

# Deterministic models for pressure-tilt-coupling

Geophysical Institute (GPI), Black Forest Observatory (BFO)

Thomas Forbriger and Walter Zürn | 4.11.2022



# Preview

1. The trigger for this presentation: KAGRA's seismic response to atmospheric pressure signal from Hunga Tonga eruption
2. Black Forest Observatory (BFO)
3. Levels of common tilt noise floor
4. Coupling mechanisms: LDT and TWT
5. Evidence from horizontal component data analysis
6. Application to Hunga Tonga Lamb- and sound-wave
7. The traveling wave model above 10 mHz
8. Conclusions

Background  
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BFO

Noise floor  
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Mechanisms  
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Data analysis  
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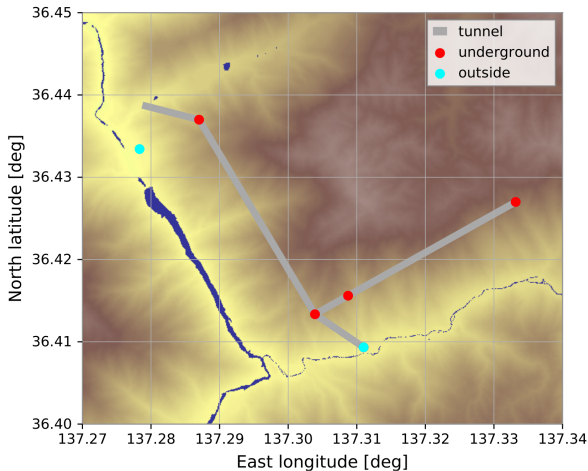
Hunga Tonga  
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TWM

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# The trigger for this presentation

## KAGRA seismic response to Hunga Tonga eruption



Washimi *et al.* (2022, arXiv:2206.14396v3)

Background



BFO

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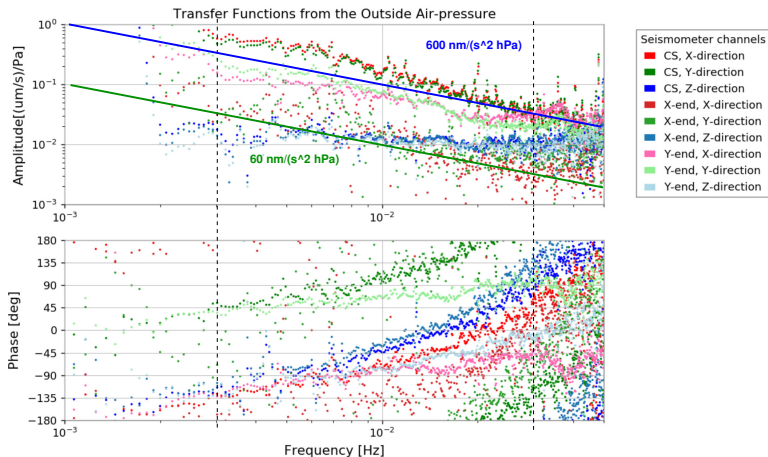
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Summary



# The trigger for this presentation

## KAGRA seismic response to Hunga Tonga eruption



Washimi *et al.* (2022, arXiv:2206.14396v3)

Background



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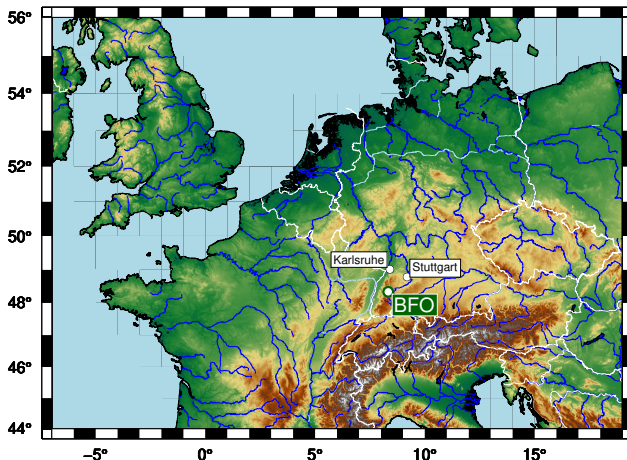
# The trigger for this presentation

- The nature of the frequency dependence (rather continuous) points to a deterministic coupling to external pressure.
- Response of KAGRA seismometers is significantly larger than expected (by at least a factor of 10).
- Due to large inter-sensor distances the phase response cannot easily be attributed to a dominating mechanism.

## Questions raised

- Is the atmospherically induced tilt-background an issue for ET?
- At which level is the tilt-background to be expected?
- What are the dominating mechanisms controlling the level of the tilt background?
- Does the effect of loading at the Earth's surface dominate?
- How strong is the effect of pressure fluctuations in the cavern?

# Black Forest Observatory (BFO)



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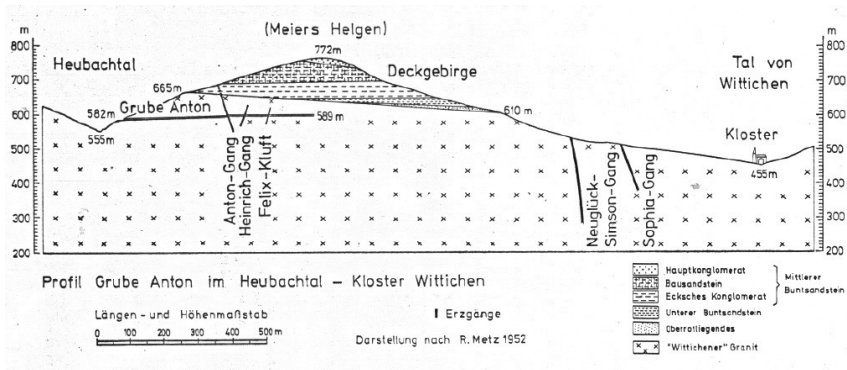
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# Black Forest Observatory (BFO)

## Cross-section of local geology



Background  
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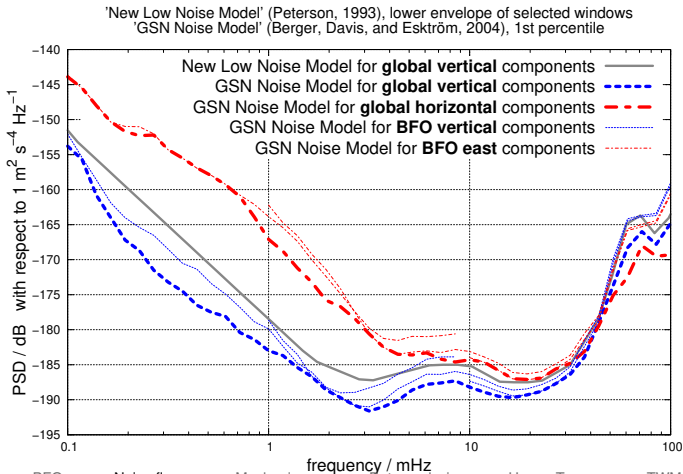
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# Common noise floor

## Low noise models



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# Common noise floor

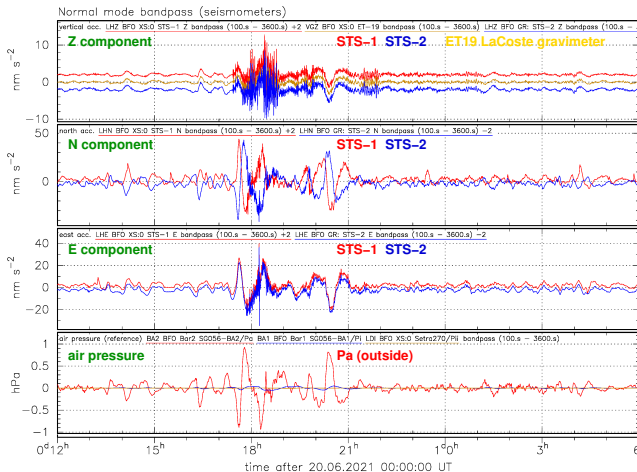
## Horizontal component background signals

Signal background as observed at BFO caused by permanent natural processes (not including earthquakes or volcanic eruptions)

- tidal deformation (12 h, 24 h)  
50 nrad equivalent to  $500 \text{ nm s}^{-2}$
- atmospherically induced tilt (0.3 mHz to 10 mHz)  
5 nrad equivalent to  $50 \text{ nm s}^{-2}$
- atmospherically induced acceleration (3 mHz to 40 mHz)  
 $20 \text{ nm s}^{-2}$
- marine microseisms (50 mHz to 200 mHz)  
 $30 \text{ nm s}^{-2}$  to  $1 \mu\text{m s}^{-2}$   
inertial acceleration dominates  
tilt does not exceed 1 nrad

# Noise caused by the atmosphere

## period band 100 s to 3600 s



20.06.2021 17:05:51 M6.4 South of Kermadec Islands

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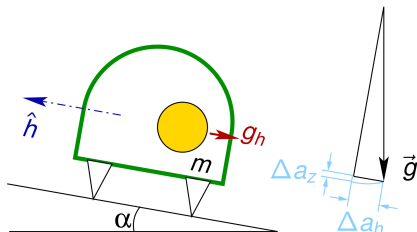
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# Coupling mechanisms: LDT and TWT

## Tilt-coupled gravity

### Response to tilt



### horizontal component

first order effect;  $\Delta a_h \approx |\vec{g}| \alpha$

### vertical component

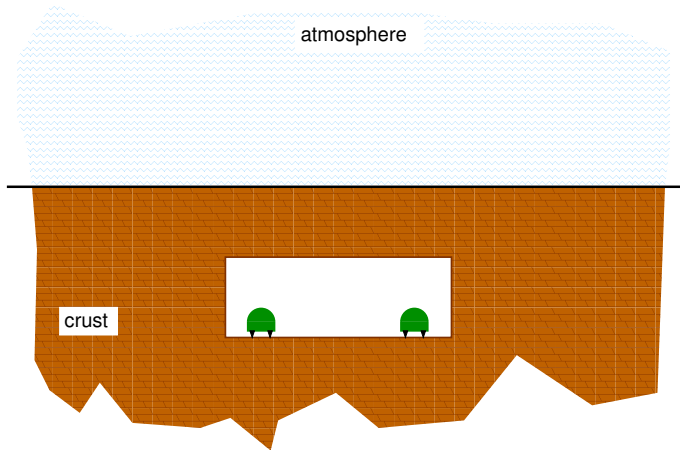
second order effect;  $\Delta a_z \approx |\vec{g}| \alpha^2/2$

$$\alpha \approx 0.00000001 \text{ rad}$$

$$|\vec{g}| \approx 1000000000 \text{ nm s}^{-2}$$

# Coupling mechanisms: LDT and TWT

## 'Local deformation tilt' (LDT): cavity deformation



LDT as defined by Zürn *et al.* (2007, GJI, vol. 171, 780–796)

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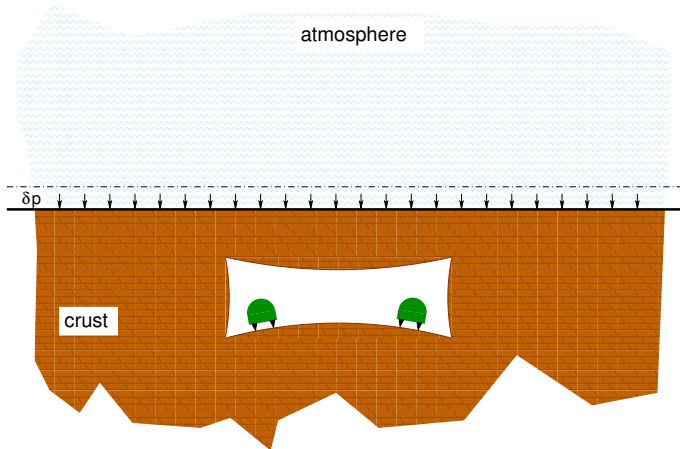
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# Coupling mechanisms: LDT and TWT

## 'Local deformation tilt' (LDT): cavity deformation



LDT as defined by Zürn *et al.* (2007, GJI, vol. 171, 780–796)

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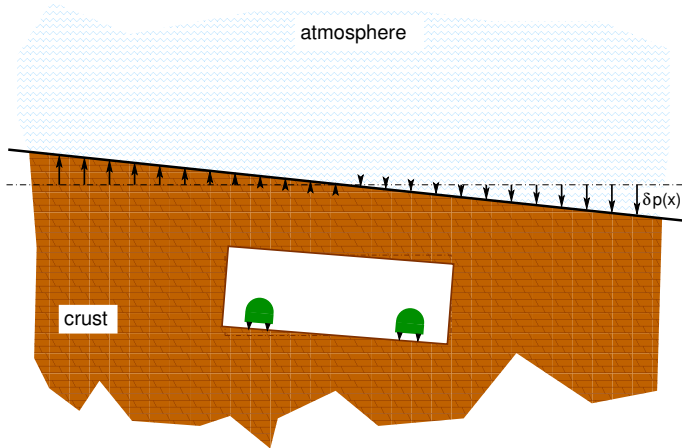
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# Coupling mechanisms: LDT and TWT

## 'Traveling wave tilt' (TWT): horizontal gradient



TWT as defined by Zürn *et al.* (2007, GJI, vol. 171, 780–796)

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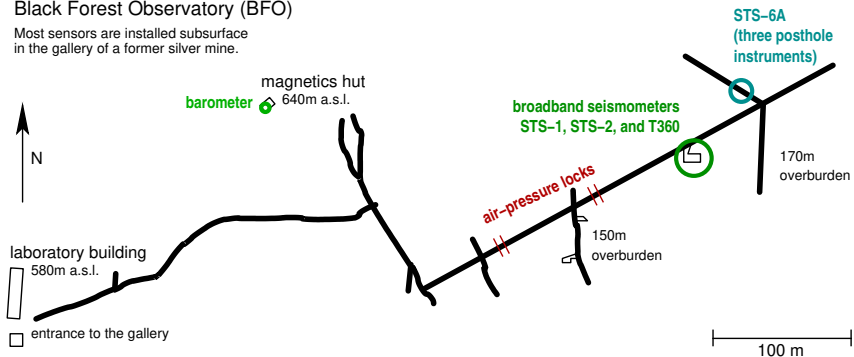
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# Evidence from recorded data

## Instruments selected for this study

### Black Forest Observatory (BFO)

Most sensors are installed subsurface in the gallery of a former silver mine.



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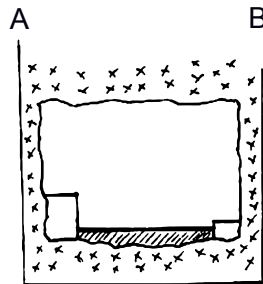
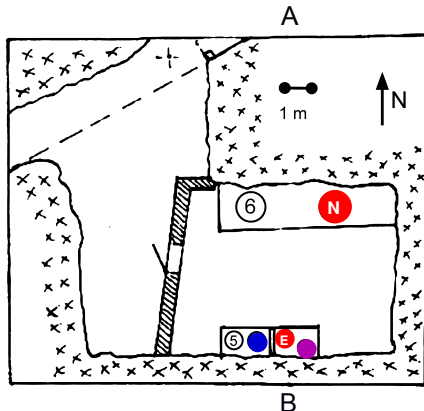
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# Evidence from recorded data

Instruments selected for this study

Seismometer vault at BFO



- STS-1
- STS-2
- Trillium T360

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# Evidence from recorded data

## Joint regression

$$a_h(t) = c_p p(t) + c_{HT} HT [p(t)] + r(t)$$

Minimize  $r(t)$  in a least squares sense by adjusting  $c_p$  and  $c_{HT}$ .

$a_h(t)$ : horizontal acceleration

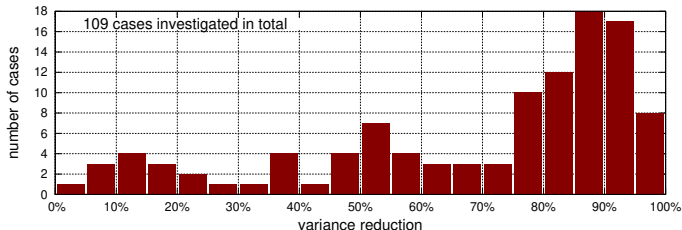
$r(t)$ : residual

$p(t)$ : air pressure

HT [ $p(t)$ ]: Hilbert transform of  $p(t)$

$c_p$ : accounts for LDT

$c_{HT}$ : accounts for TWT



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

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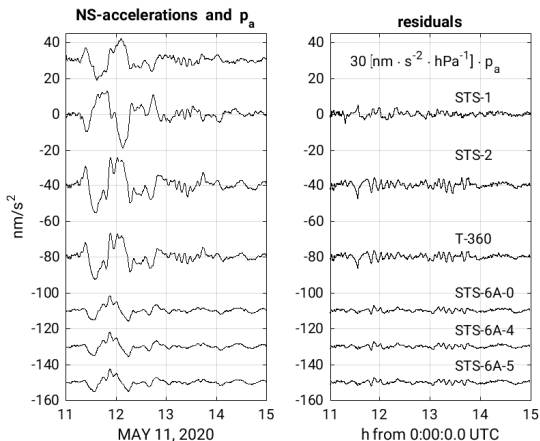
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# Evidence from recorded data

## Example of joint regression



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

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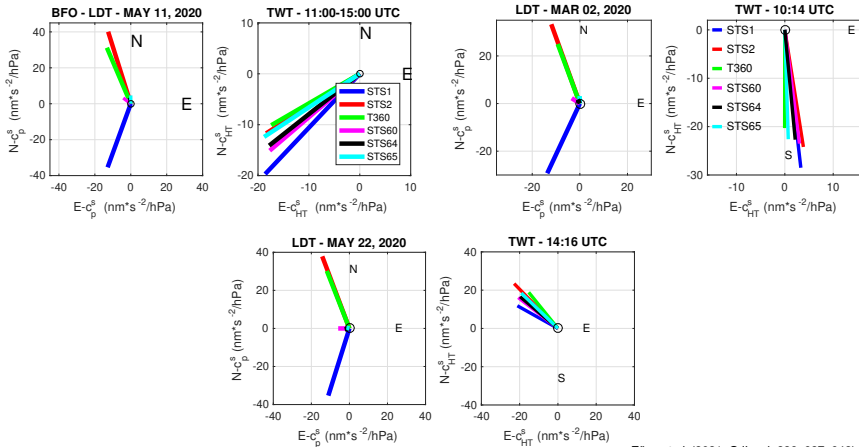
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# Evidence from recorded data

## Regression factors



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

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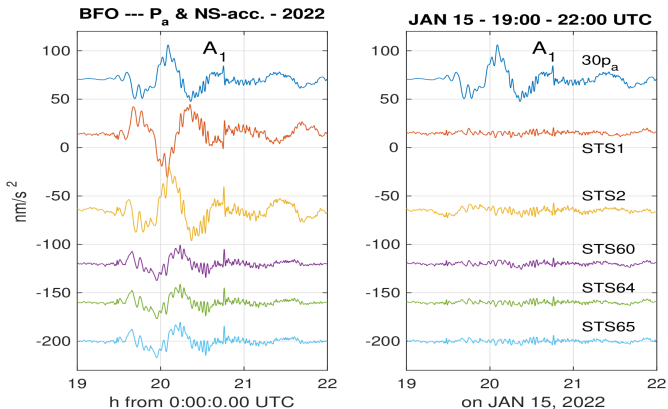
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# Application to Hunga Tonga Lamb- and sound-wave

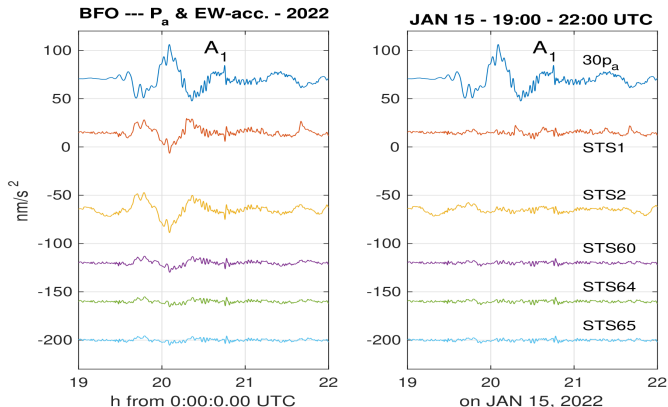
## The Lamb wave signal (1 h – 100 s)



**Fig. 10.** NS - accelerations vs. barometric pressure. Left panel: same records as in Fig. 4 but for a shorter window with A<sub>1</sub>. Right panel: pressure multiplied by 30 c-u and residuals after fitting p<sub>a</sub> and HT(p<sub>a</sub>) to the accelerograms in the left panel for this window. Broadband sensors are identified in the right panel and in Table 1.

# Application to Hunga Tonga Lamb- and sound-wave

The Lamb wave signal (1 h – 100 s)



**Fig. 11.** EW - accelerations vs. barometric pressure. Left panel: same records as in Fig. 5 but for a shorter window with  $A_1$ . Right panel: pressure multiplied by 30 c-u and residuals after fitting  $p_a$  and  $HT(p_a)$  to the accelerograms in the left panel for this window. Broadband sensors are identified in the right panel and in Table I.

Background  
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Mechanisms  
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Data analysis  
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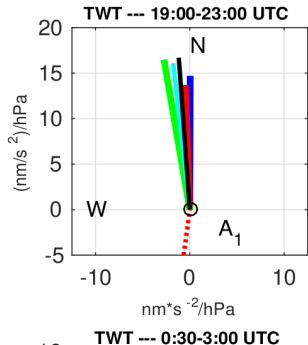
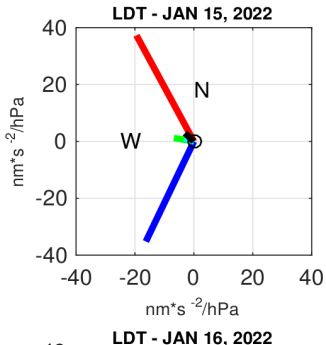
Hunga Tonga  
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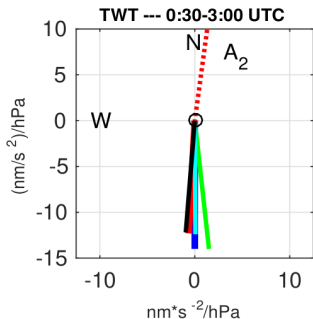
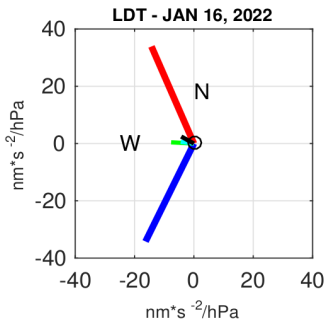
# Application to Hunga Tonga Lamb- and sound-wave

## Regression coefficients A1 wave



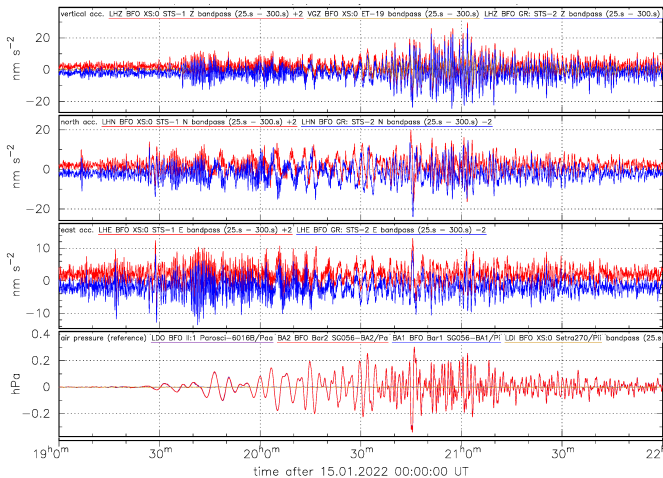
# Application to Hunga Tonga Lamb- and sound-wave

## Regression coefficients A2 wave



# Application to Hunga Tonga Lamb- and sound-wave

## The acoustic wave signal (3 mHz – 40 mHz)



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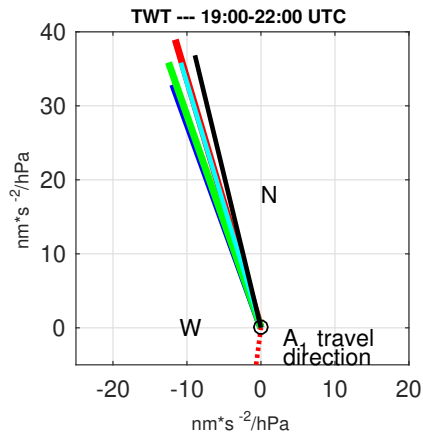
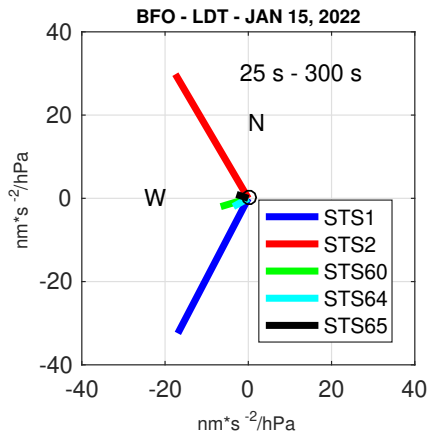
TWM

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# Application to Hunga Tonga Lamb- and sound-wave

The acoustic wave signal (3 mHz – 40 mHz)



# The traveling wave model above 10 mHz

$$\frac{\tilde{a}_h(\omega)}{\tilde{p}(\omega)} = \underbrace{2\pi i \frac{G}{g_0} \frac{1}{1 + \frac{\omega c^2}{c_h g_0}}}_{(1)} + \underbrace{i \frac{g_0}{2\mu} \frac{\lambda + 2\mu}{\lambda + \mu}}_{(2)} + \underbrace{i \frac{1}{2\mu} \frac{\mu}{\lambda + \mu} \omega c_h}_{(3)}$$

(1): Newtonian attraction      (2): tilt-coupled gravity      (3): inertial acceleration

$\tilde{a}_h(\omega)$ : Fourier transform of horizontal acceleration

$\tilde{p}(\omega)$ : Fourier transform of air pressure

$G$ : gravitational constant ( $= 6.6743 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$ )

$g_0$ : local gravity ( $\approx 9.81 \text{ m s}^{-2}$ )

$c$ : speed of sound ( $\approx 330 \text{ m s}^{-1}$ )

$\lambda, \mu$ : Lamé's parameters of homogeneous halfspace

$c_h$ : horizontal phase velocity ( $= \omega/k_h$ )

Zürn *et al.* (2007, GJI, vol. 171, 780–796)

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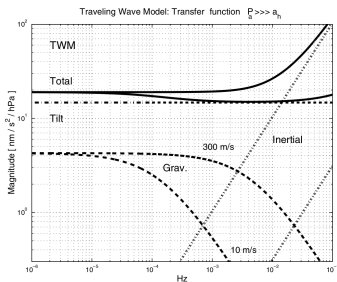
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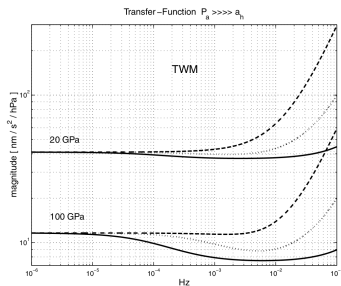
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# The traveling wave model above 10 mHz



**Figure 8.** Magnitudes of the transfer-functions between local air pressure and horizontal accelerations for the TWM. Total effect and the three contributions are shown for two values of  $c_h$ : 10 and 300  $\text{m s}^{-1}$ .  $\lambda = \mu = 50$  GPa. The tilt effect is identical for the two cases.



**Figure 7.** Magnitudes of the transfer-functions between local barometric pressure and horizontal acceleration for the TWM. Solid, dotted, and dashed lines were calculated for  $c_h = 10, 80$  and  $330 \text{ m s}^{-1}$ , respectively. Two values of  $\lambda = \mu$  were used as indicated.

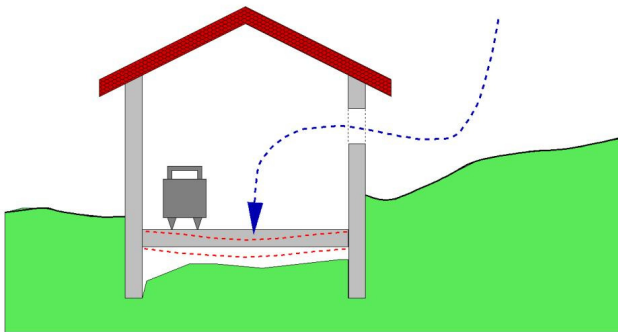
Zürn *et al.* (2007, GJI, vol. 171, 780–796)

# Conclusions

- LDT-effect and TWT-effect can be separated in data
- Joint regression with simple models provides variance reduction of up to 97 per cent
- Admittance reported by KAGRA is surprisingly large, even if inertial acceleration at frequencies larger than 10 mHz are taken into account
- LDT-type noise can be reduced by method of installation (e. g. posthole)
- With deterministic models local deformation (LDT-type noise) could be identified

# Conclusions

## Beware of membranes



Courtesy of Erhard Wielandt (software-for-seismometry.de)

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# Thank you for your attention

We are grateful to

- Ruedi Widmer-Schmidrig, Peter Duffner (BFO)
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- Sarvesh Upadhyaya (Nanometrics), Steffen Uhlmann (IGM)
- Karl-Heinz Jäckel (GFZ)
- Karin Hafner (IRIS)

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References

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Zürn, W., J. Exß, S. H., C. Kroner, T. Jahr, and M. Westerhaus (2007). “On reduction of long-period horizontal seismic noise using local barometric pressure”. In: *Geophys. J. Int.* 171, pp. 780–796. DOI: 10.1111/j.1365-246X.2007.03553.x.



Zürn, W., T. Forbriger, R. Widmer-Schmidrig, and P. Duffner (2022). “The Hunga Tonga  $A_1$ -wave at BFO”. In: *Geophys. J. Int.* in preparation.

References

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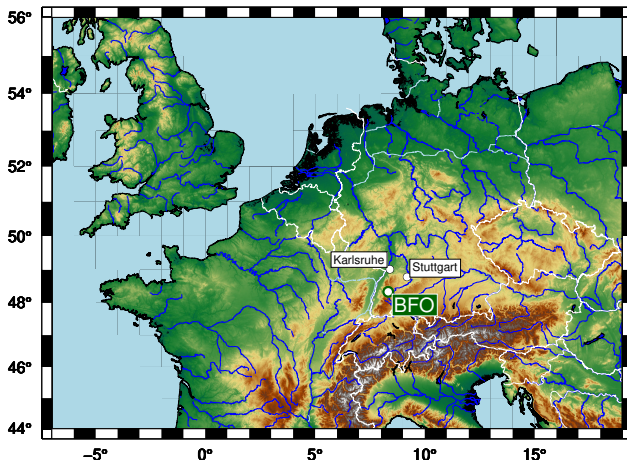


# References III



Zürn, W., T. Forbriger, R. Widmer-Schmidrig, P. Duffner, and A. T. Ringler (2021). “Modelling tilt noise caused by atmospheric processes at long periods for several horizontal seismometers at BFO — a reprise”. In: *Geophys. J. Int.* 228.2, pp. 927–943. DOI: 10.1093/gji/ggab336.

# Black Forest Observatory (BFO)



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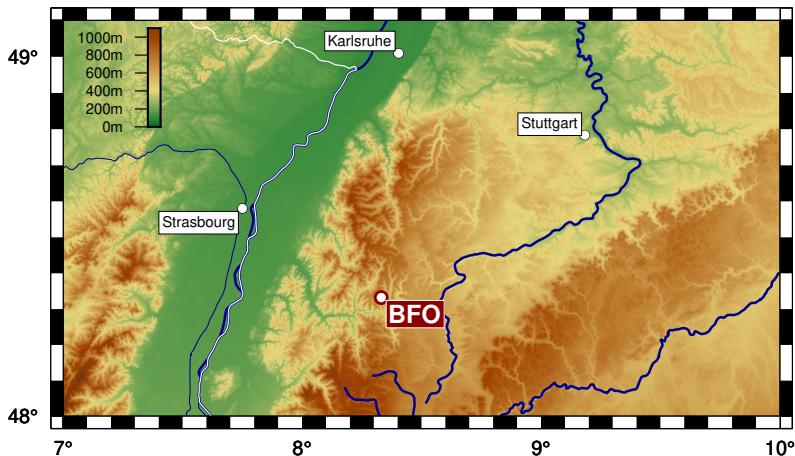
Data analysis



Hunga Tonga



# Black Forest Observatory (BFO)



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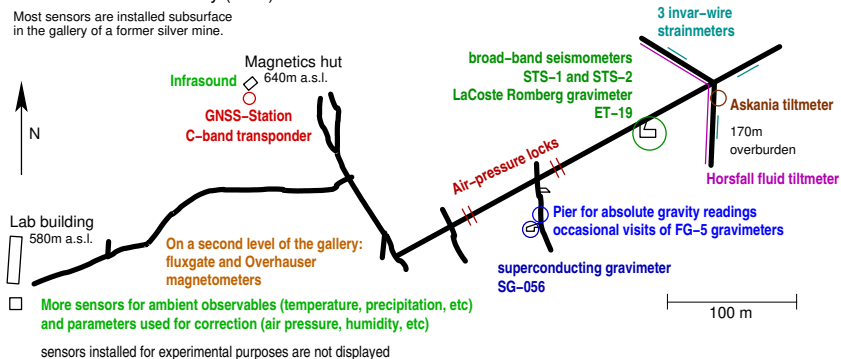
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# Black Forest Observatory (BFO)

## Black Forest Observatory (BFO)

Most sensors are installed subsurface in the gallery of a former silver mine.



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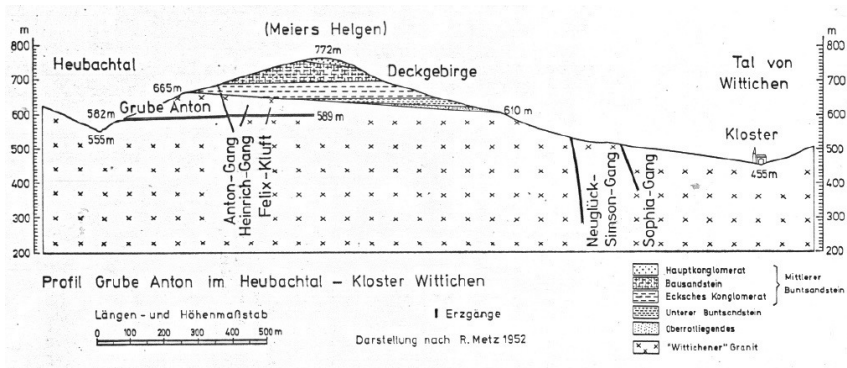
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# Black Forest Observatory (BFO)

## Cross-section of local geology



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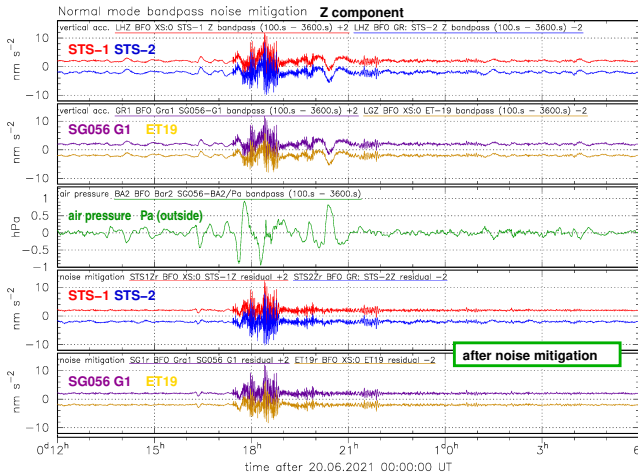
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# Mitigation of atmospheric noise

## period band 100 s to 3600 s



20.06.2021 17:05:51 M6.4 South of Kermadec Islands

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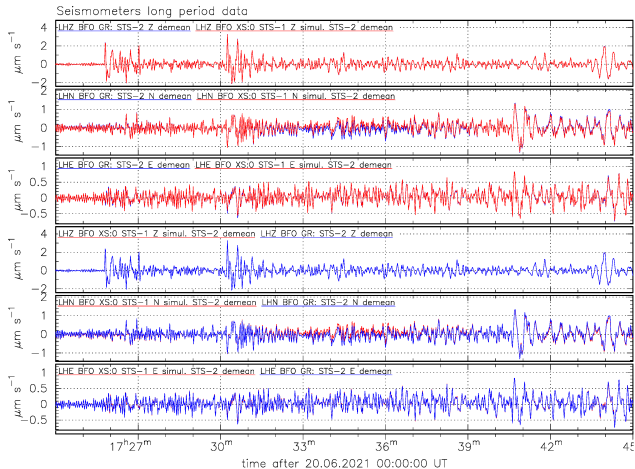
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# Noise caused by the atmosphere

## Consistency of polarity



20.06.2021 17:05:51 M6.4 South of Kermadec Islands

References

BFO

Mechanisms

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Data analysis

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Hunga Tonga

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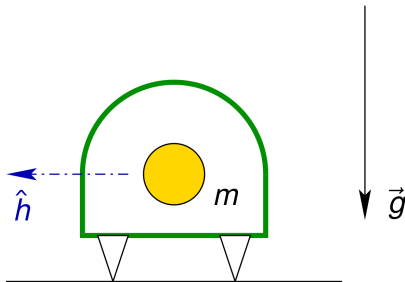




# Coupling mechanisms

## Tilt-coupled gravity

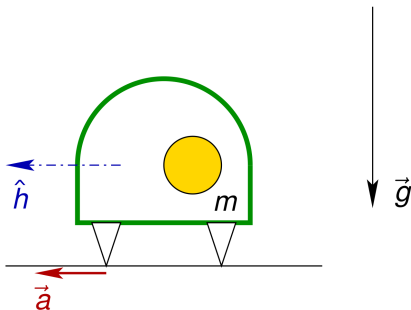
### Definition of horizontal component



# Coupling mechanisms

## Tilt-coupled gravity

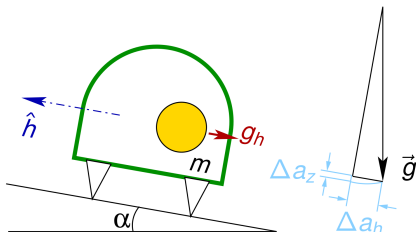
### Response to inertial acceleration



# Coupling mechanisms

## Tilt-coupled gravity

### Response to tilt



### horizontal component

first order effect;  $\Delta a_h \approx |\vec{g}| \alpha$

### vertical component

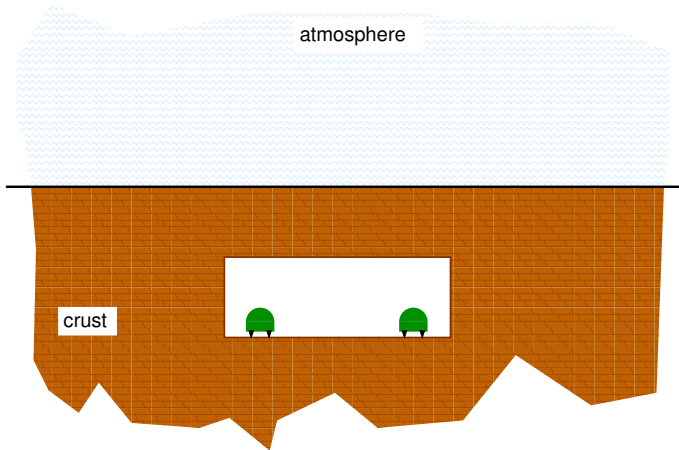
second order effect;  $\Delta a_z \approx |\vec{g}| \alpha^2/2$

$$\alpha \approx 0.00000001 \text{ rad}$$

$$|\vec{g}| \approx 1000000000 \text{ nm s}^{-2}$$

# Coupling mechanisms

## 'Local deformation tilt' (LDT): cavity deformation



LDT as defined by Zürn *et al.* (2007, GJI, vol. 171, 780–796)

References

BFO

Mechanisms

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Data analysis

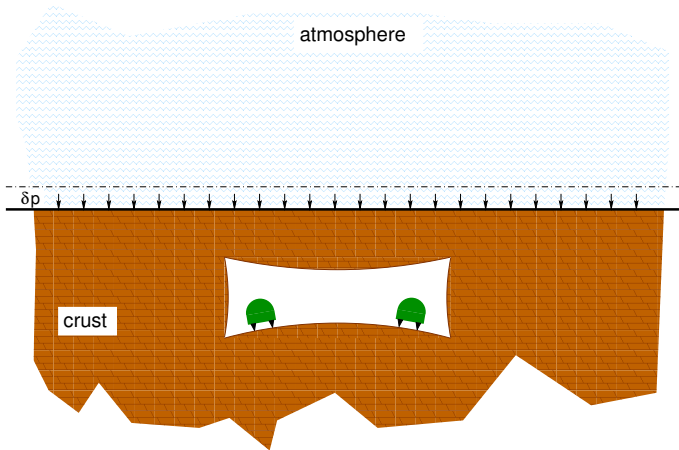
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Hunga Tonga

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# Coupling mechanisms

## 'Local deformation tilt' (LDT): cavity deformation



LDT as defined by Zürn *et al.* (2007, GJI, vol. 171, 780–796)

References

BFO

Mechanisms

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Data analysis

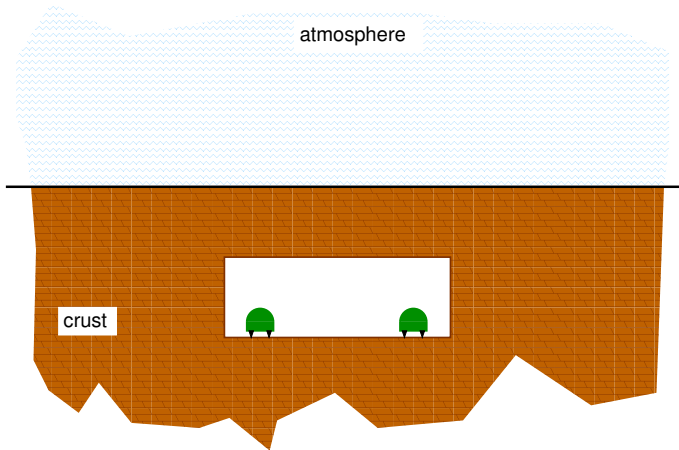
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Hunga Tonga

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# Coupling mechanisms

## 'Traveling wave tilt' (TWT): horizontal gradient



TWT as defined by Zürn *et al.* (2007, GJI, vol. 171, 780–796)

References

BFO

Mechanisms

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Data analysis

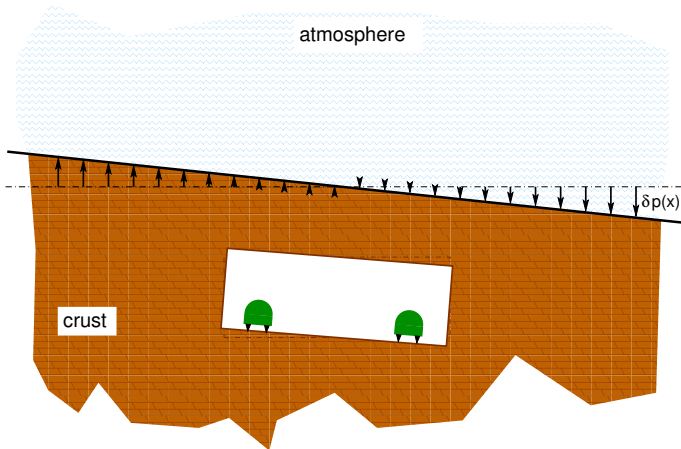
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# Coupling mechanisms

## 'Traveling wave tilt' (TWT): horizontal gradient



TWT as defined by Zürn *et al.* (2007, GJI, vol. 171, 780–796)

References

BFO

Mechanisms

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Data analysis

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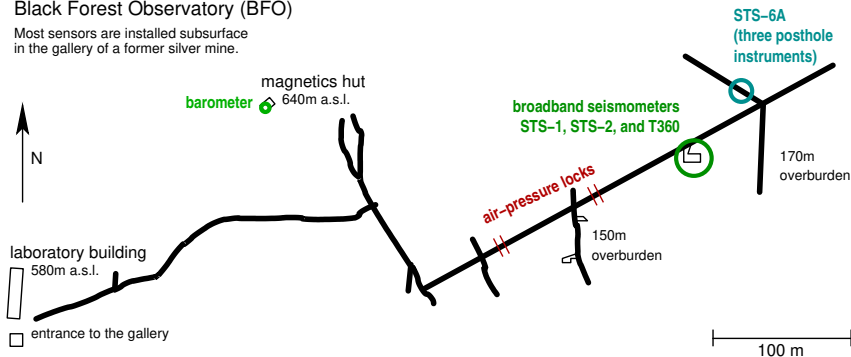


# Evidence from recorded data

## Instruments selected for this study

### Black Forest Observatory (BFO)

Most sensors are installed subsurface in the gallery of a former silver mine.



References

BFO

Mechanisms



Data analysis



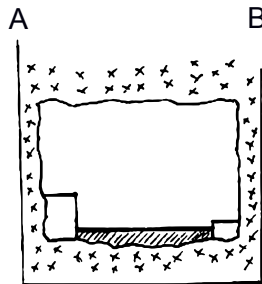
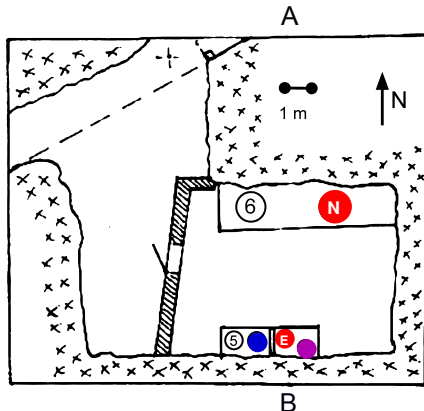
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# Evidence from recorded data

Instruments selected for this study

Seismometer vault at BFO



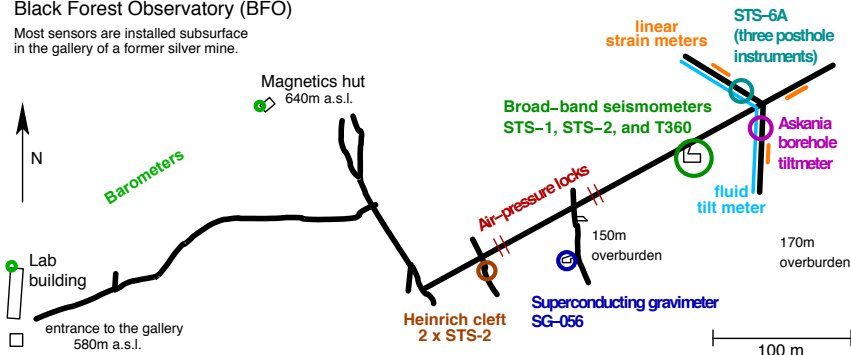
- STS-1
- STS-2
- Trillium T360

# Evidence from recorded data

## Instruments selected for this study

### Black Forest Observatory (BFO)

Most sensors are installed subsurface in the gallery of a former silver mine.



References

BFO

Mechanisms



Data analysis



Hunga Tonga



# Evidence from recorded data

## Joint regression

$$a_h(t) = c_p p(t) + c_{HT} HT [p(t)] + r(t)$$

Minimize  $r(t)$  in a least squares sense by adjusting  $c_p$  and  $c_{HT}$ .

$a_h(t)$ : horizontal acceleration

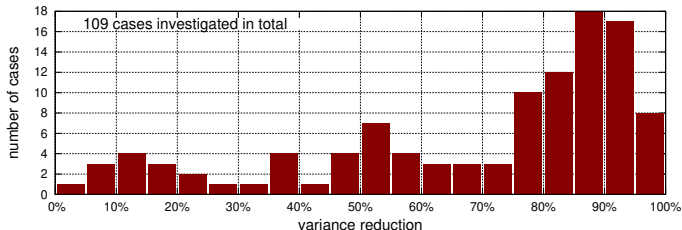
$r(t)$ : residual

$p(t)$ : air pressure

HT [ $p(t)$ ]: Hilbert transform of  $p(t)$

$c_p$ : accounts for LDT

$c_{HT}$ : accounts for TWT



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

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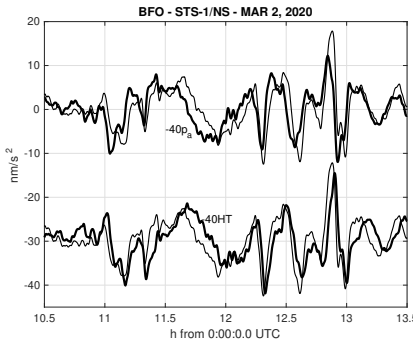
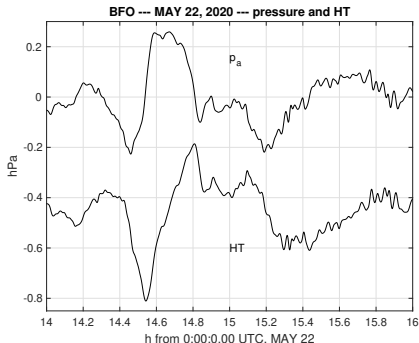
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# Evidence from recorded data

## Joint regression

$$a_h(t) = c_p p(t) + c_{HT} HT [p(t)] + r(t)$$

Minimize  $r(t)$  in a least squares sense by adjusting  $c_p$  and  $c_{HT}$ .



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

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Hunga Tonga

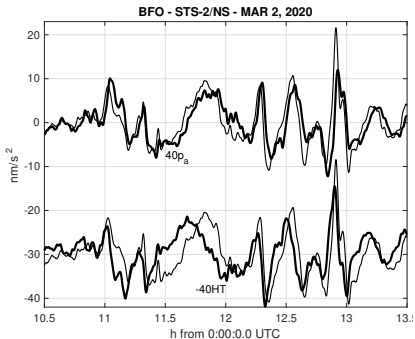
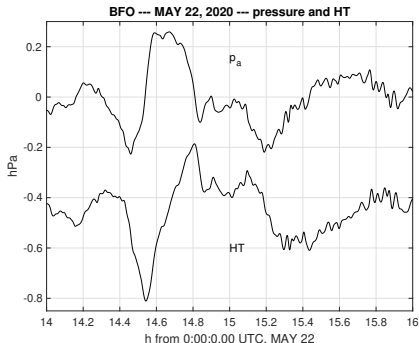
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# Evidence from recorded data

## Joint regression

$$a_h(t) = c_p p(t) + c_{HT} HT [p(t)] + r(t)$$

Minimize  $r(t)$  in a least squares sense by adjusting  $c_p$  and  $c_{HT}$ .



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

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# Evidence from recorded data

## Variance reduction

Date	Windows	Components	STS-1	STS-2	T360	STS-6A-0	STS-6A-4	STS-6A-5	Remarks
04/03/2017	1	N,E	90	94	-	-	-	-	
01/03/2018	3	N	79	77	-	-	-	-	
11/03/2018	4	N	97	97	-	-	-	-	
15/03/2018	2	N	96	93	-	-	-	-	
01/06/2018	1	N,E	84	83	-	-	-	-	ASK 52%
20/10/2019	1	N	88	91	-	-	-	-	
15/11/2019	1	N,E	87	86	84	74	-	-	
16/11/2019	3	N,E	89	89	85	78	-	-	
17/11/2019	1	N,E	89	92	90	78	-	-	STS1/N #2 79%
25/02/2020	3	N	54	74	72	17	19	27	
26/02/2020	1	N	38	47	44	9	10	15	
27/02/2020	1	N	9	22	23	10	10	12	
29/02/2020	2	N,E	57	53	53	63	65	67	
01/03/2020	1	N,E	61	55	52	57	54	49	
02/03/2020	1	N,E	89	88	84	85	85	84	
07/03/2020	1	N	39	55	35	2	31	35	STS1/N #2 9%
11/05/2020	5	N,E	93	95	94	95	95	96	S112 93%, S113 94%
22/05/2020	4	N,E	93	94	95	93	93	93	
23/05/2020	5	N,E	84	87	84	83	83	85	
03/08/2020	1	N,E	93	87	-	86	87	88	
25/09/2020	1	N,E	93	89	-	76	77	77	
05/11/2020	1	N,E	63	66	-	46	50	46	
15/11/2020	1	N,E	78	78	-	83	83	84	

Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

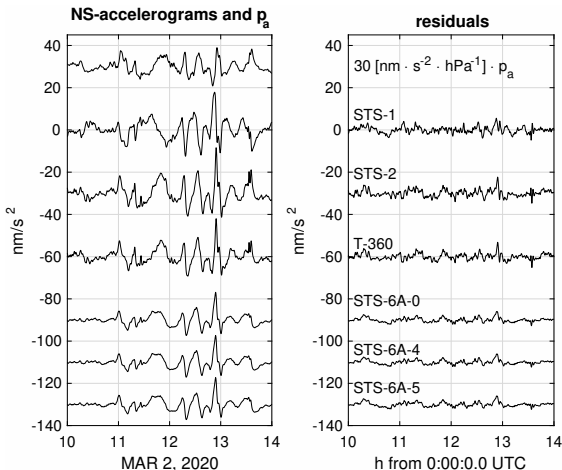
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# Evidence from recorded data

## Example of joint regression



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

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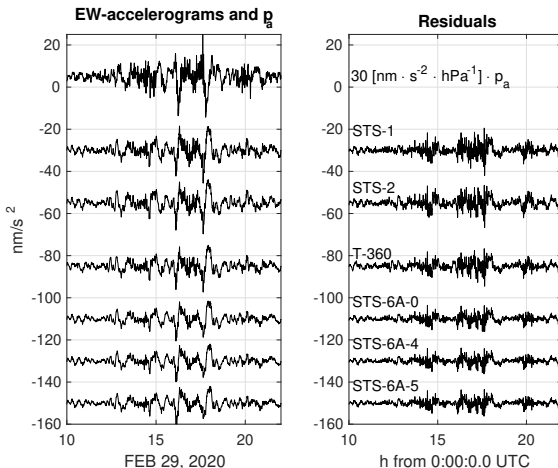
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# Evidence from recorded data

## Example of joint regression



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

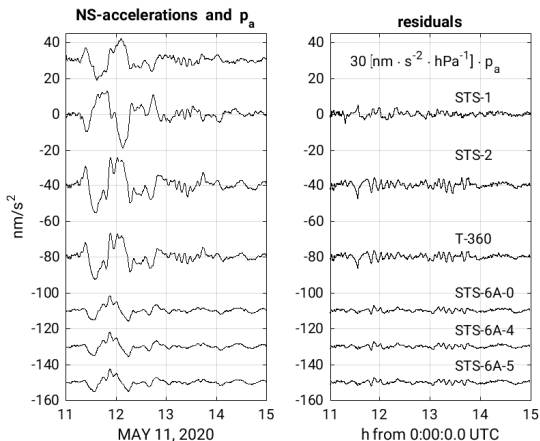
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# Evidence from recorded data

## Example of joint regression



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

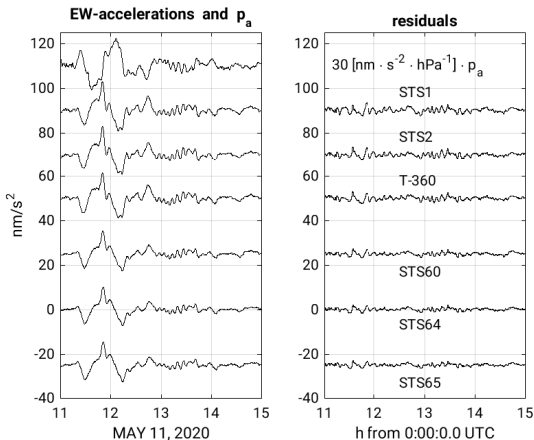
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# Evidence from recorded data

## Example of joint regression



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

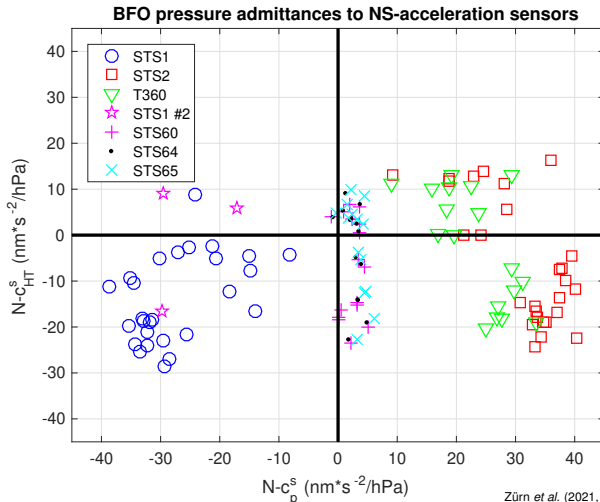
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# Evidence from recorded data

## Regression factors



References

BFO

Mechanisms

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Data analysis

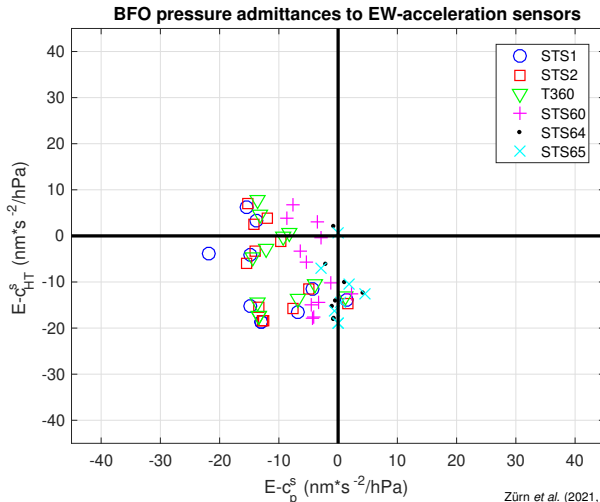
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# Evidence from recorded data

## Regression factors



References

BFO

Mechanisms

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Data analysis

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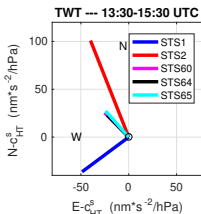
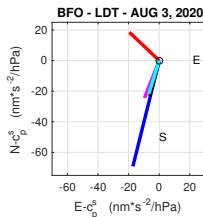
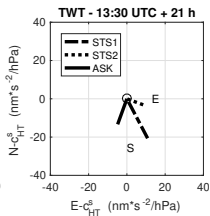
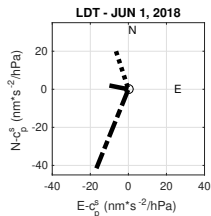
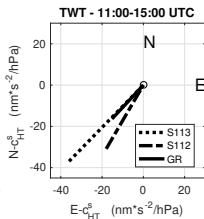
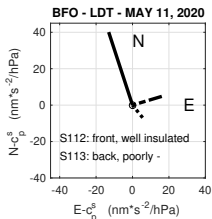
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# Evidence from recorded data

## Regression factors



Zürn *et al.* (2021, GJI, vol. 228, 927–943)

References

BFO

Mechanisms

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Data analysis

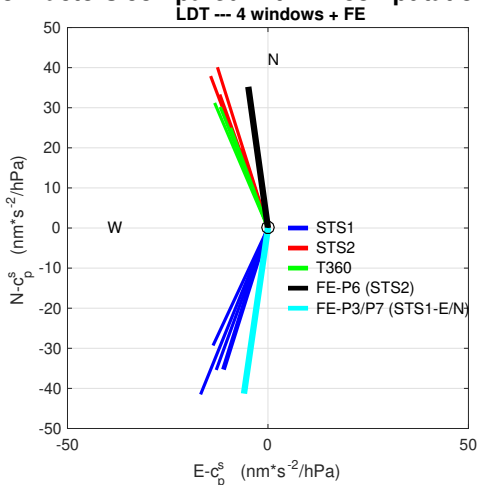
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# Evidence from recorded data

## Regression factors compared with FE computation



Zürn *et al.* (2021, GJI, vol. 228, 927–943)  
Gebauer *et al.* (2010, GJI, vol. 181, 1593–1612)

References

BFO

Mechanisms

Data analysis

Hunga Tonga

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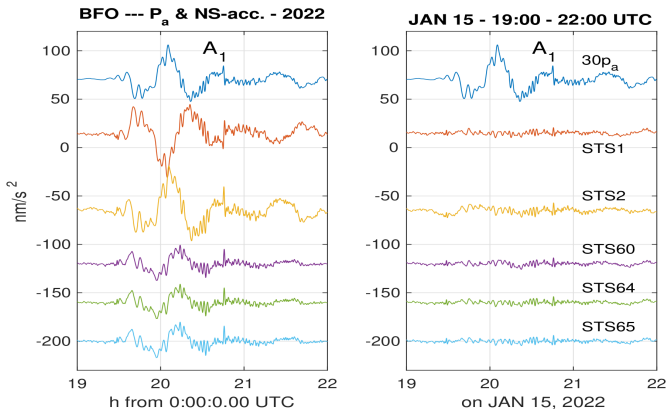
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# Hunga Tonga Lamb- and sound-wave

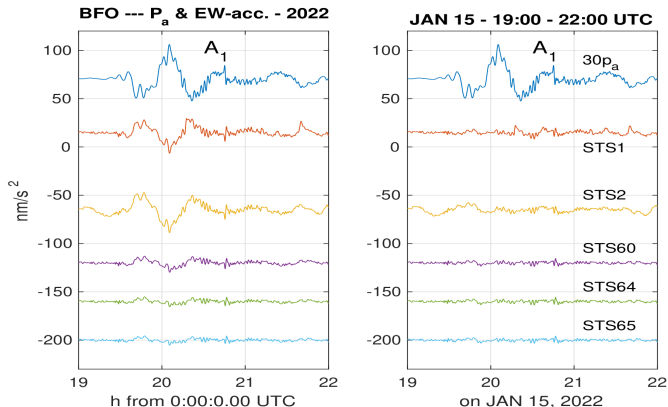
## The Lamb wave signal (1 h – 100 s)



**Fig. 10.** NS - accelerations vs. barometric pressure. Left panel: same records as in Fig. 4 but for a shorter window with A<sub>1</sub>. Right panel: pressure multiplied by 30 c-u and residuals after fitting p<sub>a</sub> and HT(p<sub>a</sub>) to the accelerograms in the left panel for this window. Broadband sensors are identified in the right panel and in Table 1.

# Hunga Tonga Lamb- and sound-wave

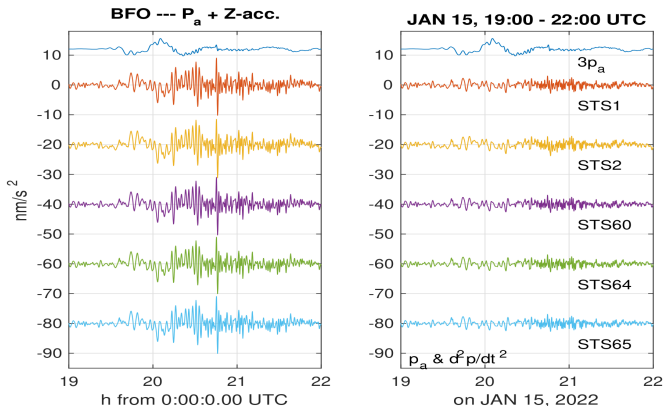
## The Lamb wave signal (1 h – 100 s)



**Fig. 11.** EW - accelerations vs. barometric pressure. Left panel: same records as in Fig. 5 but for a shorter window with  $A_1$ . Right panel: pressure multiplied by 30 c-u and residuals after fitting  $p_a$  and  $HT(p_a)$  to the accelerograms in the left panel for this window. Broadband sensors are identified in the right panel and in Table I.

# Hunga Tonga Lamb- and sound-wave

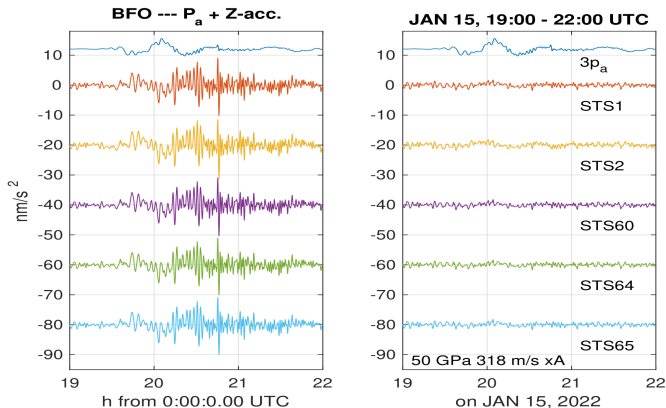
## The Lamb wave signal (1 h – 100 s)



**Fig. 13.** Vertical accelerations vs. barometric pressure. Left panel: same records as in Fig. 6 but for a shorter window with  $A_1$ . Right panel: pressure multiplied by 3.0 c-u and residuals after fitting  $p_a$  and  $d^2 p_a / dt^2$  to the accelerograms in the left panel for this window. Broadband sensors are identified in the right panel and in Table 1.

# Hunga Tonga Lamb- and sound-wave

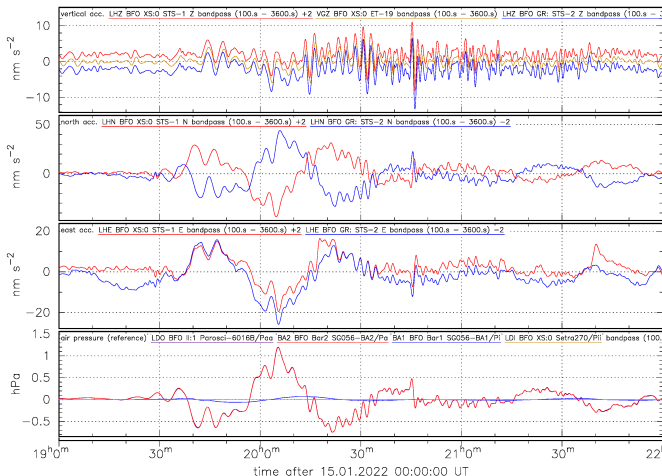
## The Lamb wave signal (1 h – 100 s)



**Fig. 15.** Same as Fig. 13: Vertical accelerations vs. barometric pressure. However, here the residuals in the right panel were obtained by subtracting a scaled version of the "pressure accelerogram" computed using the TWM with parameters  $\mu = 50$  GPa and  $c_b = 318$  m/s. The best scale factor in a least squares sense was 1.505. Broadband sensors are identified in the right panel and in Table 1.

# Hunga Tonga Lamb- and sound-wave

## The Lamb wave signal (1 h – 100 s)



References

BFO

Mechanisms

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Data analysis

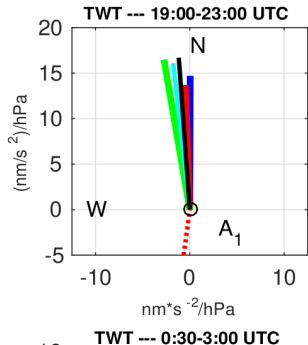
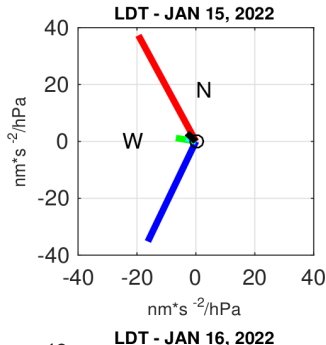
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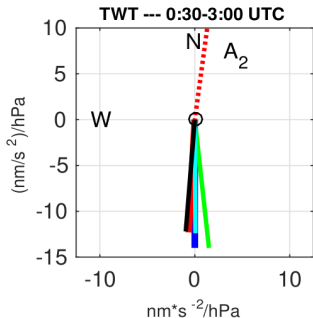
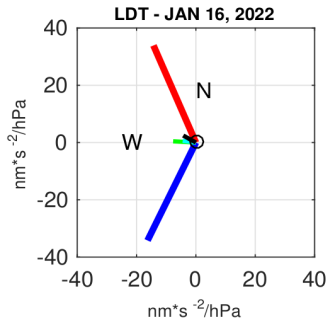
# Hunga Tonga Lamb- and sound-wave

## Regression coefficients A1 wave



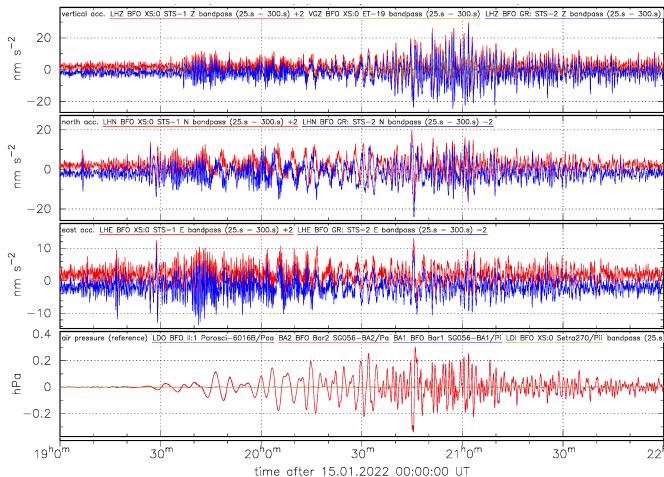
# Hunga Tonga Lamb- and sound-wave

## Regression coefficients A2 wave



# Hunga Tonga Lamb- and sound-wave

## The acoustic wave signal (3 mHz – 40 mHz)



References

BFO

Mechanisms

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Data analysis

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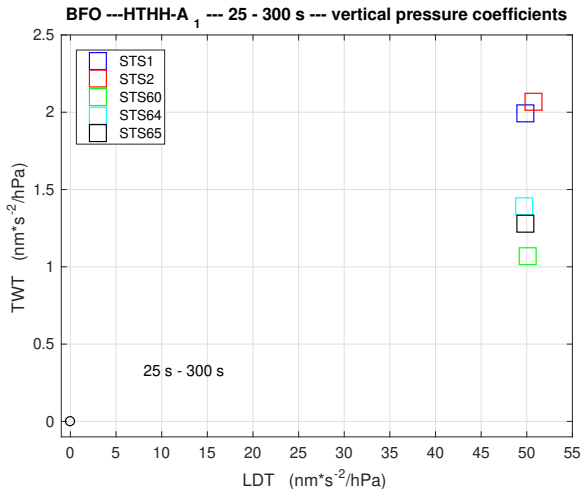
Hunga Tonga

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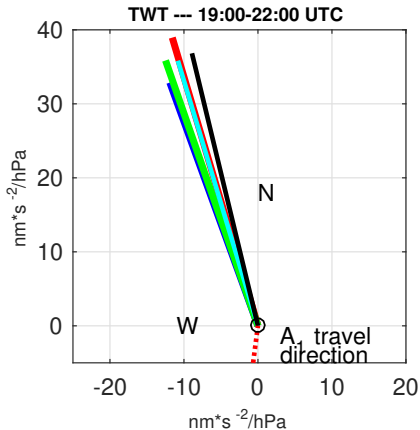
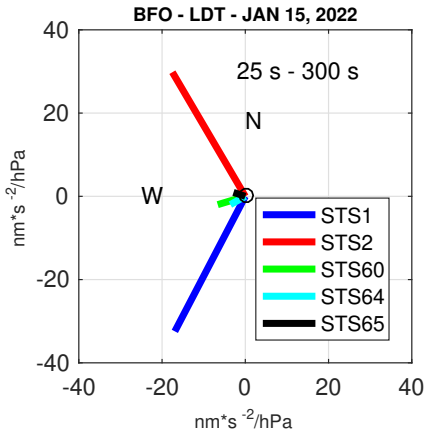
# Hunga Tonga Lamb- and sound-wave

## The acoustic wave signal (3 mHz – 40 mHz)



# Hunga Tonga Lamb- and sound-wave

The acoustic wave signal (3 mHz – 40 mHz)



References

BFO

Mechanisms

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Data analysis

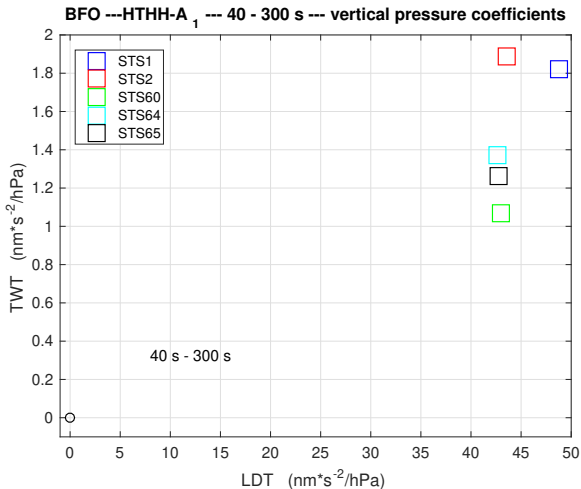
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Hunga Tonga

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# Hunga Tonga Lamb- and sound-wave

3 mHz – 25 mHz



References

BFO

Mechanisms

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Data analysis

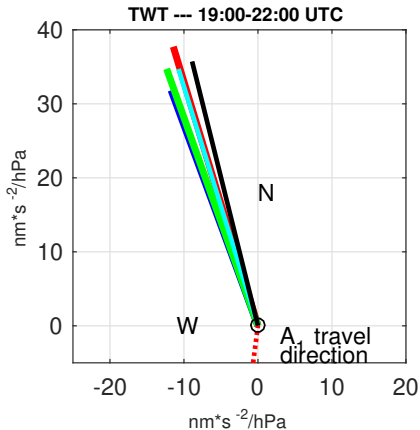
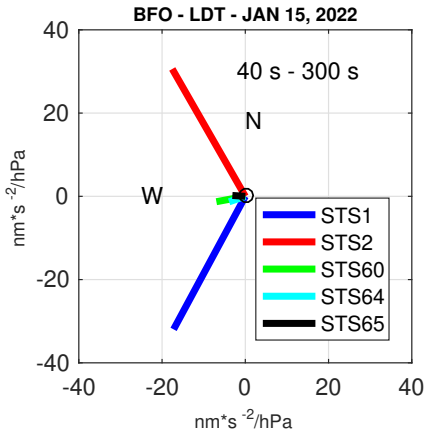
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# Hunga Tonga Lamb- and sound-wave

3 mHz – 25 mHz



References

BFO

Mechanisms

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Data analysis

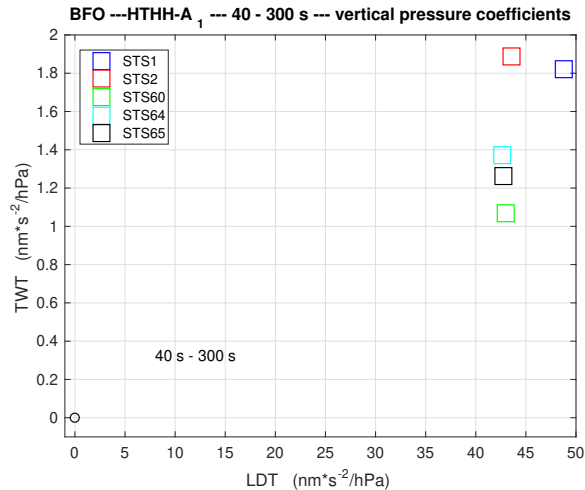
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Hunga Tonga

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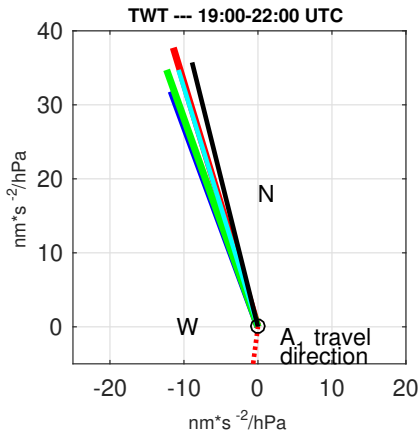
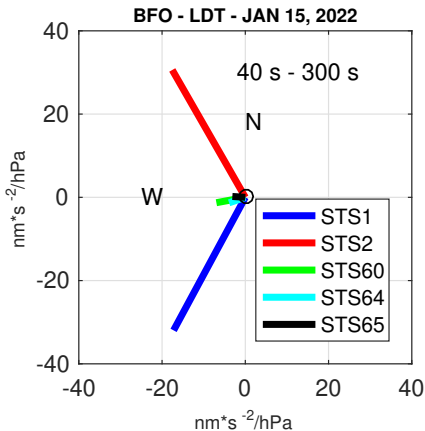
# Hunga Tonga Lamb- and sound-wave

3 mHz – 17 mHz



# Hunga Tonga Lamb- and sound-wave

3 mHz – 17 mHz



References

BFO

Mechanisms

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Data analysis

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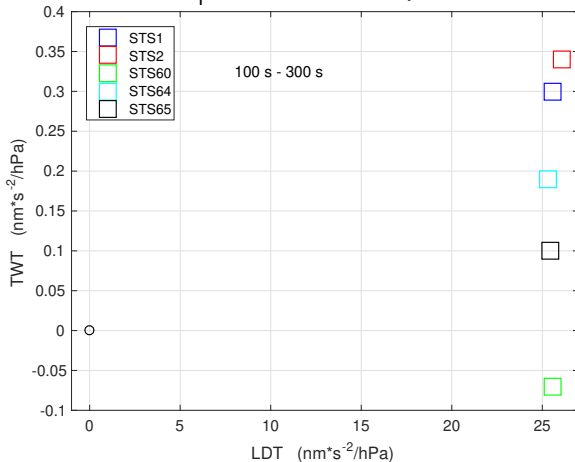
Hunga Tonga

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# Hunga Tonga Lamb- and sound-wave

3 mHz – 10 mHz

BFO ---HTHH-A<sub>1</sub> --- 100 - 300 s --- vertical pressure coefficients



References

BFO

Mechanisms

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Data analysis

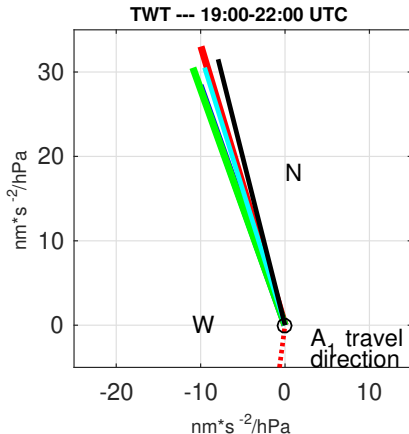
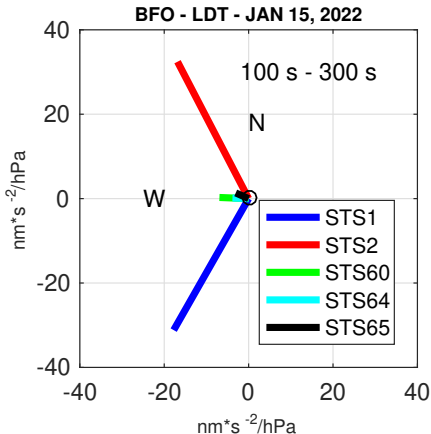
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# Hunga Tonga Lamb- and sound-wave

3 mHz – 10 mHz



References

BFO

Mechanisms

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Data analysis

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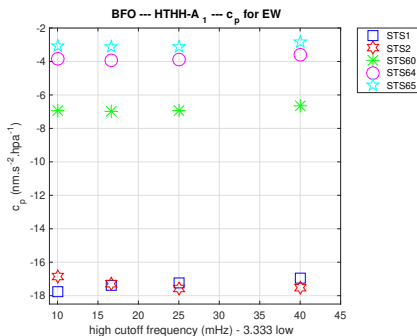
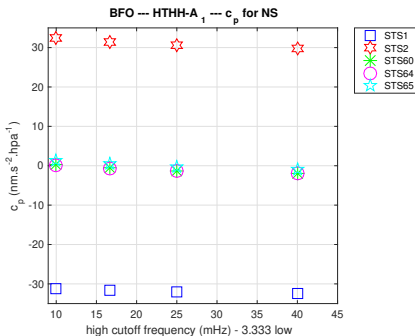
Hunga Tonga

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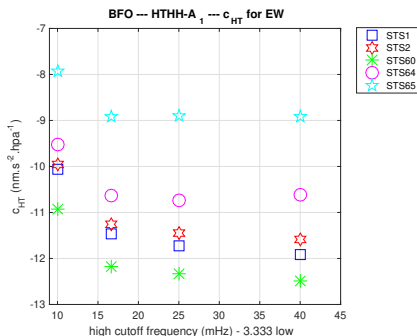
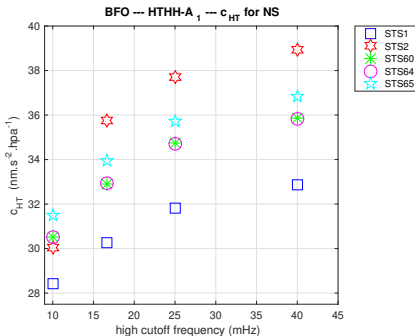
# Hunga Tonga Lamb- and sound-wave

## Frequency dependence of LDT coefficients



# Hunga Tonga Lamb- and sound-wave

## Frequency dependence of TWT coefficients



References

BFO

Mechanisms

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Data analysis

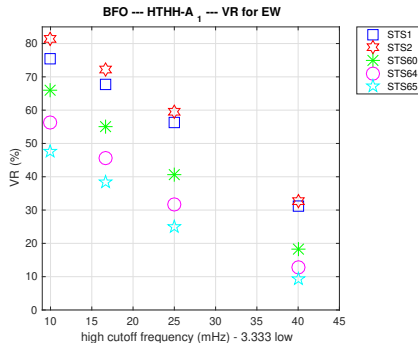
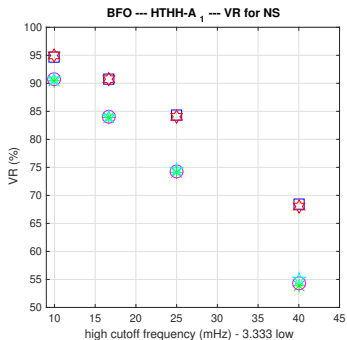
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# Hunga Tonga Lamb- and sound-wave

## Frequency dependence of variance reduction



References

BFO

Mechanisms

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Data analysis

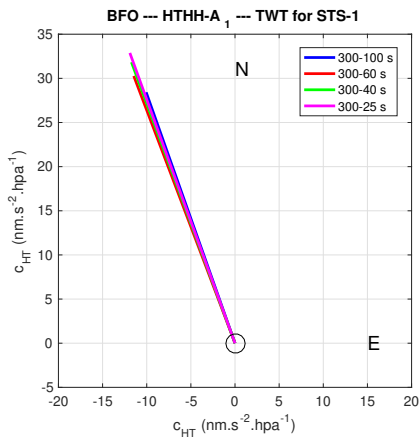
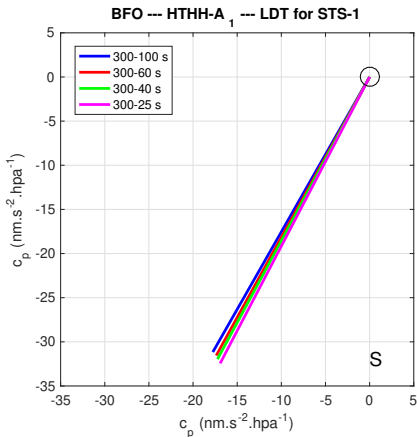
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# Hunga Tonga Lamb- and sound-wave

## Frequency dependence for STS-1



References

BFO

Mechanisms

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Data analysis

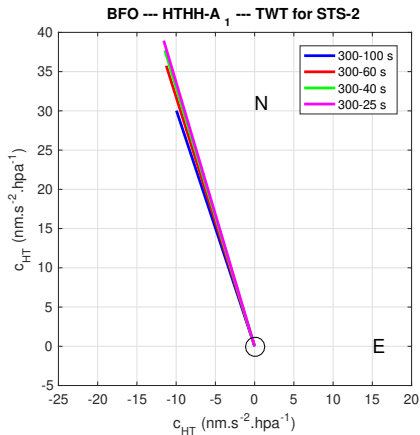
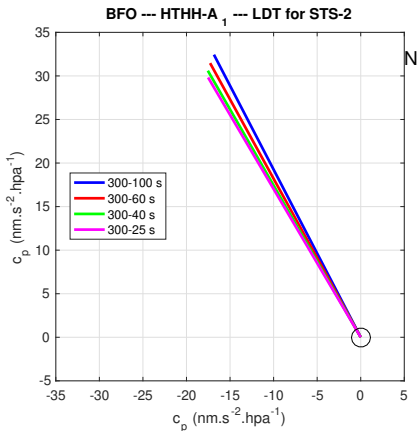
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# Hunga Tonga Lamb- and sound-wave

## Frequency dependence for STS-2



References

BFO

Mechanisms

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Data analysis

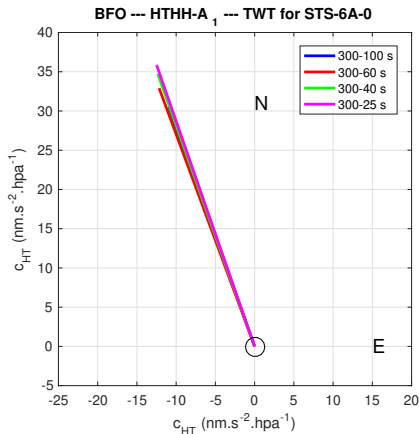
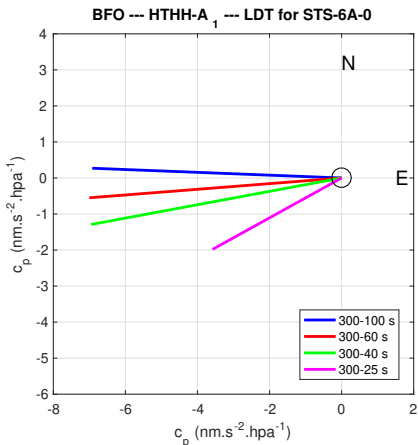
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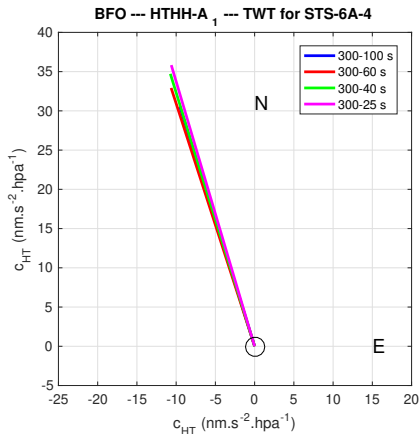
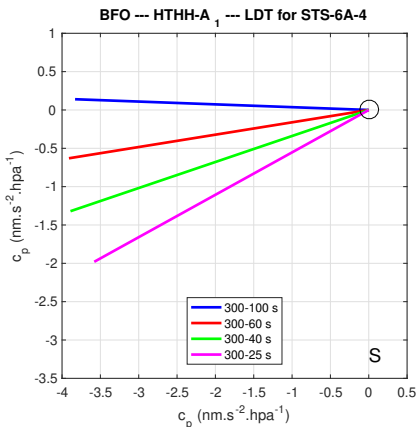
# Hunga Tonga Lamb- and sound-wave

## Frequency dependence for STS-6A-0 (SN 150804)



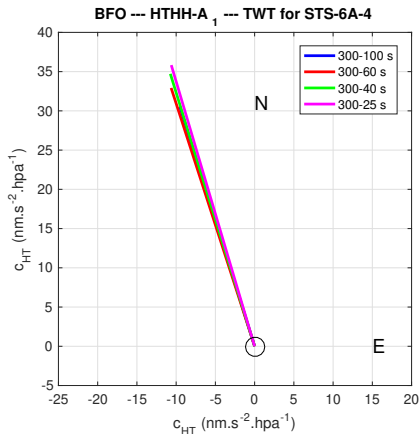
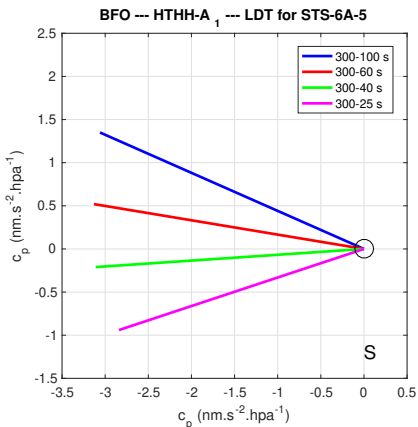
# Hunga Tonga Lamb- and sound-wave

## Frequency dependence for STS-6A-4 (SN 176241)



# Hunga Tonga Lamb- and sound-wave

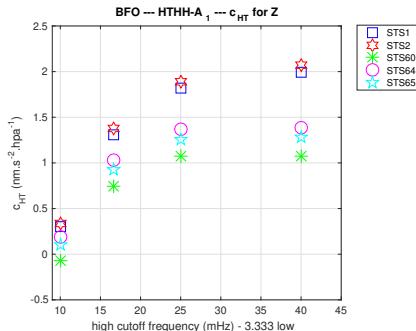
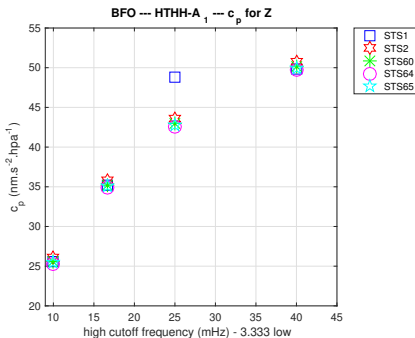
## Frequency dependence for STS-6A-5 (SN 176254)





# Hunga Tonga Lamb- and sound-wave

## Frequency dependence of Z coefficients



# Hunga Tonga Lamb- and sound-wave

## Frequency dependence of Z variance reduction

