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Optical Cavities as Springs – Anti-springs

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*A cryogenic test-mass suspension with flexures operating in
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1 - Introduction

The purpose of this report is to assess the performance attainable by a centimeter-long optical cavity proposed for a cryogenic test mass suspension system for the forthcoming Einstein Telescope [1]. The cavity would be operated as an optical anti-spring to reduce the pendulum resonant frequency of the system.

It would be placed on extensions of rigid mirror suspension beams (as illustrated in [1], Sec. 2 and Fig. 1), designed to provide mechanical attenuation. The cavities will be fiber-fed, with stabilization and control optics placed outside of the vacuum chamber. The cavities can also be used to soften the vertical springs supporting the mirror or to control other mechanical oscillators like the beam resonant modes

2 - Optical Cavities

We evaluate here the length sensitivity and anti-spring strength for a 1 cm long Fabry-Perot interferometer. The cavity would be operated in a side-locking mode, with a laser wavelength $\lambda_0=1064$ nm, and a feed intensity of 3.9mW. The cavity contains about 10^4 wavelengths.

The first step is to consider the cavity at its resonance, that is with maximum stored energy. We then detune it by a fixed amount from the resonant frequency to bring it to a state in which a small displacement produces a large stored-energy variation and hence a corresponding variation in radiation pressure applied on the mirror. Within a short range, this change is proportional to the displacement and can be assimilated to the behavior of a spring with an elastic constant either positive or negative, depending on the sign of the detuning.

The cavities are intended to be fiber-fed, with all the input-output, stabilization and control optics installed outside the vacuum vessel and the fibers penetrating in parallel to the heat links.

For an input mirror of reflectivity r_1 and an ideal end mirror ($r_2 = 1$) The finesse is

$$F = \frac{\pi\sqrt{r_1}}{1 - r_1}$$

The required reflectivity for about 10^4 finesse is $r_1 = 0.9998396$.

By considering that $T_i = t_i^2$, $R_i = r_i^2$ and that $T_i + R_i = 1$, we obtain $t_i = 0.017910$ and the stored energy is

$$W(\phi) = \frac{T_1 I_0}{1 + R_1 R_2 - 2r_1 r_2 \cos \phi}$$

The maximum energy, attained for $\cos \phi = 1$, is

$$W_{max} = \frac{0.017910^2 I_0}{1 + 0.9998396^2 - 2 \times 0.9998396} = 12467.6 I_0 = 48.62J$$

Considering, for the sake of simplicity, the energy of a 1064 nm photon is $1.87 \cdot 10^{-19}$ J such an energy is bound to

$$\frac{48.62}{1.87 \times 10^{-19}} = 2.60 \cdot 10^{20} \text{ photons/s}$$

3 - Optical Spring/Anti-spring

When operating a Fabry Perot as optical spring the FP must be fed with a fixed wavelength light. It is assumed that the frequency reference of the main interferometers or an equivalent frequency reference are available to be used to generate the feed wavelength with sufficient stability.

If the cavity is detuned by a δ_L the energy stored is reduced and, if the length is further varied with respect to the chosen detuning value the stored energy fluctuations cause a change in the light pressure on the mirrors which acts either as a spring or anti-spring depending on the sign of δ_L . We will refer to δ_L as the nominal detuning.

Such a detuning can be attained in two ways:

1. By physically changing the cavity length;
2. By changing the laser wavelength.

The two techniques are equivalent from the anti-spring operation, but since the actual distance between the mirrors cannot be controlled, laser detuning is preferred.

The spring constant is

$$K_0 = \frac{128\pi f_0 I_0}{c_0^2 T_1^2} \frac{\delta_\gamma}{(1 + \delta_\gamma^2)}; \quad \delta_\gamma = 4L_0 \frac{2\pi}{\lambda_0 T_1} \left(1 - \frac{L_0}{L}\right); \quad L = L_0 \pm \delta_L$$

To null the gravitational restoring force of the pendulum the cavity needs to attain a $K_0=3900 \text{ Nm}^{-1}$.

Note that, at a fixed finesse, K_0 can be controlled by the input power I_0 . The finesse of the cavity is determined by T_1 and its complement r_1 . Lower finesse requires higher input power to achieve the same spring constant but allows a proportionally wider dynamic range.

The optimal cavity detuning is chosen to achieve best dynamic range for the given finesse F and required rigidity K_0 . It is obtained by detuning the laser by $\delta_f/f_0 = 1.4693 \times 10^{-9}$ (Figure 1). For a cavity resonating at a wavelength of 1064nm, this optimal detuning implies to change the laser frequency by 0.4140MHz (corresponding to a wavelength change of 1.5635 pm).

The detuning is optimal in the sense that the optical rigidity isocurves attain minimum finesse, and hence any length variation of the cavity due to thermal vibrations would have the minimum effect on the K_0 value as shown in Figure 1.

Having fixed the finesse to maximize K_0 , we allow a variation of K_0 of 10% from its nominal value. This determines the allowable operational range, i.e. the change in length of the cavity due to suspension beam oscillation or to its resonant vibrations, or the vertical oscillation for the cantilever springs described in the main paper [1]. The variation of K_0 is always a decrease of the absolute value of K_0 .

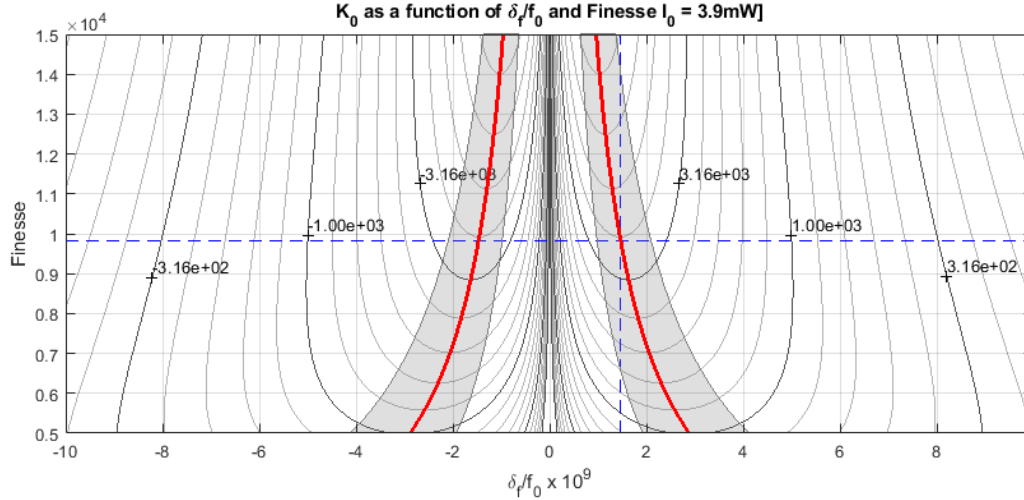


Figure 1 – K_0 as a function of finesse and detuning (in normalized frequency variation) calculated for the case of nulling the pendulum frequency.

In Fig. 1 the red curves represent the optimal detuning as a function of the cavity Finesse, the gray areas the 10% range of variation in K_0 . At the extreme of the dynamic range (intersections of the horizontal blue dashed line with the edges of the gray shadowed area) the spring constant decreases 10%, to $K_0=3512 \text{ Nm}^{-1}$.

Since the operation entails an actual length variation of the cavity it is more convenient to assess it in terms of length rather than frequency, which is done in Figure 2. The optimal detuning occurs for a variation in the cavity length $\delta_L = 15.6 \text{ pm}$. Let's consider a "detuned" cavity that is $\delta_L = 15.6 \text{ pm}$ "too long". Note that the gray shaded area width is not symmetric with respect to the red line. Fig. 2 shows that, for displacements of -5.20 pm and 6.67 pm from the optimal tuning, the K_0 drops by 10%. This is considered an acceptable behavior of the anti-spring and the dynamic range can be approximated as $\pm 5 \text{ pm}$ from the optimal detuning position.

About tuning, we remind the reader that the detuning is obtained by changing the laser frequency, which then needs to be held fixed to obtain the anti-spring effect. If there is a secular movement affecting the cavity length the laser frequency can be tuned to maintain the required detuning as long as the changes are much slower than the spring moving at its operational frequency. This detuning has the consequence of moving the usable displacement range together with the mirror, keeping the anti-spring functional. With this kind of working point tracking, the

usable displacement range of $\pm 5\mu\text{m}$ can be maintained on the correct position hence the operational range should be sufficient to attenuate the residual noise from the upper stages.

The case discussed above corresponds to an optical elastic constant matching that of a 2 m long pendulum with 50 kg load. The allowable dynamic range is diminished if the optical anti-spring is applied to stiffer springs. Note that the characteristic time of the cavity is much shorter than the mechanical oscillation, therefore, in principle, by extracting the position information from the reflected signal, it is possible to actively correct the spring constant by suitably modulating the cavity input power, or by using the information to operate actuators, thus extending the dynamic range on stiffer springs.

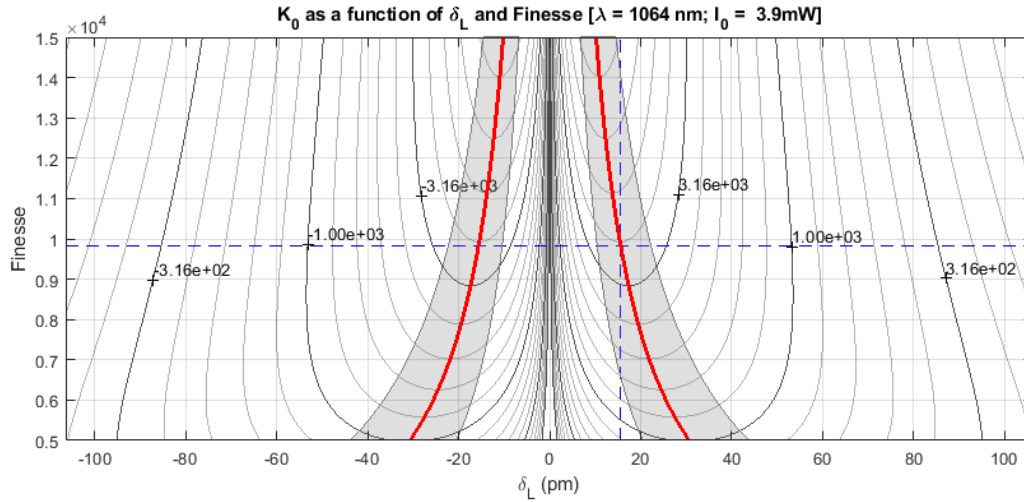


Figure 2 – K_0 as a function of finesse and detuning (in pm for a given wavelength of 1064nm)

The detuned cavity at its nominal length holds less energy than at the peak and displacements from this nominal position increases or decreases the amount of energy (hence effectively providing the spring/anti-spring behavior).

At the nominal length the stored energy and photons are

$$W_{\text{detuned}} = 20.90\text{J} \rightarrow 1.12 \cdot 10^{20} \text{ photons/s}$$

The dynamic range corresponds to an energy fluctuation of

$$\begin{aligned} W_{\text{min}} &= 13.14\text{J} \rightarrow 7.04 \cdot 10^{19} \text{ photons/s} \\ W_{\text{max}} &= 30.55\text{J} \rightarrow 1.63 \cdot 10^{20} \text{ photons/s} \end{aligned}$$

Since each 1064 nm photon carries a momentum

$$p = \frac{hf}{c} = 6.2275 \cdot 10^{-28} \text{ kg m/s}$$

The momentum applied to the end mirrors

$$P = N_{ph}p = \begin{cases} 4.38 \cdot 10^{-08} \text{ kg m/s} & \text{min} \\ 6.97 \cdot 10^{-08} \text{ kg m/s} & \text{nominal} \\ 1.02 \cdot 10^{-09} \text{ kg m/s} & \text{max} \end{cases}$$

4 - Conclusions

Within the displacement limitations due to the wavelength used and cavity finesse, a cm optical cavity can provide values of anti-spring constant useful to reduce the suspension's test mass pendulum frequency within a displacement interval of 10pm. This dynamic range is strongly reduced if the anti-spring is applied to stiffer springs, like the vertical cantilever springs supporting the mirror weight. A useful dynamic range can be recovered by extracting the cavity length information and use it as a control signal for suitable force actuators. The same sensor-actuator configuration can be used to control the vibration amplitude of beam resonances.

5 - References

[1] Fabián E. Peña Arellano *et al.*, “A cryogenic test-mass suspension with flexures operating in compression for third-generation gravitational-wave detectors,” <https://apps.et-gw.eu/tds/ql/?c=17764>