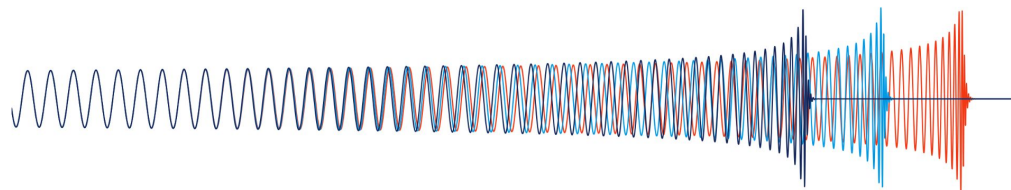


Ready for what lies ahead? Waveform opportunities and challenges towards the Einstein Telescope

Lorenzo Pompili

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**University of
Nottingham**
UK | CHINA | MALAYSIA

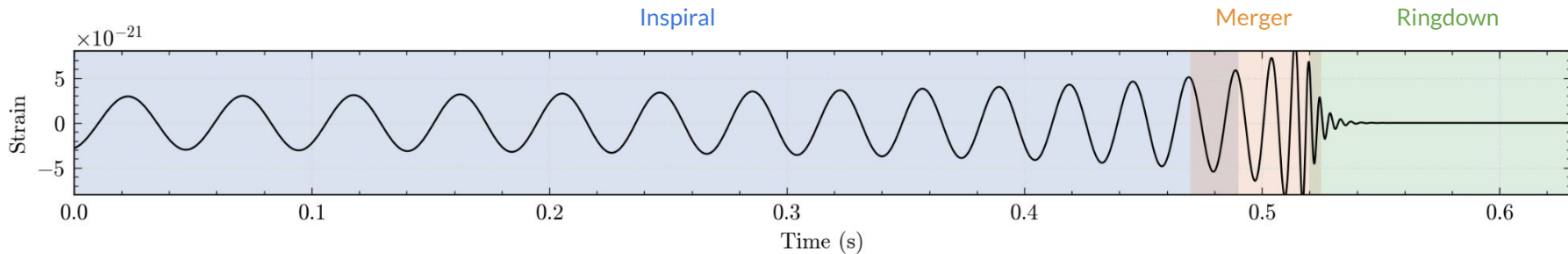
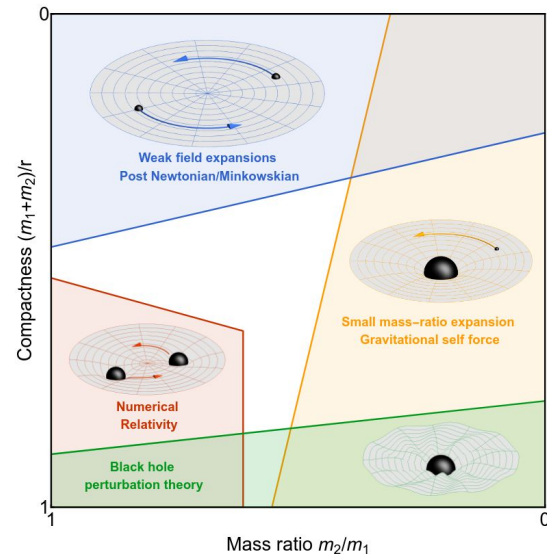
Einstein Telescope science workshop for early career researchers
Sapienza University of Rome
20.02.2026

Modeling compact binary coalescences

Many modeling frameworks:

- **Post-Newtonian theory**: slowly moving binaries (expansion in v/c).
- **Post-Minkowskian theory**: weakly gravitating sources (expansion in G).
- **Gravitational self-force**: mass-asymmetric systems (expansion in m_2/m_1).
- **Black-hole perturbation theory**: ringdown emission of remnant black hole.
- **Numerical relativity**: “exact” but computationally expensive - limitations in signal length and parameter space coverage.

Credit: LISA Waveform White paper



Inspiral–merger–ringdown waveform models

Waveform modeling: combine information from numerical and perturbative approaches. Main goals are:

- **Generality**: cover the full parameter space of astrophysical signals (spin precession + eccentricity + tides + ...)
- **Efficiency**: Bayesian parameter estimation requires $O(10^7\text{--}10^8)$ waveform evaluations.
- **Accuracy**: Keep systematic errors below statistical uncertainty.

Inspiral–merger–ringdown waveform models

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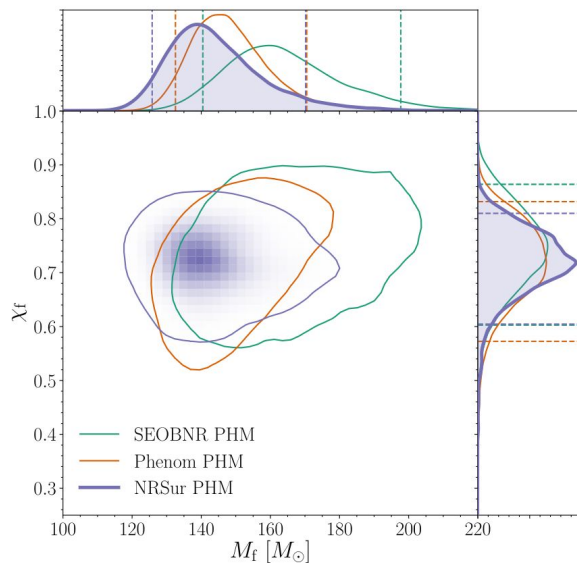
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Main waveform model families:

- **Effective-one-body (EOB)**: semi-analytical models calibrated to NR. Model binary dynamics *and* waveform $\rightarrow$ “easy” to include extra physics. Time domain + ODEs $\rightarrow$ speed is challenging.
- **IMRPhenom**: piecewise closed-form expressions for the waveform calibrated to PN/EOB – NR hybrids $\rightarrow$ fastest models available, slightly less accurate than EOB.
- **NR surrogate**: directly interpolate NR waveforms $\rightarrow$ most accurate models, but limited to NR length and parameter-space coverage.

DISCLAIMER: only talking about waveforms from compact binaries!

Credit: LIGO-Virgo-KAGRA; GW190521



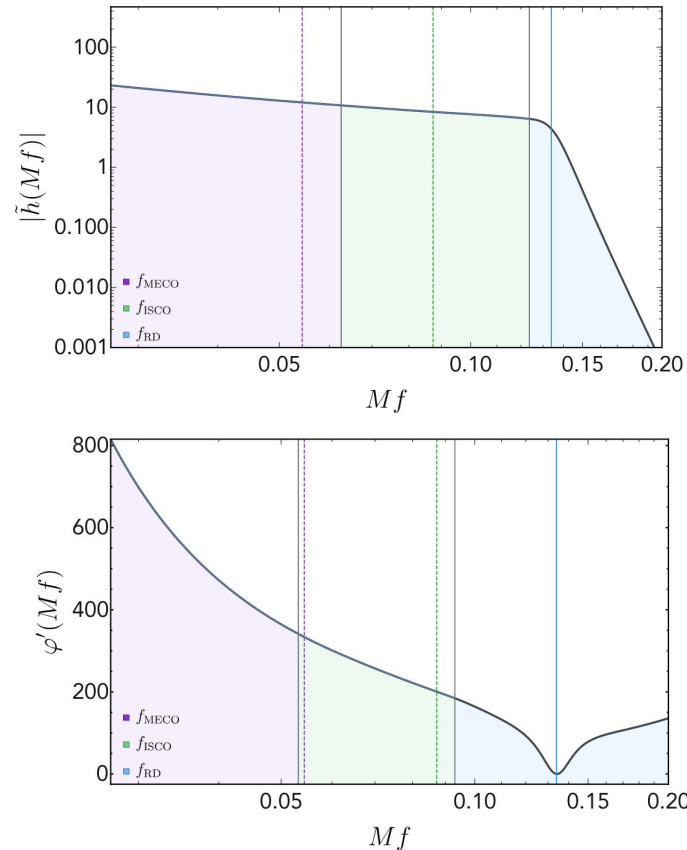
IMRPhenom: phenomenological modeling

Phenomenological modeling of the amplitude and phase of the spin-weighted spherical harmonic modes of the signal.

Ajith+07, Santamaria+10, Husa+15, Khan+15

$$h(t; \iota, \varphi) = \sum_{\ell \geq 2} \sum_{|m| \leq \ell} {}_{-2}Y_{\ell m}(\iota, \varphi) h_{\ell m}(t)$$

- Closed-form piece-wise expressions (in frequency-domain or time-domain) for amplitude and phase of each harmonic.
- Closed-form or numerical evolution of spin-precession of the orbital plane.

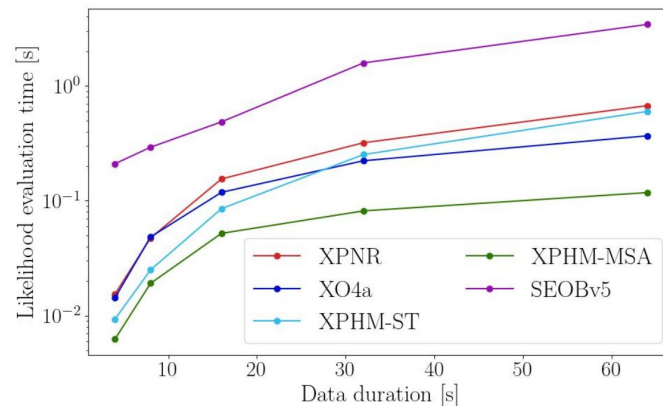


Credit: Pratten+ 2020

IMRPhenom: frequency domain

Frequency-domain models: highly efficient, direct application with current data-analysis methods, but shortcomings for modeling complex signals (eccentricity, spin-precession).

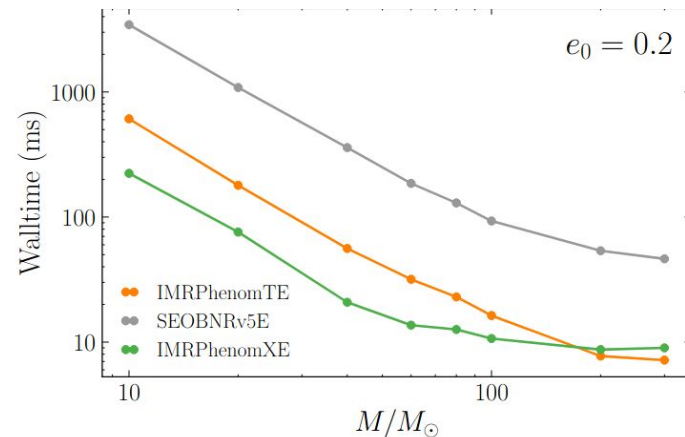
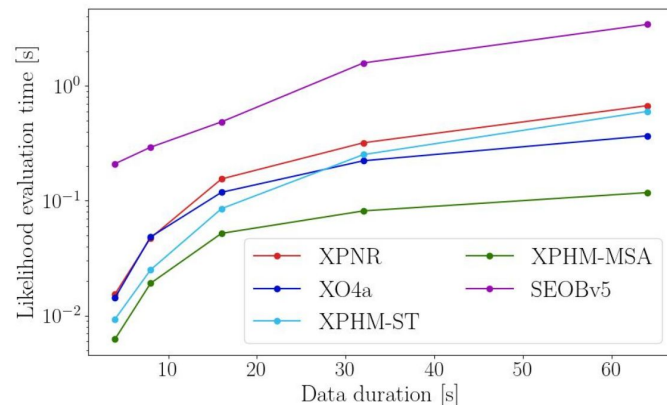
- **IMRPhenomXPHM:** multipolar spin-precessing model. Pratten+ 2021, 2020; Garcia-Quiros+ 2020
- Recent improvements in modeling spin precession: **XO4a**; **XPHM-SpinTaylor**; **XPNR**. Thompson+ 2024; Colleoni+ 2025; Hamilton+ 2025



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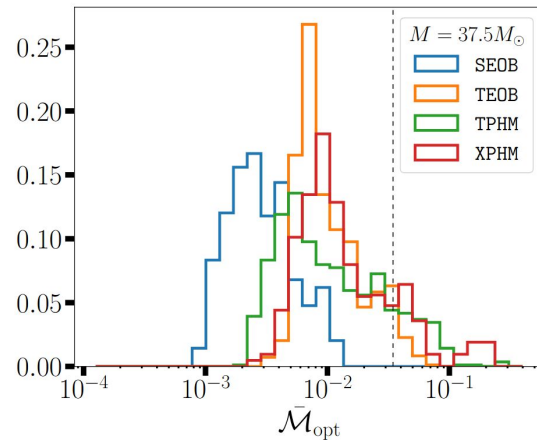
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- **IMRPhenomXE:** aligned-spin, (2,2) mode, eccentric generalization. Ramos-Buades+ 2026
- BBH baseline extended to model BNS/NSBHs by adding NR-calibrated tidal correction to phase / amplitude (**NRTidalV***). Dietrich+ 2017, 2019; Colleoni+ 2023; Abac+ 2023, 2025



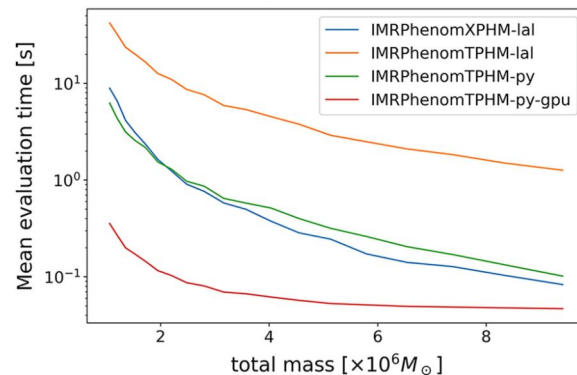
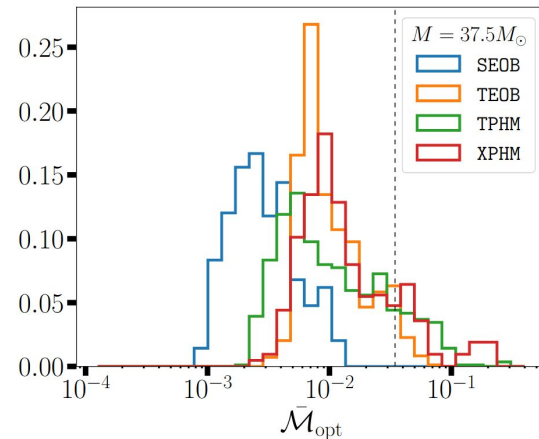
IMRPhenom: time domain

- **Time-domain models:** still efficient, more direct connection with binary dynamics (improved modeling of spin precession and eccentricity), higher accuracy than frequency-domain models.
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 - Recent developments: inclusion of non-linear memory and generalization to aligned-spin eccentric binaries (**IMRPhenomTEHM**). Planas+ 2025; Rossello-Sastre+ 2025, 2024



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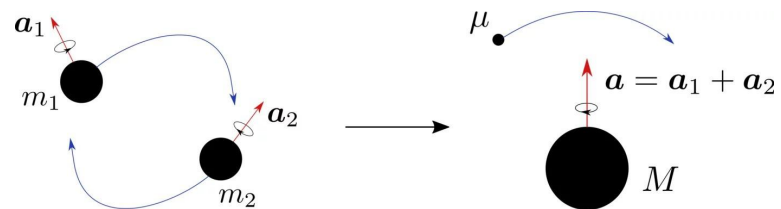
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- Original implementation in [LALSuite](#) (C99).
- New implementation of TPHM and eccentric models in standalone python package [phenomxpy](#), with GPU support. Garcia-Quiros+2025



Effective-one-body waveforms

Two-body dynamics mapped into dynamics of one effective body moving in deformed BH spacetime, deformation being the mass ratio.

Buonanno and Damour 1999, 2000, ...



Effective-one-body waveforms

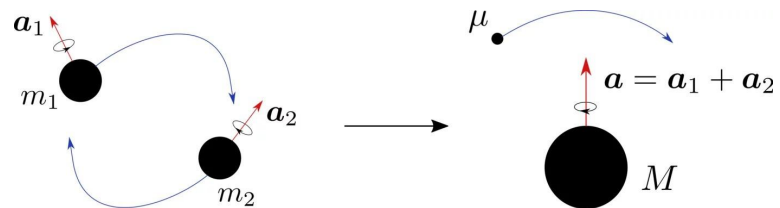
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1. **EOB Hamiltonian** describing the conservative binary dynamics

EOB Hamiltonian

$$H^{\text{EOB}} = M \sqrt{1 + 2\nu \left(\frac{H_{\text{eff}}}{\mu} \right)}$$

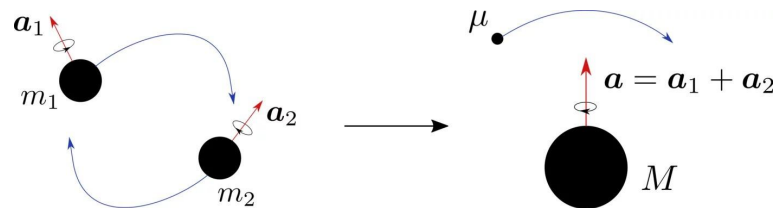


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Radiation reaction

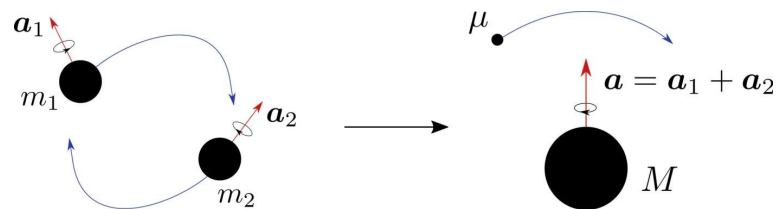
$$\mathcal{F} = \frac{\Omega}{16\pi} \frac{p}{L} \sum_{\ell, m} m^2 |h_{\ell m}|^2$$

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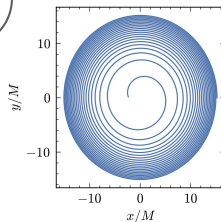
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Equations of motion (ODE)

$$\begin{aligned} \dot{\vec{r}} &= \frac{\partial H^{\text{EOB}}}{\partial \vec{p}} \\ \dot{\vec{p}} &= -\frac{\partial H^{\text{EOB}}}{\partial \vec{r}} + \vec{\mathcal{F}} \end{aligned}$$

Orbital dynamics

$$\vec{r}, \vec{p}$$



Radiation reaction

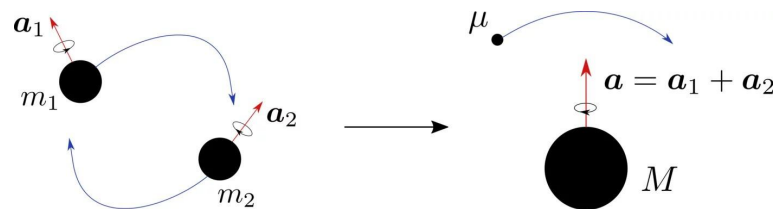
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1. **EOB Hamiltonian** describing the conservative binary dynamics
2. **Radiation reaction (RR) force** to account for loss of energy and angular momentum via emission of GWs
3. **Gravitational waveform modes** for inspiral, merger, and ringdown



EOB Hamiltonian

$$H^{\text{EOB}} = M \sqrt{1 + 2\nu \left(\frac{H_{\text{eff}}}{\mu} \right)}$$

Radiation reaction

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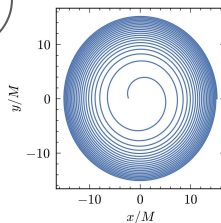
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Orbital dynamics

$$\vec{r}, \vec{p} \Rightarrow$$

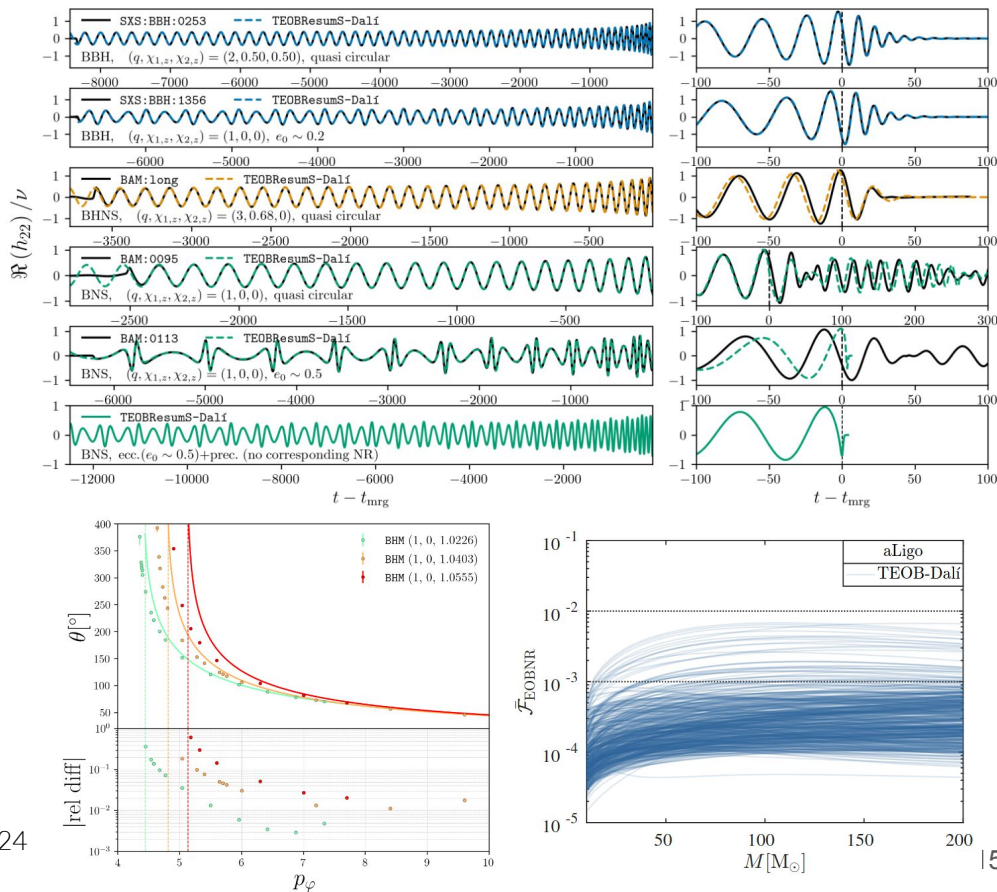
Waveform modes

$$h_{\ell m} = \begin{cases} h_{\ell m}^{\text{insp-plunge}}, & t < t_{\text{peak}}^{22} \\ h_{\ell m}^{\text{merg-RD}}, & t > t_{\text{peak}}^{22} \end{cases}$$



Effective-one-body waveforms: TEOBResumS

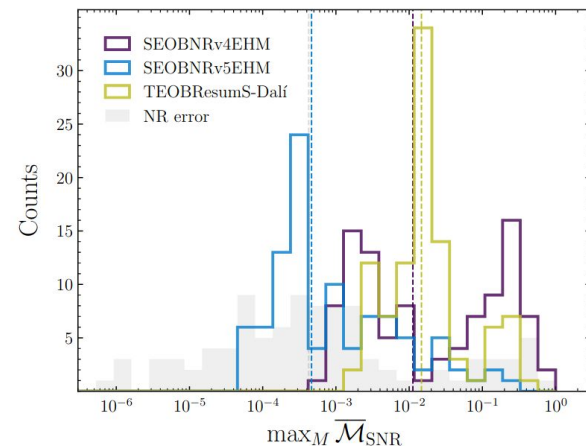
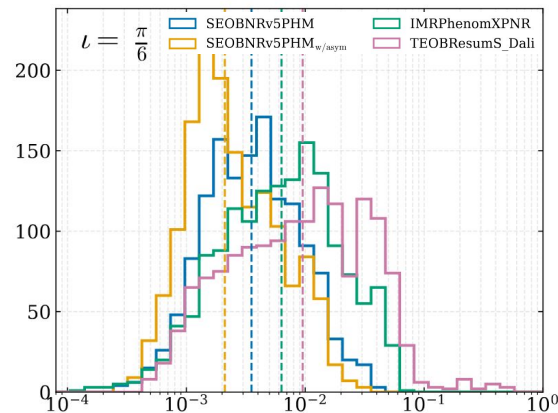
- State-of-the-art model **TEOBResumS-Dalí**: generic orbits (bound & unbound) + spin precession + tides within a single model. Albanesi+ 2025; Gonzales+ 2025; Gamba+ 2024; Nagar+ 2024, 2023
- Efficient frequency domain version for quasi-circular binaries based on SPA. Gamba+ 2021
- Parametrized beyond GR corrections for BBHs. Chiamarello+ 2025
- [C-code](#) with python bindings; interfaced through LAL routines via [gwsignal](#).



Credit: Albanesi+ 2025; Nagar+ 2024

Effective-one-body waveforms: SEOBNR

- **SEOBNRv5PHM**: quasi-circular spin precessing BBHs. Estelles+ 2025, Ramos-Buades+ 2023, Pompili+ 2023
- **SEOBNRv5EHM**: eccentric aligned-spin BBHs. Gamboa+ 2024
- **SEOBNRv5THM**: quasi-circular aligned-spin BNSs. Haberland+ 2025
- Parametrized and theory specific beyond GR extensions. Pompili+ 2025, Julie+ 2024
- Most accurate semi-analytical models for quasi-circular spin precessing and eccentric aligned-spin BBHs.
- Python package [pyseobnr](#); interfaced through LAL routines via [gwsignal](#).



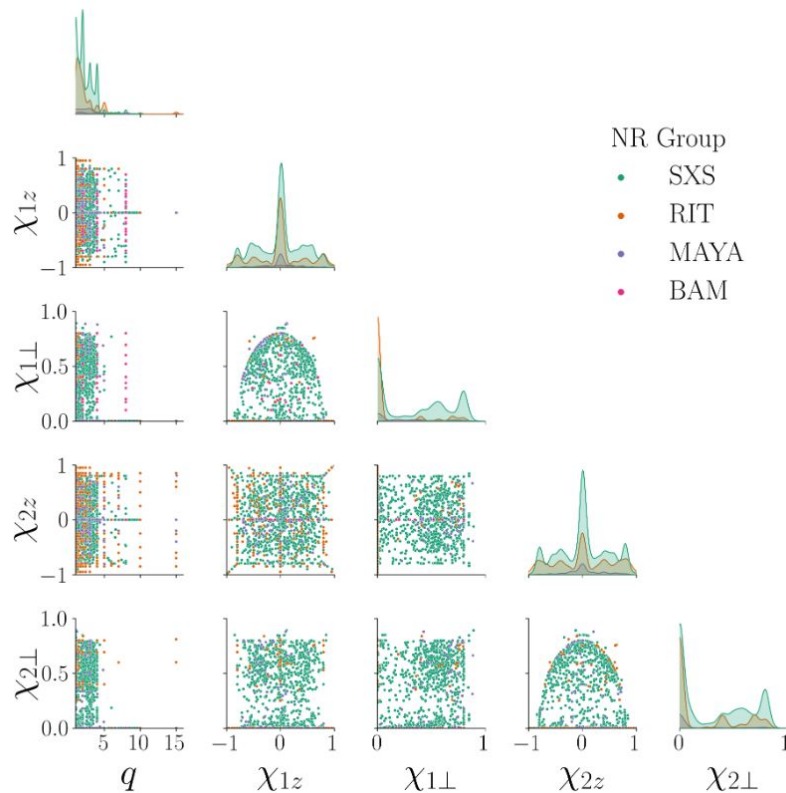
Credit: Estelles+ 2025; Gamboa+ 2024

Numerical relativity

- $O(8000)$ NR simulations across catalogs: [SXS](#), [RIT](#), [MAYA](#), [BAM](#), ...
- Progress with new generation NR codes: [SpECTRE](#), [AthenaK](#) / [Athena++](#), ... – improved scalability and waveform extraction (CCE/CCM).
- Progress in extending parameter space: eccentricity, hyperbolic, beyond GR, ...

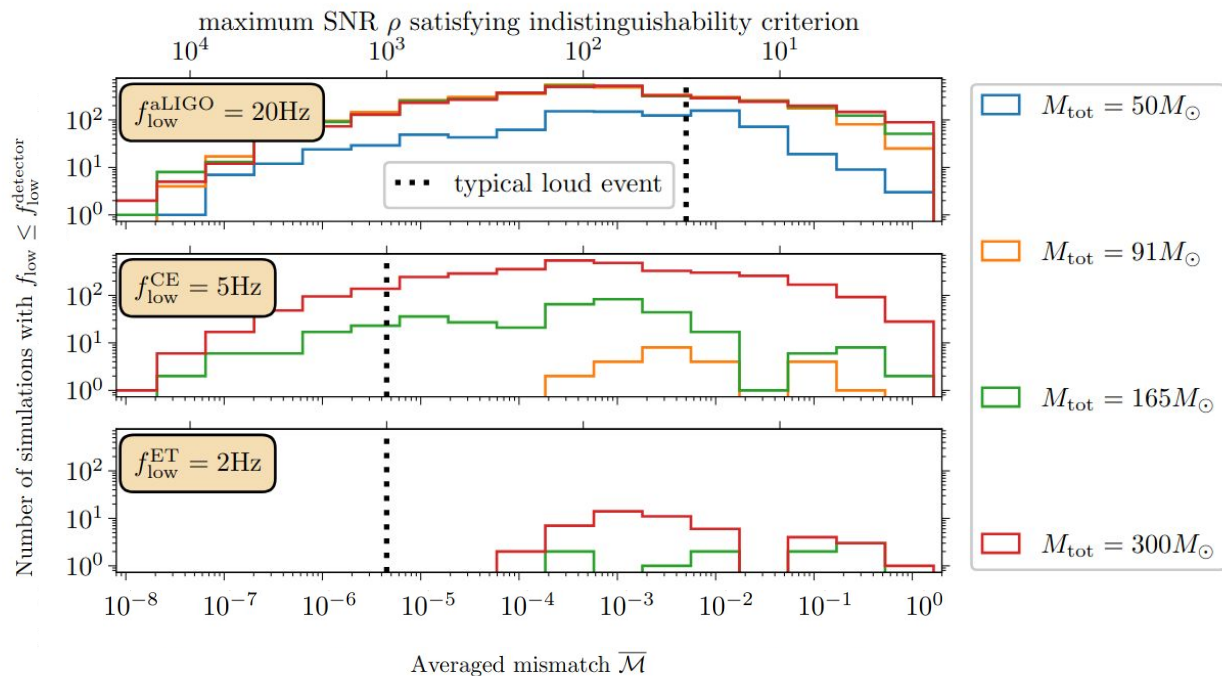
Credit: LISA Waveforms White Paper

NOTE: only updated until 2023, many more simulations since then!



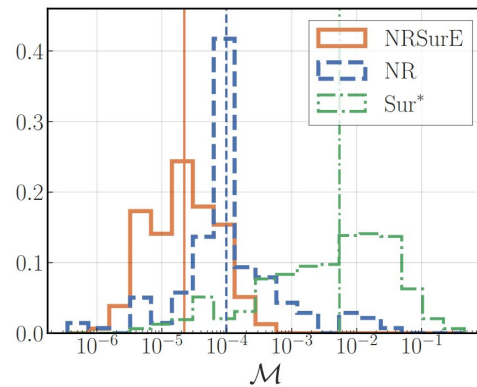
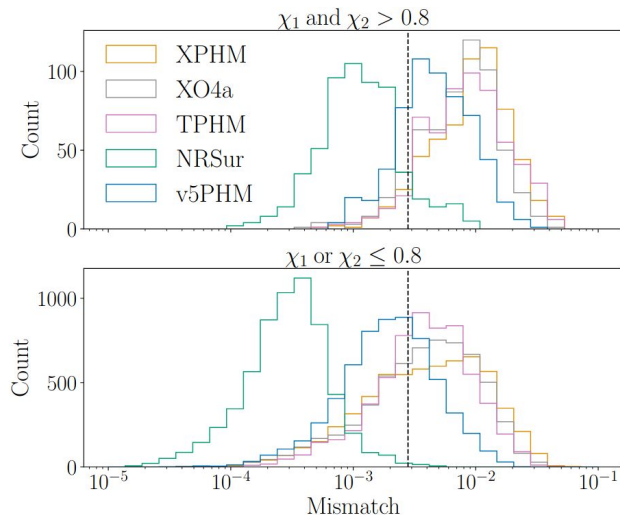
Numerical relativity

Most BBH simulations don't reach the length and accuracy required by ET. Simulations with matter face even larger challenges.

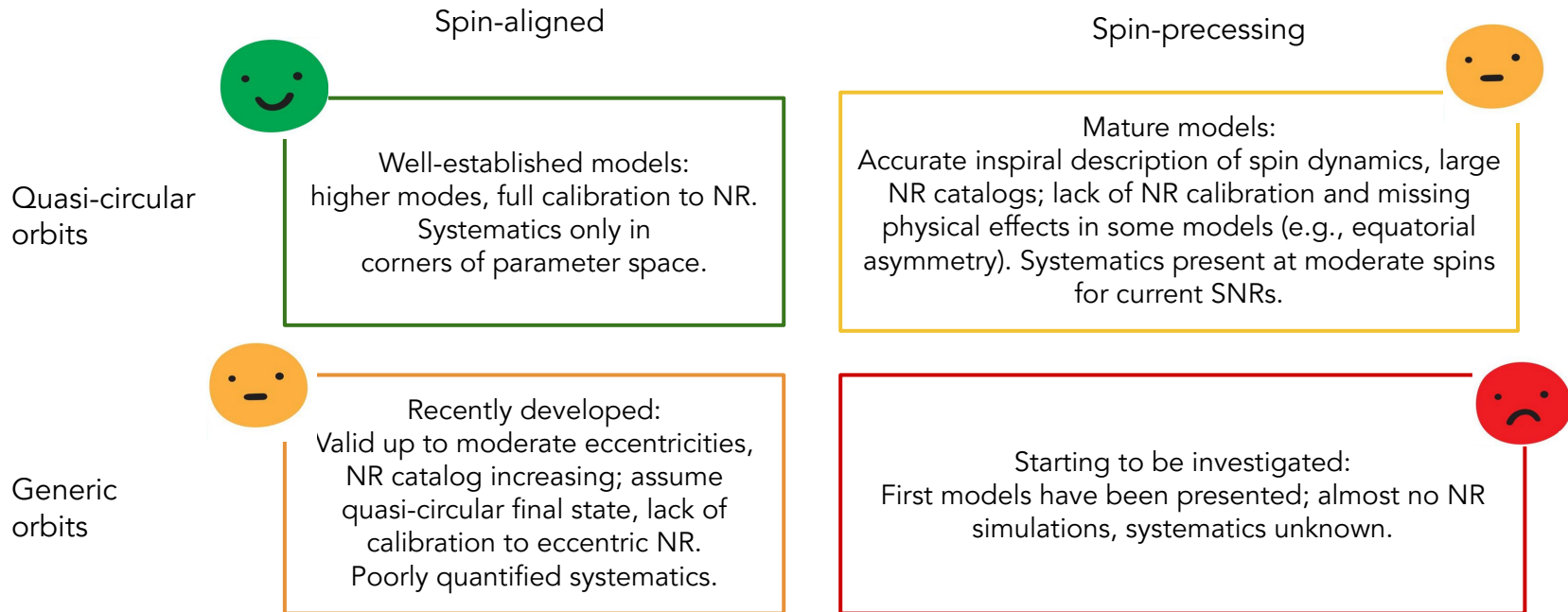


Numerical relativity surrogates

- **NR surrogates**: data-driven interpolated models built directly from NR datasets. More accurate than semi-analytical models within domain of applicability.
- **NRSur7dq4**: spin-precessing model trained on $q \leq 4$, $|\chi| \leq 0.8$. Length covers $\sim 60M_\odot$ starting from 20Hz. Varma+ 2019; Blackman+ 2015; Field+ 2013
- Aligned-spin model up to $q < 8$ hybridized with PN/EOB for inspiral, incl. GW memory. Yoo+ 2023; Varma+ 2018
- Eccentric surrogates techniques demonstrated for non-spinning binaries. Nee+ 2025
- Implemented in LALSuite and standalone package [gwsurrogate](#).



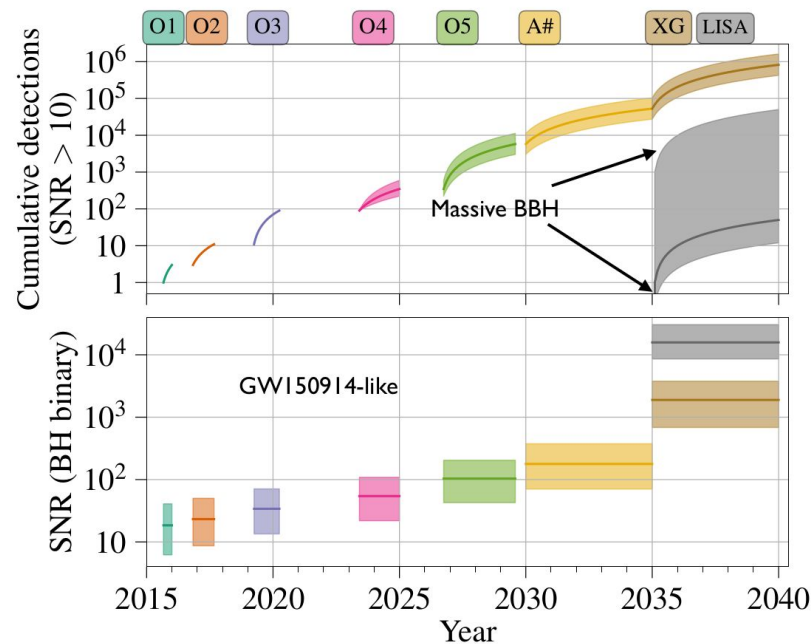
Modeling the BBH parameter space



Opportunities...

- ET will observe "golden binaries" at SNR of hundreds to thousands: we can finally resolve subtle physical effects in the waveform (spin precession dynamics, eccentricity, GW memory, ...) that are currently difficult to measure or disentangle.
- High-precision tests of general relativity.
- Neutron star astrophysics.
- Cosmology; populations; ...

However ...



Credit: Arnab Dhani

... and challenges ...

Mismatch:
$$\mathcal{M} = 1 - \max_{\varphi_0, t_0} \frac{(h_1 | h_2)}{\sqrt{(h_1 | h_1)(h_2 | h_2)}}$$

$$(h_1 | h_2) \equiv 4 \operatorname{Re} \int_{f_l}^{f_h} df \tilde{h}_1(f) \tilde{h}_2^*(f) / S_n(f)$$

Allowed waveform errors:

$$\mathcal{M} \lesssim \frac{N}{2\rho^2}$$

See Lindblom+ 2008; Chatziioannou+ 2017; Toubiana+ 2024;
Thompson+ 2025

... and challenges ...

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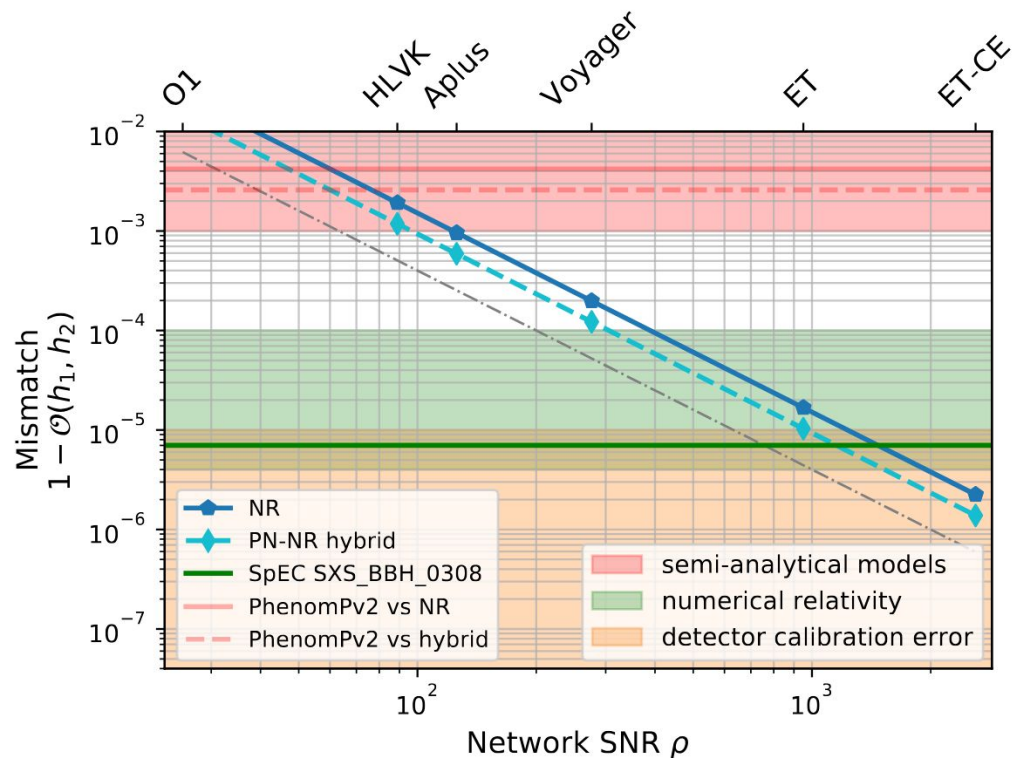
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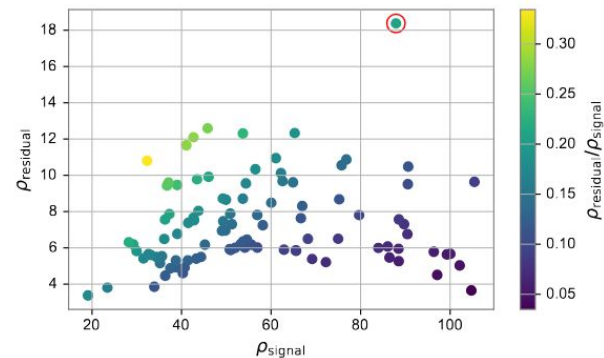
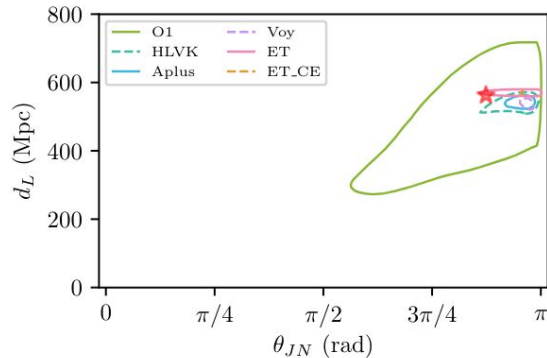
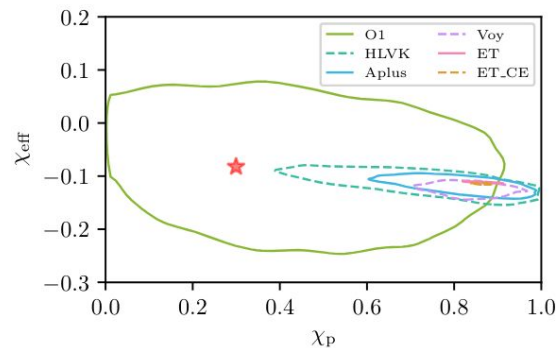
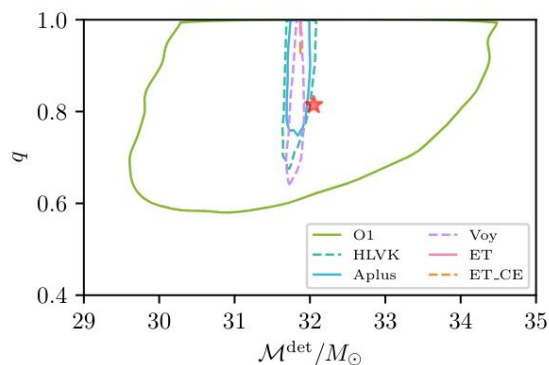
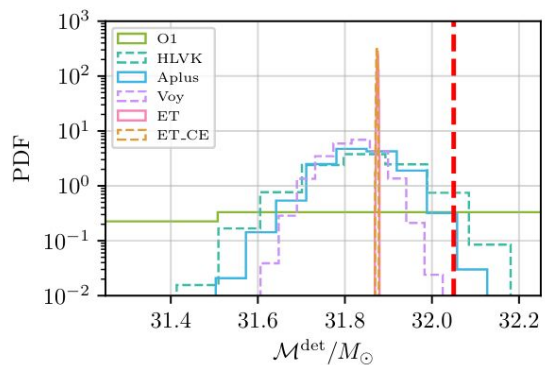
$$\mathcal{M} \lesssim \frac{N}{2\rho^2}$$

See Lindblom+ 2008; Chatziioannou+ 2017; Toubiana+ 2024; Thompson+ 2025

Mismatch for semi-analytical models to be reduced by ~ two to three orders of magnitude, and for NR waveforms by ~ one order of magnitude, for unbiased PE with “golden” binaries.



... and challenges ...



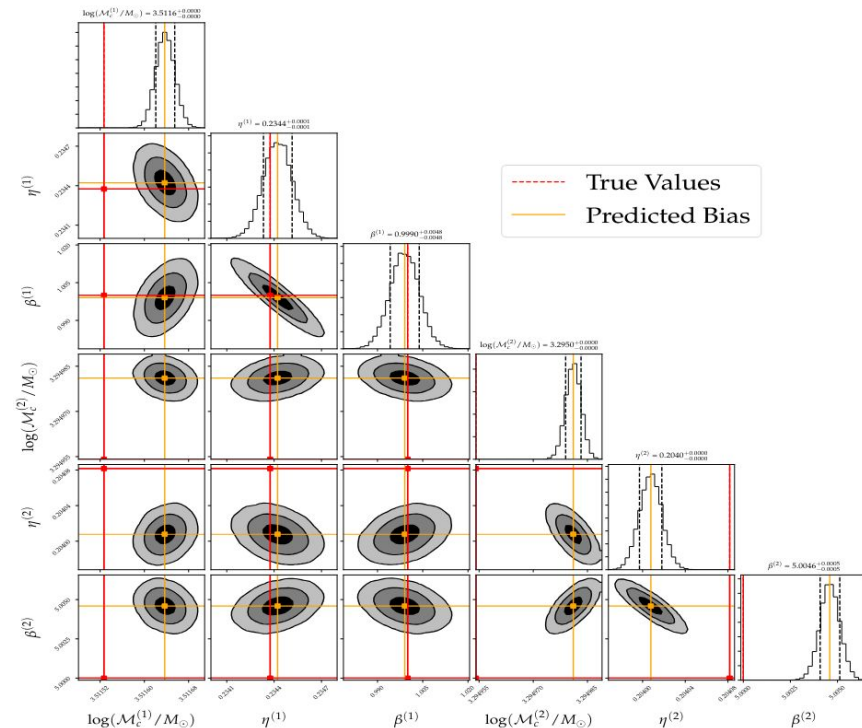
- Biased parameter estimates
- Significant residuals → potentially absorbed by false GR deviations

... and more challenges

Accuracy requirements for overlapping signals (either confusion noise from missed signals or incorrectly fitted waveforms) even more stringent.

Some preliminary studies, but poorly quantified in realistic analyses. Antonelli+ 2021; Hu+ 2023

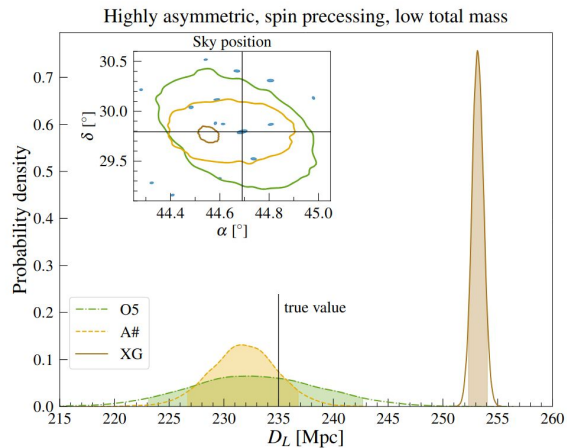
$$\begin{aligned}\Delta\Theta^i &= (\Gamma^{-1})^{ij} \left(\frac{\partial h}{\partial \Theta^j} \Big| n(t) + \delta h + \Delta H_{\text{conf}} \right) \\ &= \Delta\Theta_{\text{noise}}^i + \Delta\Theta_{\text{sys}}^i + \Delta\Theta_{\text{conf}}^i,\end{aligned}$$



Challenges: cosmology

“Golden” dark sirens

Dhani+ 2024; Benetti+ 2026

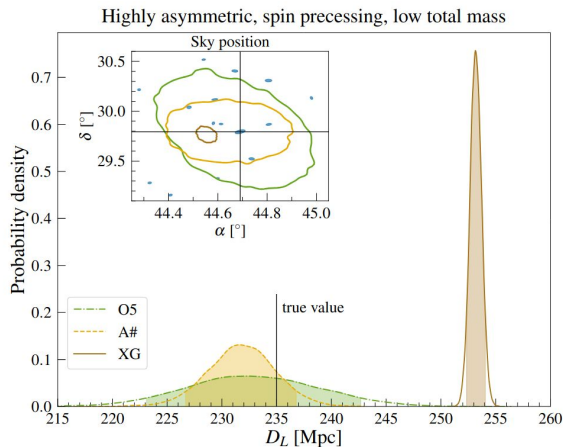


Model	Correct H_0	Incorrect H_0	Total
Correct host association			
IMRPhenomXPNR	29 (28%)	12 (12%)	41 (40%)
SEOBNRv5PHM	11 (12%)	6 (6%)	17 (18%)
Incorrect host association			
IMRPhenomXPNR	33 (32%)	29 (28%)	62 (60%)
SEOBNRv5PHM	28 (30%)	48 (52%)	76 (82%)
Total			
IMRPhenomXPNR	62 (60%)	41 (40%)	103 (100%)
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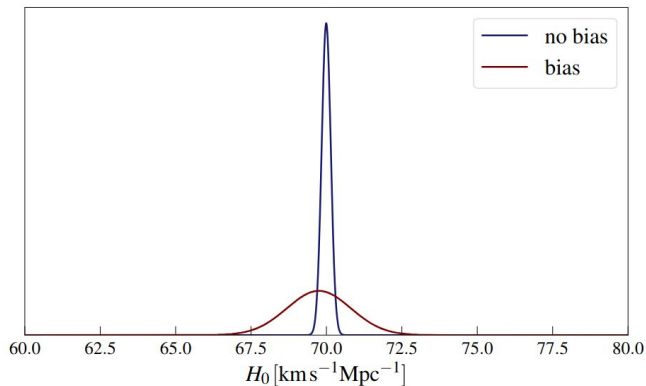
Dhani+ 2024; Benetti+ 2026



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“Statistical” dark sirens

Dhani+ 2025

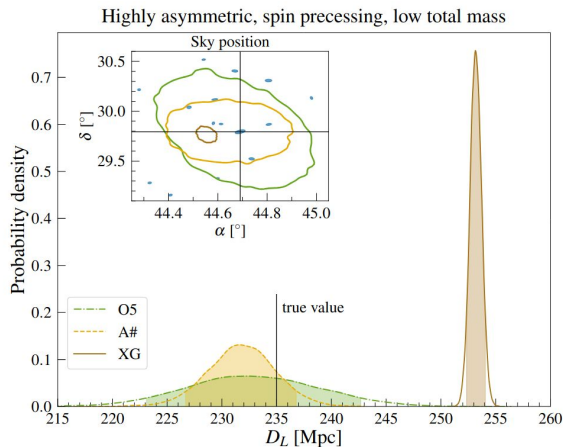


- For population analyses, even when systematics do not lead to “biases”, they can reduce precision (additional source of variance across events).
- Potentially identifiable in hierarchical analysis (variance in measurement of H_0 across events does not match noise expectation) but hard to correct.

Challenges: cosmology

“Golden” dark sirens

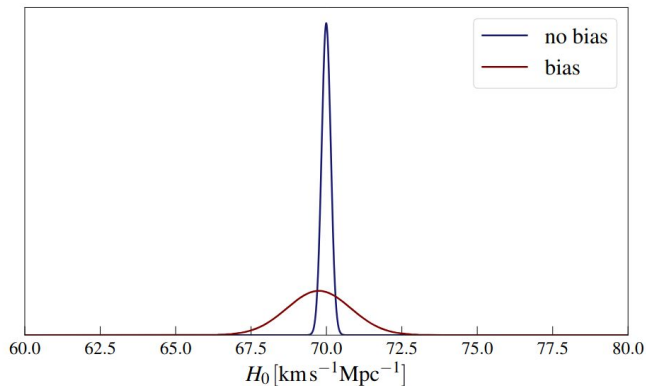
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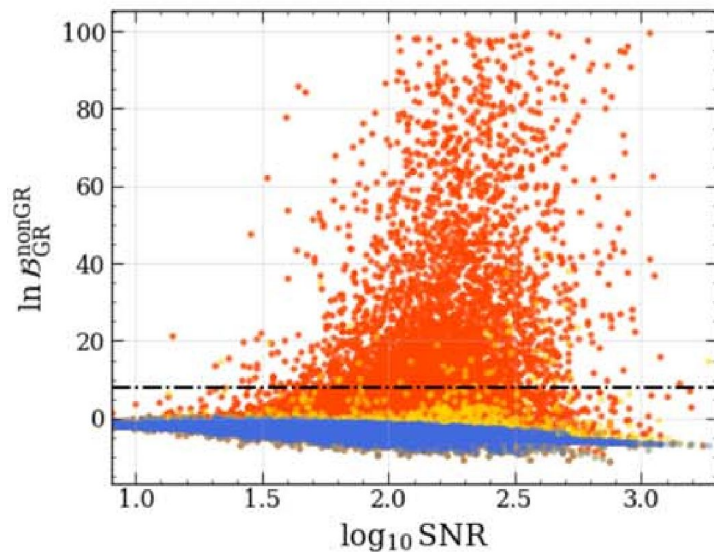
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- Potentially identifiable in hierarchical analysis (variance in measurement of H_0 across events does not match noise expectation) but hard to correct.

Challenges: tests of GR

Cause	O4	A+	A [#]	XG
Non-Stationary Noise	✓	✓	✓	✓
Non-Gaussian Noise/Glitches	✓	✓	✓	✓
Overlapping Signals	✗	✗	✗	✓
Data Gaps	✗	✗	✗	✓
Detector Calibration	✗	✗	✗	✓
Eccentricity	✓	✓	✓	✓
Tidal Effects	✗	✓	✓	✓
Kick-induced Effects	✗	✗	✗	✓
Ringdown Modes	✓	✓	✓	✓
Precession and Higher-order Modes	✓	✓	✓	✓
Memory	✗	✗	✓	✓
Sub-optimal Waveform Calibration	✗	✗	✓	✓
Lensing	✗	✗	✗	✓
Environmental Effects	✗	✗	✗	✓
Source Misclassification	✓	✓	✓	✓
Astrophysical Population Assumptions	✓	✓	✓	✓

Credit: Gupta+ 2024

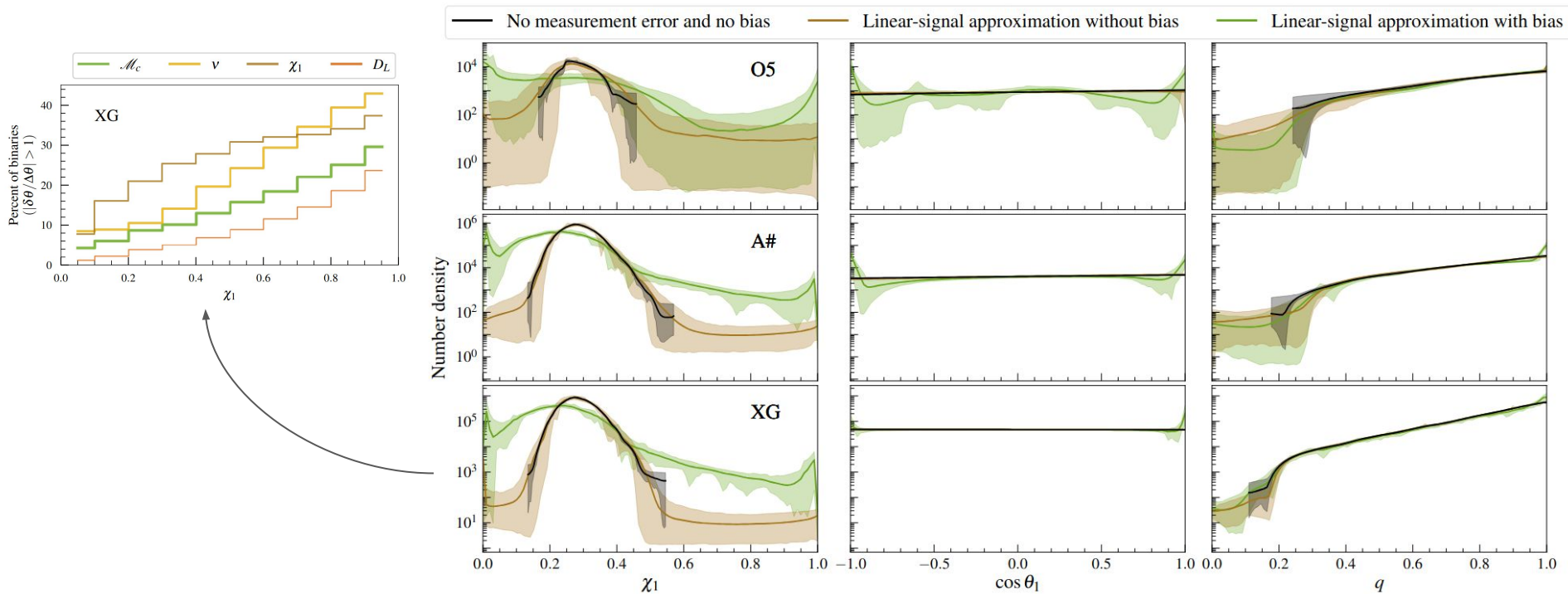
No error – error $O(10^{-3})$ – error $O(10^{-6})$



Credit: Hu and Veitch 2022

Challenges: populations

Waveform systematics potentially relevant for population analyses (but less obvious to predict – do errors accumulate or average out?)



Conclusions

- **Waveform development is progressing well**: first eccentric and spin-precessing models, next-generation NR codes, eccentric NR surrogate techniques, accelerated waveform implementations, ...
- Still, accuracy falls ~ 2 orders of magnitude short of ET requirements. **Waveform systematics threaten key ET science cases** — parameter estimation of golden binaries, tests of GR, cosmology — though the impact on population analyses may be less severe.
- Besides improving models, developing methods to marginalize over uncertainties is also important (more in Rossella's talk and the discussion session!)
- **Accuracy** and **speed** remain in tension: achieving *both* simultaneously is a key open challenge for ET data analysis.