y']
3
4 A = np.column_stack((2 * x_coords, 2 * y_coords, np.ones_like(x_coords)))
5 B = x_coords**2 + y_coords**2
6
7
8 params, residuals, rank, s = np.linalg.lstsq(A, B, rcond=None)
9 a, b, c = params
10
11 radius_of_curvature = np.sqrt(a**2 + b**2 + c)
12
13
14 print("Radius of Curvature:", radius_of_curvature, "m")

```

Running the code yields the following output:

Radius of Curvature: 13.63 m

3 Conclusion

Keeping in mind our initial proposal of keeping the stress distribution under 55 MPa, ensuring a minimum safety factor of 3, we propose the following case for future exploration.

Blade (mm)	Max Stress Top Face (MPa)	Min Stress Top Face (MPa)	Safety Factor	First Normal Mode (Hz)	Spring Constant (kN/m)
10.5	52.59	36.41	3.1	13.613	330

Table 5: Main attributes of a blade spring with thickness of 10.5 mm.

As described above, a conservative case to consider is the one described a thickness of 10.5 mm, with a radius of curvature of 13.63 m, a base tilt 12.453 mrad in which the sag of the tip is 1.3 mm when loaded. The upper face experiences an almost uniform tensional stress, with a maximum value of 52.69 MPa. The overall maximum stress was estimated with Ansys to be 79.55 MPa, occurring along the fillets at the transitions from the thinner to the thicker sections. Thinner and more stressed blades can be designed if tests show that a higher stress is tolerable with a good quality surface finish.

Anti-springs with optical sensing and electrostatic actuation are necessary to soften the elastic constant, over a short displacement range, to provide attenuation for the vertical thermal noise of the seismic attenuation chain.

Given that the fragility of silicon largely arises from surface defects that initiate catastrophic failure, further investigation is necessary. This should include an experimental testing of the actual breaking points under various surface treatments, along with a holistic analysis of the interferometer as a whole.

References

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A Appendix

A.1 Linearity check of the spring constant calculation

By imposing an external force to our blade spring and calculating the generated displacement at the same point, the loading area, we can see that the force and the displacement follow a linear dependency, as easily seen in Figure 19.

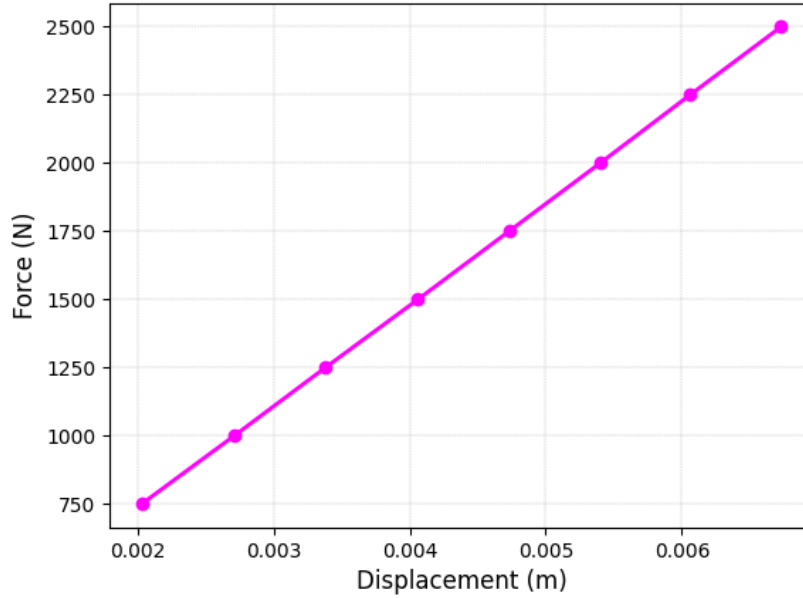


Figure 19: Linearity check for 10.5 mm case, as an example.

We additionally compared results for different strength inputs, and different blade spring thicknesses, overlapping of all the values ensured that we stayed within the linear regime, in which Hooke's law holds, found in Figure 20.

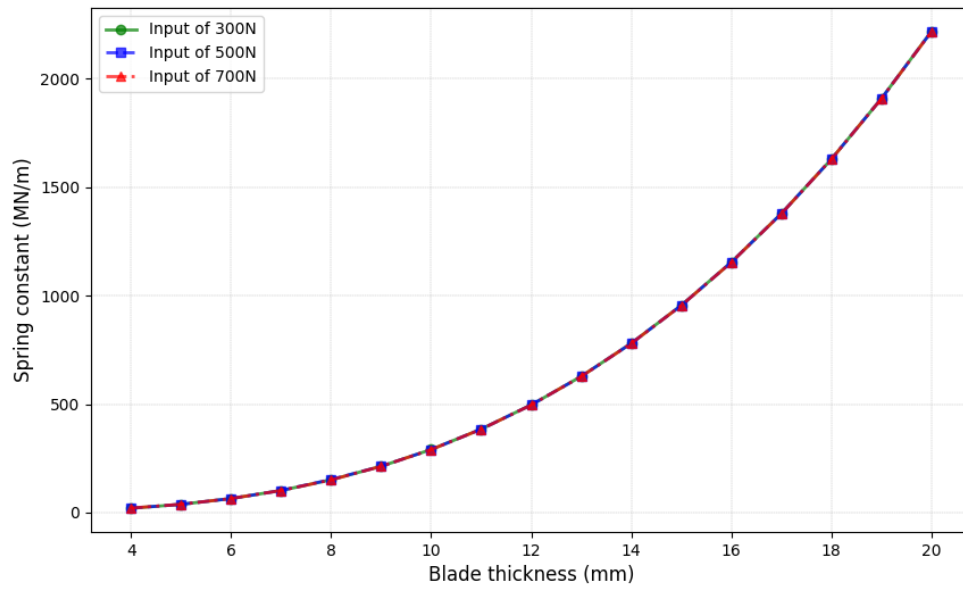


Figure 20: Linearity check of the spring constant calculation, in which the value of the spring constant, overlaps for different forces and configurations.